CONTENTS

FOREWORD

This booklet contains reports written by Examiners on the work of candidates in certain papers. **Its contents are primarily for the information of the subject teachers concerned**.

FURTHER MATHEMATICS

GCE Advanced Level

Paper 9231/01
Paper 1

General comments

The majority of candidates were able to make some progress with at least ten questions and few appeared to have time problems. The number of misreads was negligibly small and most candidates set out their work in an orderly and legible way. The accuracy of the working was generally impressive, especially in cases where good candidates got involved in unnecessarily extended strategies. As is usually the case, the calculus based questions generated the best work, though some Centres also produced excellent work in non-calculus areas.

Nevertheless, candidates' syllabus coverage appeared to be incomplete to the extent that many candidates had a poor understanding of curve sketching, both in the x-y and r-θ domains, summation of series, induction, change of independent variable in the context of differential equations, special theoretical results appertaining to the eigenvalue problem and especially the vector product. This last named deficiency led to time consuming, and usually inaccurate, working in the application of extensive alternative strategies. It seems therefore that future candidates who are unable to absorb all the syllabus material should at least ensure that they have a complete knowledge of the topics that they do study. In this way they will avoid involvement in suboptimal strategies which can waste examination time.

Finally, attention must be drawn to two important, but frequently ignored, aspects of the rubric. Firstly, the requirement that where a numerical answer has no exact decimal representation, it should be given to 3 significant figures, or 1 decimal place in the case of angles. Secondly, the requirement that no credit is given for unexplained results obtained directly from a graphic calculator.

Comments on specific questions

Question 1

Most candidates produced a complete and correct response to this question. The most popular strategy was to reduce the given matrix, **A** to the row-echelon form and then to use the dimension theorem to argue that if K is the null space of T, then dim(K) = 4 – r(**A**) = 1. However, some candidates, in defiance of the rubric, obtained the echelon form of **A** directly from a graphic calculator and so did not gain full credit. Yet others did not obtain an echelon form, but instead stopped the reduction process immediately a row of zeros had been obtained. This too led to loss of marks.

A popular alternative strategy was to show from the echelon form of **A** that $\begin{pmatrix} 4 \\ 0 \\ 1 \\ -1 \end{pmatrix}$ spans K, and hence that dim(K) = 1. Actually, this vector can easily be obtained by working with linear equations based on the original form of **A**. However, among the few candidates who employed this strategy, there were some who did not complete their working by showing that within a multipicative constant no other non-zero vector satisfies **Ax = 0**.

Answer: The dimension of the null space of T is 1.

Question 2

This question was answered correctly and in full by almost all candidates. Very few did not begin with a correct integral representation of S, the surface area, in some form. Actually, in view of the symmetries of C, the required result can equally well be obtained from $6\pi a^2 \int_0^{\pi/2} x\sqrt{\dot{x}^2+\dot{y}^2}\,\mathrm{d}t$, though explanation of why this is correct in this context was expected but not always supplied.

Few failed to obtain $S = 6\pi a^2 \int_0^{\pi/2} \cos t \sin^4 t\,\mathrm{d}t$, or a similar integral, and to carry out the integration accurately.

Answer: Area of surface of revolution = $\dfrac{6\pi a^2}{5}$.

Question 3

Most candidates understood the basic ideas involved, though elementary errors did some damage. It was usual to see a correct argument such as $x^3 + bx + c = 0 \Rightarrow 2b = \left(\sum \alpha\right)^2 - \sum \alpha^2 = 0 - 14 \Rightarrow b = -7$. On the other hand, the evaluation of c frequently went off the rails with '$\sum \alpha^3 - 7\sum \alpha + c = 0$' appearing as a starting point. This leads to c = 18 (incorrect) and so to no chance of obtaining the possible values of α, β and γ. Almost all of those who did obtain the required cubic equation went on to produce a correct response to the last part of the question.

Answer: $x^3 - 7x + 6 = 0$; possible values of α, β and γ are 1, 2, –3.

Question 4

(i) Most responses showed a spiral starting at some point of the line $\theta = 0$, though not everyone made it clear what the coordinates of this point actually are. Beyond that, a persistent error was failure to show increasing intercepts, in approximately correct proportions, on the lines $\theta = \dfrac{n\pi}{2}$ for n = 0, 1, 2, 3.

(ii) Most responses to this part of the question started correctly with:

length of C = $\displaystyle\int_0^{3\pi/2} \left(e^{2\theta/5} + \frac{1}{25}e^{2\theta/5}\right)^{\frac{1}{2}} \mathrm{d}\theta$.

