UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/02**

Paper 2 **May/June 2009**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\text{m}\,\text{s}^{-2}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

UNIVERSITY of CAMBRIDGE International Examinations

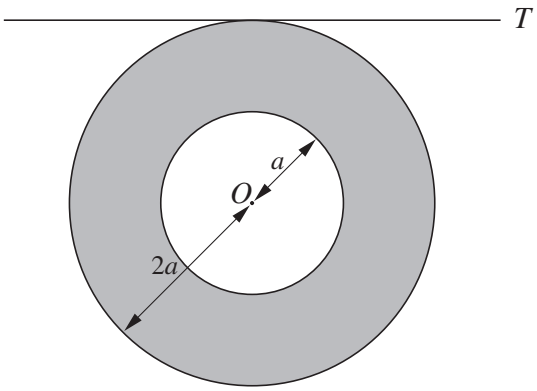
[Turn over

1 A line OP of fixed length l rotates in a plane about the fixed point O. At time $t = 0$, the line is at the position OA. At time t, angle $AOP = \theta$ radians and $\frac{\mathrm{d}\theta}{\mathrm{d}t} = \sin\theta$. Show that, for all t, the magnitude of the acceleration of P is equal to the magnitude of its velocity. [5]

2 The tip of a sewing-machine needle oscillates vertically in simple harmonic motion through a distance of 2.10 cm. It takes 2.25 s to perform 100 complete oscillations. Find, in $\mathrm{m\,s^{-1}}$, the maximum speed of the tip of the needle. [5]

Show that the speed of the tip when it is at a distance of 0.5 cm from a position of instantaneous rest is $2.50\,\mathrm{m\,s^{-1}}$, correct to 3 significant figures. [2]

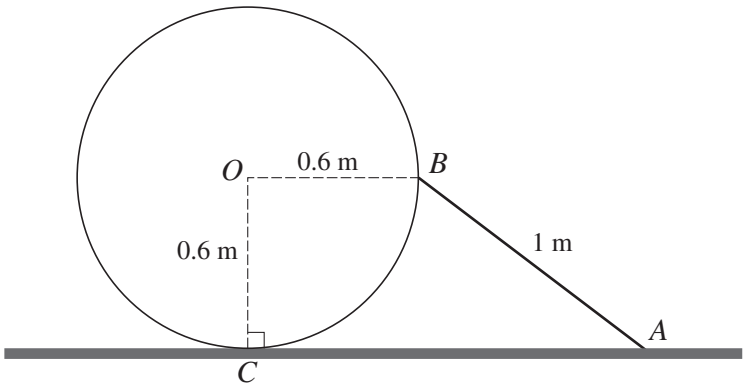
3



A uniform lamina of mass m is bounded by concentric circles with centre O and radii a and $2a$. The lamina is free to rotate about a fixed smooth horizontal axis T which is tangential to the outer rim (see diagram). Show that the moment of inertia of the lamina about T is $\frac{21}{4}ma^2$. [5]

When hanging at rest, with O vertically below T, the lamina is given an angular speed ω about T. The lamina comes to instantaneous rest in the subsequent motion. Neglecting air resistance, find the set of possible values of ω. [3]

4



A uniform sphere rests on a horizontal plane. The sphere has centre O, radius 0.6 m and weight 36 N. A uniform rod AB, of weight 14 N and length 1 m, rests with A in contact with the plane and B in contact with the sphere at the end of a horizontal diameter. The point of contact of the sphere with the plane is C, and A, B, C and O lie in the same vertical plane (see diagram). The contacts at A, B and C are rough and the system is in equilibrium. By taking moments about C for the system, show that the magnitude of the normal contact force at A is 10 N. [2]

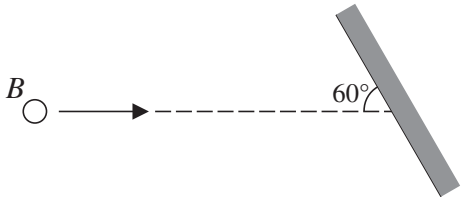
Show that the magnitudes of the frictional forces at A, B and C are equal. [2]

The coefficients of friction at A, B and C are all equal to μ. Find the smallest possible value of μ. [7]

5 Two spheres A and B, of equal radius, have masses m_1 and m_2 respectively. They lie at rest on a smooth horizontal plane. Sphere A is projected directly towards sphere B with speed u and, as a result of the collision, A is brought to rest. Show that

(i) the speed of B immediately after the collision cannot exceed u, [2]

(ii) $m_1 \leqslant m_2$. [2]



After the collision, B hits a smooth vertical wall which is at an angle of 60° to the direction of motion of B (see diagram). In the impact with the wall B loses $\frac{2}{3}$ of its kinetic energy. Find the coefficient of restitution between B and the wall and show that the direction of motion of B turns through 90°. [8]

6 The times taken by employees in a factory to complete a certain task have a normal distribution with mean μ seconds and standard deviation σ seconds, both of which are unknown. Based on a random sample of 20 employees, the symmetric 95% confidence interval for μ is (481, 509). Calculate a symmetric 90% confidence interval for μ. [6]

