Cambridge Assessment International Education

# Cambridge International AS & A Level

**FURTHER MATHEMATICS** **9231/03**

Paper 3 Further Mechanics **For examination from 2020**

MARK SCHEME

Maximum Mark: 50

**Specimen**

This document has **10** pages. Blank pages are indicated.



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## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

| |
|---|
| GENERIC MARKING PRINCIPLE 1:<br><br>Marks must be awarded in line with:<br><br>• the specific content of the mark scheme or the generic level descriptors for the question<br>• the specific skills defined in the mark scheme or in the generic level descriptors for the question<br>• the standard of response required by a candidate as exemplified by the standardisation scripts. |
| GENERIC MARKING PRINCIPLE 2:<br><br>Marks awarded are always **whole marks** (not half marks, or other fractions). |
| GENERIC MARKING PRINCIPLE 3:<br><br>Marks must be awarded **positively**:<br><br>• marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate<br>• marks are awarded when candidates clearly demonstrate what they know and can do<br>• marks are not deducted for errors<br>• marks are not deducted for omissions<br>• answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous. |
| GENERIC MARKING PRINCIPLE 4:<br><br>Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors. |
| GENERIC MARKING PRINCIPLE 5:<br><br>Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen). |

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

## Mark Scheme Notes

Marks are of the following three types.

| | |
|---|---|
| M | Method mark, given for a valid method applied to the problem. Method marks can still be given even if there are numerical errors, algebraic slips or errors in units. However the method must be applied to the specific problem, e.g. by substituting the relevant quantities into a formula. Correct use of a formula without the formula being quoted earns the M mark and in some cases an M mark can be implied from a correct answer. |
| A | Accuracy mark, given for an accurate answer or accurate intermediate step following a correct method. Accuracy marks cannot be given unless the relevant method mark has also been given. |
| B | Mark for a correct statement or step. |
| DM or DB | M marks and B marks are generally independent of each other. The notation DM or DB means a particular M or B mark is dependent on an earlier M or B mark (indicated by *). When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given. |

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT below).
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures (sf) or would be correct to 3 sf if rounded (1 decimal place (dp) in the case of an angle in degrees). As stated above, an A or B mark is not given if a correct numerical answer is obtained from incorrect working.
- Common alternative solutions are shown in the Answer column as: '**EITHER** Solution 1 **OR** Solution 2 **OR** Solution 3 …'. Round brackets appear in the Partial Marks column around the marks for each alternative solution.
- The total number of marks available for each question is shown at the bottom of the Marks column in bold type.
- Square brackets [ ] around text show extra information not needed for the mark to be awarded.

The following abbreviations may be used in a mark scheme.

| | |
|---|---|
| AG | Answer given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid). |
| CAO | Correct answer only (emphasising that no 'follow through' from an error is allowed). |
| CWO | Correct working only |
| FT | Follow through after error (see Mark Scheme Notes for further details). |
| ISW | Ignore subsequent working |
| OE | Or equivalent form |
| SC | Special case |
| SOI | Seen or implied |

| Question | Answer | Marks | Partial Marks | Guidance |
|---|---|---|---|---|
| 1 | Density $\rho$<br>Volume — COM from vertex<br>Cone: $\frac{1}{3}\pi r^2 \times 3r$ — $\frac{3}{4} \times 3r$<br>Hemisphere: $\frac{2}{3}\pi r^3$ — $3r + \frac{3}{8}r$<br>Combined: $\frac{5}{3}\pi r^3$ — $\overline{x}$ | 1 | **B1** | For COM (centre of mass) $\frac{3}{4}.3r$ or $3r + \frac{3}{8}r$ |
| | Take moments about vertex: (allow $\rho$ omitted)<br>$\frac{5}{3}\pi r^3 \times \overline{x} = \pi r^3 \times \frac{9}{4}r + \frac{2}{3}\pi r^3 \times \frac{27}{8}r$ | 1 | **M1** | Taking moments equation |
| | leading to $\overline{x} = \frac{27}{10}r$ | 1 | **A1** | |
| | distance of COM from vertex = $\frac{27}{10}r$ | 1 | **A1** | AG Correct answer, with convincing working |
| | | **4** | | |

| Question | Answer | Marks | Partial Marks | Guidance |
| --- | --- | --- | --- | --- |
| 2(a) | In equilibrium, $2mg = 24mg.\dfrac{x}{a}$ | 1 | **M1** | Use Hooke's law |
| | $x = \dfrac{a}{12}$ | 1 | **A1** | |
| | | **2** | | |
| 2(b) | Let $d$ be the distance below $A$<br>Loss in gravitational potential energy (GPE) of particle = $2mgd$ | 1 | **B1** | |
| | Gain in elastic potential energy (EPE) of string<br>$= \frac{1}{2} \times \dfrac{24mg}{a} \times (d - a)^2$ | 1 | **B1** | |
| | No change in kinetic energy (KE), so<br>$2mgd = \frac{1}{2} \times \dfrac{24mg}{a} \times (d - a)^2$ | 1 | **M1** | Equate energy loss to energy gain |
| | $6(d - a)^2 = ad$ | 1 | **M1** | Attempt to solve |
| | $d = \frac{3}{2}a$ or $\frac{2}{3}a$ | 1 | **A1** | |
| | Since $d > a$, distance below $A = \frac{3}{2}a$ | 1 | **A1** | |
| | | **6** | | |