Usually this integral was evaluated accurately at least as far as the obtaining of $\sqrt{26}\mathrm{e}^{\frac{3\pi}{10}} - 1$, though some candidates then failed to provide a decimal answer in accordance with the requirements of the rubric.

Answer: **(ii)** Length of C = 7.99.

Question 5

In contrast to the earlier questions, most candidates found this question to be difficult so that complete answers were very much in a minority.

Some responses began with a decomposition of the form $S_{2N} = \sum_{n=1}^{2N} n^3 - k\sum_{n=1}^{N} n^3$ and this was followed by sensible attempts to sum the two series involved by means of the standard result $\sum_{n=1}^{N} n^3 = \frac{1}{4}n^2(n+1)^2$. However, more often than not, the summation limits were incorrect in at least one part of the decomposition and/or the implied value of k was wrong. Even less successful was the strategy of writing $S_{2N} = \sum_{n=1}^{N}\left[(2n-1)^3 - (2n)^3\right]$ (*) followed by separate attempts to sum each series. A few candidates, however, did see that (*) is equivalent to $\sum_{n=1}^{N}\left(-12n^2 + 6n - 1\right)$ and hence that $S_{2N} = -2N(N+1)(2N+1) + 3N(N+1) - N$, etc.

For the second part of the question, many innovative but incorrect results for S_{2N+1} were written down. Only about half of all candidates appeared to understand that $S_{2N+1} = S_{2N} + (2N+1)^3$ and only about half of these went on to produce a result such as $\frac{S_{2N+1}}{N^3} = \left(2 + \frac{1}{N}\right)^3 - \left(4 + \frac{3}{N}\right)$ from which the required limit can be obtained immediately.

Answer: $S_{2N} = -N^2(4N+3)$; $\lim_{N\to\infty}\left(\frac{S_{2N+1}}{N^3}\right) = 4$.

Question 6

Some candidates failed to write down all the 8th roots of unity without omission and/or duplication. Thus, for example, it was common for $\exp(\pm \pi i)$ to appear, yet for there to be no mention of 1.

For the rest, the factors $z - 1$, $z + 1$, and to a lesser extent $z^2 + 1$, appeared on most scripts. About half of all candidates identified the remaining (quadratic) factors, though sometimes without the use of a 'hence' method, in contradiction to the question. Some also left the quadratic factors in a trigonometric form contrary to what was required.

Answer: $\exp\left(\frac{k\pi i}{4}\right)$, $k = 0, 1, \ldots, 7$; $z - 1$, $z + 1$, $z^2 + 1$, $z^2 \sqrt{2}z + 1$, $z^2 + \sqrt{2}z + 1$.

Question 7

The majority of responses to both parts of this question were complete and correct.

(i) The best candidates differentiated the given equation with respect to x without first implementing any expansion or rearrangement. The careful working of this strategy leads almost immediately to the required result. However, some candidates, for reasons only known to them, first expanded $(x + y)^5$ so as to obtain a total of 7 terms on the left-hand side. They then differentiated with respect to x so as to obtain an equation with a total of 12 terms. Such a strategy demands extreme accuracy and persistence, especially when carried over into the next part of the question. In consequence, very few candidates obtained full credit in this way. Yet again therefore, it must be emphasised how important it is for candidates to look for strategies which are not especially error prone and which do not squander valuable examination time.

(ii) For those proceeding in a time optimum way, that is with a strategy which did not involve expansion of $(x + y)^5$ and/or expressing $\frac{dy}{dx}$ in terms of x and y, the next stage is clearly devoid of difficulty. In this respect, much of the working shown was impressive. On the other hand, suboptimal strategies almost invariably got nowhere.

Answer: **(ii)** $\frac{5}{27}$.

Question 8

By a long way, this was the least well answered question of the paper. In fact, a typical response did no more than set out the inductive hypothesis, H_k, and to verify it to be correct for k = 2, and then to go on to make no further progress in either part of the question.

In part **(i)** it was first required that H_k be defined as $a_k > 2^{g(k)}$ for some k and that subsequently it would be clearly stated that as $\frac{1}{a_k} > 0$, then $a_{k+1} > (a_k)^\lambda$, and hence that $H_k \Rightarrow a_{k+1} > 2^{\lambda g(k)} = 2^{\left(\lambda^k\right)} = 2^{g(k+1)}$, so that $H_k \Rightarrow H_{k+1}$.