7 An experiment was carried out to determine how much weedkiller to apply per $100\,m^2$ in a large field. Ten $100\,m^2$ areas of the field were randomly chosen and sprayed with predetermined volumes of the weedkiller. The volume of the weedkiller is denoted by x litres and the number of weeds that survived is denoted by y. The results are given in the table.

x	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55
y	48	40	44	35	39	24	10	13	9	6

$[\Sigma x = 3.25,\ \Sigma x^2 = 1.2625,\ \Sigma y = 268,\ \Sigma y^2 = 9548,\ \Sigma xy = 66.10.]$

It is given that the product moment correlation coefficient for the data is −0.951, correct to 3 decimal places.

(i) Calculate the equation of a suitable regression line, giving a reason for your choice of line. [5]

(ii) Estimate the best volume of weedkiller to apply, and comment on the reliability of your estimate. [3]

8 Part of a research study of identical twins who had been separated at birth involved a random sample of 9 pairs, in which one twin had been raised by the natural parents and the other by adoptive parents. The IQ scores of these twins were measured, with the following results.

Twin pair	1	2	3	4	5	6	7	8	9
IQ of twin raised by natural parents	82	92	115	132	88	95	112	83	123
IQ of twin raised by adoptive parents	92	88	115	134	97	94	107	88	130

It may be assumed that the difference in IQ scores has a normal distribution. The mean IQ scores of separated twins raised by natural parents and by adoptive parents are denoted by μ_N and μ_A respectively. Obtain a 90% confidence interval for $\mu_N - \mu_A$. [6]

One of the researchers claimed that there was no evidence of a difference between the two population means. State, giving a reason, whether the confidence interval supports this claim. [2]

9 The proportions of blood types A, B, AB and O in the Australian population are 38%, 10%, 3% and 49% respectively. In order to test whether the population in Sydney conforms to these figures, a random sample of 200 residents is selected. The table shows the observed frequencies of these types in the sample.

Blood Type	A	B	AB	O
Frequency	57	24	9	110

Carry out a suitable test at the 5% significance level. [7]

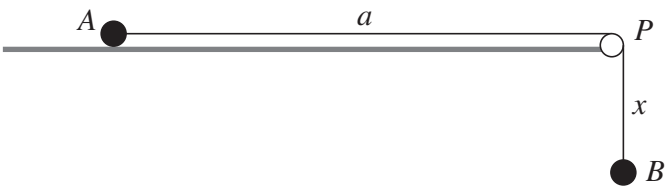
Find the smallest sample size that could be used for the test. [2]

10 The number of hits per minute on a particular website has a Poisson distribution with mean 0.8. The time between successive hits is denoted by T minutes. Show that $\mathrm{P}(T > t) = \mathrm{e}^{-0.8t}$ and hence show that T has a negative exponential distribution. [5]

Using a suitable approximation, which should be justified, find the probability that the time interval between the 1st hit and the 51st hit exceeds one hour. [7]

11 Answer only **one** of the following two alternatives.

EITHER



Two particles A and B, of equal mass m, are connected by a light elastic string of natural length a and modulus of elasticity $4mg$. Particle A rests on a rough horizontal table at a distance a from the edge of the table. The string passes over a small smooth pulley P fixed at the edge of the table. At time $t = 0$, B is released from rest at P and falls vertically. At time t, B has fallen a distance x, without A slipping (see diagram). Show that

$$\ddot{x} = -\frac{g}{a}(4x - a). \quad [3]$$

Deduce that, while A does not slip, B moves in simple harmonic motion and identify the centre of the motion. [3]

Given that the coefficient of friction between A and the table is $\frac{1}{3}$, find the value of x when A starts to slip, and the corresponding value of t, expressing this answer in the form $k\sqrt{\left(\frac{a}{g}\right)}$. Give the value of k correct to 3 decimal places. [8]

OR

A study was made of the acidity levels in farmland on opposite sides of an island. The levels were measured at six randomly chosen points on the eastern side and at five randomly chosen points on the western side. The values obtained, in suitable units, are denoted by x_E and x_W respectively. The sample means $\bar{x}_E$ and $\bar{x}_W$, and unbiased estimates of the two population variances, s_E^2 and s_W^2, are as follows.

$$\bar{x}_E = 5.035,\ s_E^2 = 0.0231,\ \ \bar{x}_W = 4.782,\ s_W^2 = 0.0195.$$

The population means on the eastern and western sides are denoted by μ_E and μ_W respectively. State suitable hypotheses for a test for a difference between the mean acidity levels on the two sides of the island. [1]

Stating any required assumptions, obtain the rejection region for a test at the 5% significance level of whether the mean acidity levels differ on the two sides of the island. Give the conclusion of the test. [9]

Find the largest value of a for which the samples above provide evidence at the 5% significance level that $\mu_E - \mu_W > a$. [4]

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