| Question | Answer | Marks | Partial Marks | Guidance |
|---|---|---|---|---|
| 3(a) | $v\dfrac{\mathrm{d}v}{\mathrm{d}x} = g - kv^2$ | 1 | **B1** | Use Newton's 2nd law |
| | $\displaystyle\int \frac{v}{g - kv^2}\,\mathrm{d}v = \int \mathrm{d}x$ | 1 | **M1** | Separate variables and attempt to integrate |
| | $-\dfrac{1}{2k}\ln(g - kv^2) = x + c$ | 1 | **A1** | Correct |
| | When $x = 0$, $v = 0$: $c = -\dfrac{1}{2k}\ln g$ | 1 | **M1** | Use initial condition |
| | $x = \dfrac{1}{2k}\ln\left(\dfrac{g}{g - kv^2}\right)$ | 1 | **A1** | |
| | $\mathrm{e}^{2kx} = \dfrac{g}{g - kv^2}$ | 1 | **M1** | Rearrange, removing log correctly |
| | $v^2 = \dfrac{g}{k}(1 - \mathrm{e}^{-2kx})$ | 1 | **A1** | AG Correct answer, with convincing working |
| | | **7** | | |
| 3(b)(i) | $V = 31.62\ldots$ | **1** | **B1** | |
| 3(b)(ii) | When $v = \dfrac{1}{2} \times 31.62\ldots$, $x = \dfrac{1}{2k}\ln\dfrac{4}{3}$ | 1 | **M1** | |
| | distance = 14.4 m | 1 | **A1** | |
| | | **2** | | |

| Question | Answer | Marks | Partial Marks | Guidance |
|---|---|---|---|---|
| 4(a) | Speeds after collision: A $\to v$, B $\to w$<br>Momentum: $2mw + mv = mu\cos 30°$ | 1 | **M1** | Arrows or equivalent to show it is the horizontal component of the velocity. |
| | Restitution: $w - v = eu\cos 30°$ | 1 | **M1** | |
| | Solve to give: $w = \frac{\sqrt{3}}{6}u(1+e)$. | 1 | **A1** | AG |
| | $v = \frac{\sqrt{3}}{6}u(1-2e)$ | 1 | **A1** | |
| | $\uparrow u\sin 30°$<br>Speed of $A = \sqrt{(u\sin 30°)^2 + \left(\frac{\sqrt{3}}{6}u(1-2e)\right)^2}$ | 1 | **M1** | Speed of $A$ perpendicular to line of centres unchanged: used in expression for speed |
| | $= u\sqrt{\frac{1-e+e^2}{3}}$ | 1 | **A1** | |
| | | **6** | | |
| 4(b) | Loss in kinetic energy (KE) =<br>$\frac{1}{2}mu^2 - \frac{1}{2}\times 2m\left(\frac{\sqrt{3}}{6}\times\frac{4u}{3}\right)^2 - \frac{1}{2}m\times\frac{7}{27}u^2$ | 2 | **M1A1FT** | Difference in KEs with $e = \frac{1}{3}$<br>FT their speed from part **(a)** |
| | $= \frac{2}{9}mu^2$ | 1 | **A1** | Correct answer |
| | | **3** | | |

| Question | Answer | Marks | Partial Marks | Guidance |
|---|---|---|---|---|
| 5(a) | $T\cos 60° = mg$ | 1 | **B1** | Vertically |
| | $T\sin 60° = m \times a\sin 60° \times \omega^2$ | 2 | **M1A1** | Horizontally |
| | Divide: $\omega^2 = \dfrac{2g}{a}$ | 1 | **A1** | AG |
| | | **4** | | |
| 5(b) | Energy from 60° with downward vertical to point when string goes slack: $\tfrac{1}{2}m\left(a\sqrt{\dfrac{2g}{a}}\right)^2 - \tfrac{1}{2}mv^2 = mg(a\cos 60° + a\cos\theta)$ | 2 | **M1A1** | Energy equation |
| | Leading to $v^2 = ga - 2ga\cos\theta$ | 1 | **M1** | |
| | When string goes slack, tension is zero: $mg\cos\theta = \dfrac{mv^2}{a}$ | 1 | **A1** | Use Newton's law, with tension or with tension equated to zero |
| | Combine: $\cos\theta = \tfrac{1}{3}$ | 2 | **M1A1** | |
| | | **6** | | |

| Question | Answer | Marks | Partial Marks | Guidance |
|---|---|---|---|---|
| 6(a) | $\rightarrow x = u\cos\alpha\, t$<br>$\uparrow y = u\sin\alpha\, t - \tfrac{1}{2}gt^2$ | 1 | **B1** | Both needed |
| | Eliminate $t$: $y = u\sin\alpha \times \dfrac{x}{u\cos\alpha} - \tfrac{1}{2}g\left(\dfrac{x}{u\cos\alpha}\right)^2$ | 1 | **M1** | Eliminate |
| | $y = x\tan\alpha - \dfrac{gx^2}{2u^2}\sec^2\alpha$ | 1 | **A1** | AG |
| | | **3** | | |
| 6(b) | For greatest height, using vertical motion,<br>$H = \dfrac{(u\sin\alpha)^2}{2g}$ | 1 | **M1** | |
| | $= \dfrac{2u^2}{5g}$ | 1 | **A1** | using $\tan\alpha = 2$ |
| | $\dfrac{3H}{4} = 2d - \dfrac{1}{2u^2}g \times 5d^2$ | 1 | **M1** | When $y = \dfrac{3H}{4}$, $x = d$ |
| | $\dfrac{3H}{4} = 2d - \dfrac{d^2}{H}$ | 1 | **M1** | Substitute for $u$ |
| | Rearrange: $4d^2 - 8dH + 3H^2 = 0$<br>$(2d - 3H)(2d - H) = 0$: | 1 | **A1** | Correct quadratic equation |
| | $d = \tfrac{1}{2}H,\ \tfrac{3}{2}H$ | 1 | **A1** | CAO |
| | | **6** | | |

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