The inductive argument would then be completed by stating that H_2 is true, since $a_2 = 2^\lambda = 2^{g(2)}$.

Fundamentally erroneous statements of the form $a_k + \frac{1}{a_k} \geqslant 2^{g(k)} + \frac{1}{2^{g(k)}}$ were very prevalent as also were arguments based on the binomial expansion of $\left(a_k + \frac{1}{a_k}\right)^\lambda$ as if λ is an integer. Use of the binomial series for a non-integer $\lambda > 1$ would, in the first instance, involve an infinite series which includes some negative terms. The working of such an argument into a rigorous form would be time consuming.

The proof of the displayed result in the second part of this question requires no more than a simple argument such as: $\frac{a_{n+1}}{a_n} = a_n^{\lambda-1}\left(1+\frac{1}{a_n^2}\right)^\lambda > a_n^{\lambda-1} > \left[2^{g(n)}\right]^{\lambda-1} = 2^{(\lambda-1)g(n)}$.

However, this response was produced by only a small minority of candidates, whereas the erroneous argument $\left[a_n > 2^{g(n)} \text{ and } a_{n+1} > 2^{g(n+1)}\right] \Rightarrow \frac{a_{n+1}}{a_n} > \frac{2^{g(n+1)}}{2^{g(n)}}$ appeared in some form in more than half of all scripts.

Question 9

About a half of all responses showed deficiencies of various kinds.

(i) Most responses began with a correct differentiation of $x(1 + x^3)^{-n}$, but subsequent attempts to work the expression obtained into the required form were not always successful. The basic problems here, therefore, appertained to the algebra rather than to the calculus. Nevertheless, subsequent working to show how the displayed reduction formula may be deduced was very often correct.

(ii) Most candidates attempted to draw a sketch graph of y over the interval J, [0, 1]. This usually appeared as a concave downward, monotonically decreasing graph over J. In fact, the graph is concave downwards for $x \in [0, \alpha)$ and concave upwards for $x \in (\alpha, 1]$, where $\alpha^3 = \frac{1}{2}$. Such detail was not required, but only a monotonically decreasing graph starting at (0, 1) and finishing at approximately (1, 0.5). It was, however, expected that the unit square would be included in the same diagram so that with a minimum of explanation it would be clear that $I_1 < 1$. Nevertheless, in passing, it must be observed that some candidates attempted to evaluate I_1 by the trapezium rule with [0, 0.5] and [0.5, 1] as subintervals. Actually, this underestimates I_1 and consequently some inequality arguments based on this unnecessarily complicated analysis were invalid.

(iii) Almost all candidates started with $I_3 = \frac{1}{24} + \frac{5}{6}I_2$ and $I_2 = \frac{1}{6} + \frac{2}{3}I_1$ and so obtained $I_3 = \frac{1}{24} + \frac{5}{6}\left(\frac{1}{6}+\frac{2}{3}I_1\right)$. Use of the result of part **(ii)** then led to the required inequality for I_3. Some candidates started here with $I_1 = 1$ and then went on to obtain $I_3 = \frac{53}{72}$. This, as such, does not establish the displayed inequality result. Further detail is necessary for that purpose, but almost always this did not appear in this context.

Question 10

This question was well answered by the majority of candidates. Lack of insight into the algebra led to erroneous conclusions in part **(iii)**. Some sketches were poorly drawn and did not include all the important aspects.

(i) Few candidates failed to establish the equations of both asymptotes. For the obtaining of the diagonal asymptote, there were occasional errors in the division of $x^2 + 2x - 3$ by $x + 4$. A small minority of candidates worked from $mx + c = \frac{x^2+2x-3}{x+4}$ to obtain $m - 1 = 0$, $4m + c - 2 = 0$ and hence $m = 1$, $c = -2$.

(ii) Most candidates wrote down the equations of the two vertical asymptotes, but some appeared to be unaware that in the current context C has a horizontal asymptote.

(iii) A significant minority of candidates did not effect any cancellation. This led to the supposition that the line $x = 1$ is an asymptote and hence to a completely wrong right hand branch of C. The rest, the majority, did first reduce the equation to $y = -\frac{x+3}{x+4}$ and went on to produce a satisfactory sketch graph. However, even some of the better candidates failed to mark in the coordinates of the points of intersection of C with the coordinate axes. Moreover, bad forms at infinity appeared on many scripts.

Answers: **(i)** $x = -4$, $y = x - 2$; **(ii)** $x = -4$, $x = -\frac{1}{\lambda}$, $y = \frac{1}{\lambda}$; **(iii)** C crosses axes at $(-3, 0)$ and $(0, -0.75)$.

