

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS  
General Certificate of Education Ordinary Level

**ADDITIONAL MATHEMATICS**

**4037/01**

Paper 1

May/June 2005

**2 hours**

Additional Materials: Answer Booklet/Paper  
Graph paper  
Mathematical tables

**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.  
Write your Centre number, candidate number and name on all the work you hand in.  
Write in dark blue or black pen on both sides of the paper.  
You may use a soft pencil for any diagrams or graphs.  
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.  
Write your answers on the separate Answer Booklet/Paper provided.  
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.  
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.  
The total number of marks for this paper is 80.  
The use of an electronic calculator is expected, where appropriate.  
You are reminded of the need for clear presentation in your answers.

**1. ALGEBRA***Quadratic Equation*

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

*Binomial Theorem*

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{(n-r)!r!}$ .

**2. TRIGONOMETRY***Identities*

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

*Formulae for  $\triangle ABC$* 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

- 1 Given that  $\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$ , find  $(\mathbf{A}^2)^{-1}$ .
- 2 A student has a collection of 9 CDs, of which 4 are by the Beatles, 3 are by Abba and 2 are by the Rolling Stones. She selects 4 of the CDs from her collection. Calculate the number of ways in which she can make her selection if
- (i) her selection must contain her favourite Beatles CD, [2]
- (ii) her selection must contain 2 CDs by one group and 2 CDs by another. [3]
- 3 Given that  $\theta$  is acute and that  $\sin\theta = \frac{1}{\sqrt{3}}$ , express, without using a calculator,  $\frac{\sin\theta}{\cos\theta - \sin\theta}$  in the form  $a + \sqrt{b}$ , where  $a$  and  $b$  are integers. [5]
- 4 The position vectors of points  $A$  and  $B$  relative to an origin  $O$  are  $-3\mathbf{i} - \mathbf{j}$  and  $\mathbf{i} + 2\mathbf{j}$  respectively. The point  $C$  lies on  $AB$  and is such that  $\vec{AC} = \frac{3}{5}\vec{AB}$ . Find the position vector of  $C$  and show that it is a unit vector. [6]
- 5 The function  $f$  is defined, for  $0^\circ \leq x \leq 180^\circ$ , by
- $$f(x) = A + 5 \cos Bx,$$
- where  $A$  and  $B$  are constants.
- (i) Given that the maximum value of  $f$  is 3, state the value of  $A$ . [1]
- (ii) State the amplitude of  $f$ . [1]
- (iii) Given that the period of  $f$  is  $120^\circ$ , state the value of  $B$ . [1]
- (iv) Sketch the graph of  $f$ . [3]
- 6 Given that each of the following functions is defined for the domain  $-2 \leq x \leq 3$ , find the range of
- (i)  $f: x \mapsto 2 - 3x$ , [1]
- (ii)  $g: x \mapsto |2 - 3x|$ , [2]
- (iii)  $h: x \mapsto 2 - |3x|$ . [2]
- State which of the functions  $f$ ,  $g$  and  $h$  has an inverse. [2]

- 7 (a) Variables  $l$  and  $t$  are related by the equation  $l = l_0(1 + \alpha)^t$  where  $l_0$  and  $\alpha$  are constants.

Given that  $l_0 = 0.64$  and  $\alpha = 2.5 \times 10^{-3}$ , find the value of  $t$  for which  $l = 0.66$ .

- (b) Solve the equation  $1 + \lg(8 - x) = \lg(3x + 2)$ . [4]

8

$x$	10	100	1000	10 000
$y$	1900	250	31	4

The table above shows experimental values of the variables  $x$  and  $y$  which are related by an equation of the form  $y = kx^n$ , where  $k$  and  $n$  are constants.

- (i) Using graph paper, draw the graph of  $\lg y$  against  $\lg x$ . [3]

- (ii) Use your graph to estimate the value of  $k$  and of  $n$ . [4]

- 9 (i) Determine the set of values of  $k$  for which the equation

$$x^2 + 2x + k = 3kx - 1$$

has no real roots. [5]

- (ii) Hence state, giving a reason, what can be deduced about the curve  $y = (x + 1)^2$  and the line  $y = 3x - 1$ . [2]

- 10 The remainder when  $2x^3 + 2x^2 - 13x + 12$  is divided by  $x + a$  is three times the remainder when it is divided by  $x - a$ .

- (i) Show that  $2a^3 + a^2 - 13a + 6 = 0$ . [3]

- (ii) Solve this equation completely. [5]

- 11 A particle travels in a straight line so that,  $t$  seconds after passing a fixed point  $A$  on the line, its acceleration,  $a \text{ ms}^{-2}$ , is given by  $a = -2 - 2t$ . It comes to rest at a point  $B$  when  $t = 4$ .

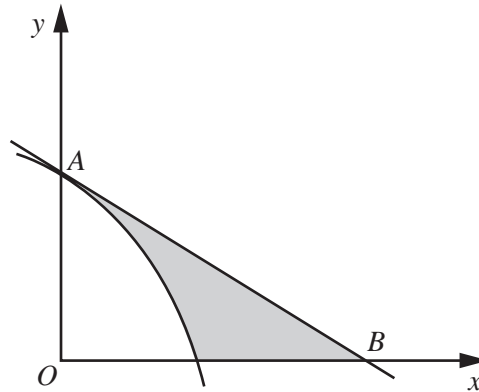
- (i) Find the velocity of the particle at  $A$ . [4]

- (ii) Find the distance  $AB$ . [3]

- (iii) Sketch the velocity-time graph for the motion from  $A$  to  $B$ . [1]

12 Answer only **one** of the following two alternatives.

**EITHER**



The diagram, which is not drawn to scale, shows part of the graph of  $y = 8 - e^{2x}$ , crossing the  $y$ -axis at  $A$ . The tangent to the curve at  $A$  crosses the  $x$ -axis at  $B$ . Find the area of the shaded region bounded by the curve, the tangent and the  $x$ -axis. [10]

**OR**

A piece of wire, of length 2 m, is divided into two pieces. One piece is bent to form a square of side  $x$  m and the other is bent to form a circle of radius  $r$  m.

(i) Express  $r$  in terms of  $x$  and show that the total area,  $A$  m<sup>2</sup>, of the two shapes is given by

$$A = \frac{(\pi + 4)x^2 - 4x + 1}{\pi}. \quad [4]$$

Given that  $x$  can vary, find

(ii) the stationary value of  $A$ , [4]

(iii) the nature of this stationary value. [2]