Question 11

The general standard of responses was below that for similar questions in previous examinations. Some candidates made no use of the vector product and moreover understanding of the essential geometry was sometimes defective.

(i) The majority of responses began with $(\mathbf{i} - \mathbf{j} - 4\mathbf{k}) \times (3\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}) = 14\mathbf{i} - 14\mathbf{j} + 7\mathbf{k}$ or equivalent, and then proceeded along standard lines. In this context, a persistent error was the supposition that if $\mathbf{y}$ and $\mathbf{z}$, are any 2 vectors, then $\frac{\mathbf{y}\times\mathbf{z}}{|\mathbf{y}\times\mathbf{z}|} = \frac{\mathbf{y}\times\mathbf{z}}{|\mathbf{y}|\times|\mathbf{z}|}$.

In contrast, there was a substantial majority of candidates who specified a general point $P(\lambda)$ on l_1, together with a general point $Q(\mu)$ on l_2, and hence obtained $\overrightarrow{PQ}$ in terms of λ and μ. Use of orthogonality conditions will then lead (eventually) to $\lambda = 1$, $\mu = 2$. This extended strategy was generally worked accurately, but relative to working with the vector product it must have absorbed a lot of examination time.

(ii) The use of orthogonality conditions in preference to use of the vector product appeared even here. This suggests that some candidates did not have a complete knowledge of the relevant material of the syllabus. Among the majority, who did use the vector product, there were more errors than usual to be seen.

(iii) Only about half of all candidates started with $\frac{|(\mathbf{i}-8\mathbf{j}-9\mathbf{k}).(-8\mathbf{i}-\mathbf{j}+14\mathbf{k})|}{\sqrt{261}} = \frac{126}{\sqrt{261}}$ in an attempt to obtain the required perpendicular distance.

In contrast, the rest first obtained the scalar equation of Π_1 and then attempted to apply the standard formula for the perpendicular distance of a point from a plane or to use more complicated methods. Again, the employment of such suboptimal strategies proved to be very error prone.

(iv) The most direct method here is to observe, first of all, that the angle between the planes Π_1 and Π_2 is equal to the angle between the line l_1 and l_2. Since these directions are part of the data, then the required result may be obtained immediately.

The alternative, and certainly the most popular strategy in responses, is to attempt to find the angle between the normals to the 2 planes. Thus most responses began with an evaluation of the vector product of the direction of $\overrightarrow{PQ}$, as obtained in part **(i)**, and $\mathbf{i} - \mathbf{j} - 4\mathbf{k}$, the direction of l_2. If up to this point all relevant working is correct, then $9\mathbf{i} + 9\mathbf{j}$ is obtained. The angle between this vector and $8\mathbf{i} + \mathbf{j} - 14\mathbf{k}$ can then be determined by means of the standard formula.

Answers: **(i)** 3; **(ii)** $8\mathbf{i} + \mathbf{j} - 14\mathbf{k}$; **(iii)** 7.80; **(iv)** 66.8° (or 113.2°).

Question 12 EITHER

This question was answered satisfactorily by most of those who attempted it, although few responses were complete and correct in every detail.

The proof of the first of the displayed results appeared in almost all responses. The proof of the second and more difficult result turned out to be beyond some candidates. The root problem here was the differentiation of $x\dfrac{dy}{dx}$ with respect to t.

(i) Almost all candidates used the previous results to obtain the correct t-y second order linear differential equation.

(ii) Very few failed to solve the AQE accurately and to go on to obtain the displayed complementary function. There was much careful attention to detail at this stage.

(iii) Almost all candidates assumed (correctly) that the particular integral is of the form $at + b$ and went on to apply standard procedures for the determination of a and b.

(iv) The basic methodology for obtaining the general solution of the t-y differential equation, G(t), was understood by almost all candidates. However, some stopped there and moreover a significant minority of responses showed a general solution in the x-y domain which was inconsistent with G(t).

Answers: **(i)** $4\dfrac{d^2y}{dx^2} + 12\dfrac{dy}{dt} + 25y = 50t - 1$; **(iii)** $2t - 1$; **(iv)** $y = Rx^{-3/2}\sin[2(\ln x) + \phi] + 2\ln x - 1$.

Question 12 OR

This option enjoyed about the same popularity as its alternative. Most responses showed less coherence and completeness in parts **(i)** and **(ii)** than in the rest of the question.

(i) Only a minority of responses showed a complete proof. A simple argument such as the following was produced by a very small number of candidates: $\lambda = 0$ and $\det(\mathbf{A} - \lambda\mathbf{I}) = 0 \Rightarrow \det \mathbf{A} = 0$ which contradicts the data. Hence $\lambda \neq 0$.

(ii) The majority of responses showed a complete argument such as:

$$\mathbf{Ae} = \lambda\mathbf{e} \Rightarrow \mathbf{A}^{-1}(\mathbf{Ae}) = \mathbf{A}^{-1}(\lambda\mathbf{e}) \Rightarrow \mathbf{e} = \lambda\mathbf{A}^{-1}\mathbf{e} \Rightarrow \mathbf{A}^{-1}\mathbf{e} = \frac{1}{\lambda}\mathbf{e}$$

Others showed incomplete versions of this and yet others showed an attempt to use such arguments to prove parts **(i)** and **(ii)** at the same time. Frequently, however, there were notational inaccuracies which destroyed the utility of such strategies.

Most responses continued with an attempt to determine both the eigenvalues and eigenvectors of **A**. The majority of responses showed the correct eigenvalues to be written down without any preliminary argument. This is completely acceptable, for **A** is a triangular matrix and so, of course, the eigenvalues are the elements of the leading diagonal. In contrast, some candidates loaded themselves with the task of first obtaining the characteristic equation of **A** in polynomial form and this was eventually solved after further labour.

Most responses showed a correct strategy for the obtaining of the eigenvestors of **A** but the working was not always correct. Beyond that, most responses continued with a sound strategy for the obtaining of the eigenvalues of **B** and then went straight on to exhibit results for **P** and **D**. However, it was expected that candidates would first emphasise that the result of part **(ii)** implies that that the eigenvectors of **A** and **B** are the same.

Answers: $\mathbf{P} = \begin{pmatrix} 1 & 1 & 3 \\ 0 & 3 & 8 \\ 0 & 0 & -2 \end{pmatrix}$, $\mathbf{D}$ = diag (0.2, 0.5, 1).

Paper 9231/02

Paper 2

General comments

Virtually all candidates attempted all the eleven required questions, and excellent work was produced by some candidates in all questions. The overall impression of the Examiners was that candidates performed somewhat better on the compulsory Statistics questions (numbers 6 - 10) than on the Mechanics ones (1 - 5). The Statistics alternative in **Question 11** was chosen more often than the Mechanics one, and in general produced rather better answers, but the difference was not as marked as last year. **Question 9** and the Mechanics option in **Question 11** seemed to challenge candidates the most, while **Questions 1**, **4**, **6** and **8** produced good answers most frequently.

Comments on specific questions

Question 1

Almost all candidates used a moment equation to find the vertical force exerted by one or other of the supports on the beam, followed by either resolution of forces or a second moment equation for the force exerted by the other support. Although these forces are of course equal to those asked for in the question, namely the vertical forces exerted on the supports by the beam, remarkably few candidates stated this either explicitly or implicitly.

Answers: 1200 N, 800 N.

Question 2

This question was answered correctly in a variety of ways, some very brief and elegant, but essentially requires the magnitude and direction or equivalently the normal and parallel components of the speed of the particle before and after each of the two collisions to be related. Some candidates argued mainly through a diagram, others took the sum of the squares of the final components, and still others effectively showed the final direction was the reverse of the initial one. Among the false arguments encountered were those which considered two parallel cushions, sometimes with the ball striking perpendicularly, and those which made invalid assumptions about the directions.

Question 3

The first two given equations are perhaps obtained most easily by taking moments for the sphere about C and B respectively. The second result was also often obtained by resolving the forces on the sphere both vertically and horizontally, and combining the resulting equations, though the simplification of the trigonometric expressions was sometimes unconvincing. The inequality follows almost immediately from the two given equations, though the Examiners did require some minimal argument. By contrast many candidates were unable to deduce that equilibrium is limiting at D, or to find the value of μ by using $F_1 = \mu N_1$ in conjunction with a suitable moment or resolution equation.

Answer: 0.7.

Question 4

The given expression for the tension was usually found without difficulty by relating the initial and subsequent speeds using conservation of energy, and substituting for the latter speed in the expression for the tension which results from a radial resolution. Similarly most candidates realised that if the particle is to describe a complete circle then the tension must be non-negative when $\theta = \pi$. Conservation of energy yields the speed at the highest point, and then consideration of the vertical and horizontal components of motion of the particle produces the given result for HK.

Question 5

Most candidates were able to write down the moment of inertia $\frac{8ma^2}{3}$ of the square lamina, and realised that each of the triangular laminas must have a moment of inertia of one-quarter of this. Rather fewer applied the parallel axis theorem correctly to obtain the moment of inertia of the square lamina about V, and then simply added to it the given moment of inertia of a triangular lamina to find the combined moment of inertia I. The final part proved even more demanding, requiring that the moment about V of the weight of the lamina be equated to $-I\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$. The most common error was an incorrect distance from V of the centre of mass. Approximation of $\sin\theta$ by θ and use of the standard SHM formula produces the approximate period of oscillation.

Answers: $\frac{58ma^2}{3}$; $2\pi\sqrt{\frac{29a}{13g}}$.

Question 6

This question requires the application of a standard t-test to the differences in the hours lost before and after introduction of the regulations in the eight factories, but a small proportion of candidates mistakenly applied an unpaired sample test. Comparison of the calculated t-value of magnitude 1.38 with the tabular value 1.895 leads to the conclusion that the working time lost has not decreased. The necessary assumption is that the differences come from a normal distribution, and while many candidates made some reference to normality, their assumption was frequently expressed imprecisely.

Question 7

This question presented few difficulties to most candidates, except in some cases the second part, which simply requires the addition of the expected numbers of shots for two successive instances of shooting until the target is hit.

Answers: **(i)** $\frac{1}{p}$; **(ii)** $\frac{2}{p}$; **(iii)** $5p^2(1-p)^4$.

Question 8

This question was usually well done, with the nine expected values first calculated, preferably to at least one decimal place. The value 13.1 of χ^2 is calculated in the usual way and compared with the tabular value 9.488, leading to the conclusion that quality rating is not independent of supplier.

Question 9

Most candidates correctly used the Poisson distribution with parameter $4t$, noting that the required result is the complement of no shoppers entering in a period of t minutes. Although many then wrote down either the correct negative exponential distribution or something akin to it, very few gave a valid argument, which stems from a realisation that $P(T \leqslant t)$ is the same as the probability found in the first part of the question. The median is found by equating this probability to $\frac{1}{2}$.

Answer: $\frac{1}{4}\ln 2$.

Question 10

The correct approach was usually used to determine the confidence interval, though not all candidates chose the correct tabular t-value of 3.106, and some confused biased and unbiased estimates of the variance. The pooled estimate of the common variance was normally found correctly from the standard formula, and used in the final test. This also presented few difficulties, with comparison of the calculated value 0.564 of t with the tabular value 1.697 leading to the conclusion that the fisherman's belief is mistaken. A frequent failing was, however, to use a rounded value of the first sample mean with too few significant figures.

Answers: 0.283 ± 0.075; 0.0128.

Question 11 EITHER

The first part of the Mechanics alternative question can essentially be answered by using a suitable trigonometric rule to express PD in terms of $\sin\theta$ or $\cos\theta$ and then neglecting higher order terms in its expansion. Since PD is constant then so is the extension of the string and hence the tension in it. Noting that the triangle is isosceles, the angle at D must be θ, and this leads to the required result for φ. Thereafter most candidates attempting the question ran into difficulties, sometimes because they wrongly introduced the weight of the ring into their equation of motion. The correct equation involves $\cos\varphi$ and hence $\sin 2\theta$, and the latter may be approximated by 2θ to yield $\frac{d^2\theta}{dt^2} = -2T\theta$. This equation permits the given period to be equated to the usual SHM formula to give T. Finally the two unknown constants in a general form of the equation's solution may be determined from the given boundary conditions.

Answers: $\frac{1}{2}\pi^2 ma$; $\frac{0.03}{\pi}\cos \pi t$.

Question 11 OR

Although many candidates embarked on the alternative Statistics question by effectively noting that the regression line must be of the form $y = bx$ and pass through the mean point, some hopefully substituted $\frac{\sum x_r y_r}{\sum x_r^2}$ in place of b (without justifying this as one of the normal equations for a regression line) instead of using the correct expression from the List of Formulae and simplifying. Most were able to substitute in the tabular coordinates, however, to verify satisfaction of condition (A) and to find the regression line. On the other hand only a minority produced a diagram to satisfactorily show that S is the sum of the squares of the vertical distances of the given points from the line. The minimum can be demonstrated either by equating to zero the derivative of S with respect to k, or by expressing S as $30(k-3)^2 + 84$. A common omission with the former approach was to verify that the turning point is in fact a minimum, by noting that the second derivative is positive.

Answer: $y = 3x$.