UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS **4037/02**

Paper 2

May/June 2006

2 hours

Additional Materials: Answer Booklet/Paper
Mathematical tables

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Booklet/Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.
At the end of the examination, fasten all your work securely together.

This document consists of **6** printed pages and **2** blank pages.
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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

1 Variables x and y are connected by the equation $y = (3x - 1)\ln x$. Given that x is increas e of 3 units per second, find the rate of increase of y when $x = 1$.

2 The table shows the number of games played and the results of five teams in a football league.

	Played	Won	Drawn	Lost
Parrots	8	5	3	0
Quails	7	4	1	2
Robins	8	4	0	4
Swallows	7	2	1	4
Terns	8	1	1	6

A win earns 3 points, a draw 1 point and a loss 0 points. Write down two matrices which on multiplication display in their product the total number of points earned by each team and hence calculate these totals. [4]

3 The points A and B are such that the unit vector in the direction of $\overrightarrow{AB}$ is $0.28\mathbf{i} + p\mathbf{j}$, where p is a positive constant.

(i) Find the value of p. [2]

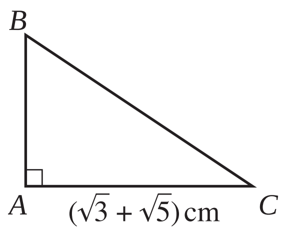
The position vectors of A and B, relative to an origin O, are $q\mathbf{i} - 7\mathbf{j}$ and $12\mathbf{i} + 17\mathbf{j}$ respectively.

(ii) Find the value of the constant q. [3]

4 **(a)** Differentiate $e^{\tan x}$ with respect to x. [2]

(b) Evaluate $\int_0^{\frac{1}{2}} e^{1-2x}\,dx$. [4]

5

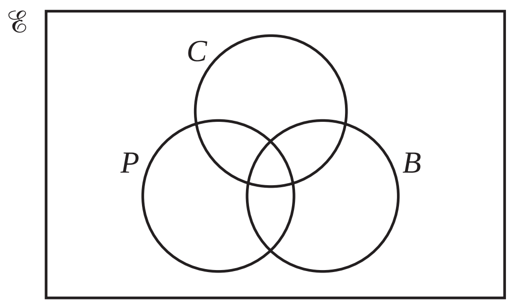


The diagram shows a right-angled triangle ABC in which the length of AC is $(\sqrt{3} + \sqrt{5})$ cm. The area of triangle ABC is $(1 + \sqrt{15})$ cm^2.

(i) Find the length of AB in the form $(a\sqrt{3} + b\sqrt{5})$ cm, where a and b are integers. [3]

(ii) Express $(BC)^2$ in the form $(c + d\sqrt{15})$ cm^2, where c and d are integers. [3]

6 **(a)**



The Venn diagram above represents the universal set ℰ of all teachers in a college. The sets *C*, *B* and *P* represent teachers who teach Chemistry, Biology and Physics respectively. Sketch the diagram twice.

(i) On the first diagram shade the region which represents those teachers who teach Physics and Chemistry but not Biology. [1]

(ii) On the second diagram shade the region which represents those teachers who teach either Biology or Chemistry or both, but not Physics. [1]

(b) In a group of 20 language teachers, *F* is the set of teachers who teach French and *S* is the set of teachers who teach Spanish. Given that $n(F) = 16$ and $n(S) = 10$, state the maximum and minimum possible values of

(i) $n(F \cap S)$,

(ii) $n(F \cup S)$.

[4]

7 **(a)** 7 boys are to be seated in a row. Calculate the number of different ways in which this can be done if 2 particular boys, Andrew and Brian, have exactly 3 of the other boys between them. [4]

(b) A box contains sweets of 6 different flavours. There are at least 2 sweets of each flavour. A girl selects 3 sweets from the box. Given that these 3 sweets are not all of the same flavour, calculate the number of different ways she can select her 3 sweets. [3]

8 **(i)** In the binomial expansion of $\left(x + \frac{k}{x^3}\right)^8$, where *k* is a positive constant, the term independent of *x* is 252.

Evaluate *k*. [4]

(ii) Using your value of *k*, find the coefficient of x^4 in the expansion of $\left(1 - \frac{x^4}{4}\right)\left(x + \frac{k}{x^3}\right)^8$. [3]

9 A cuboid has a total surface area of $120\,\text{cm}^2$. Its base measures x cm by $2x$ cm and its heig

(i) Obtain an expression for h in terms of x.

Given that the volume of the cuboid is $V\,\text{cm}^3$,

(ii) show that $V = 40x - \dfrac{4x^3}{3}$. [1]

Given that x can vary,

(iii) show that V has a stationary value when $h = \dfrac{4x}{3}$. [4]

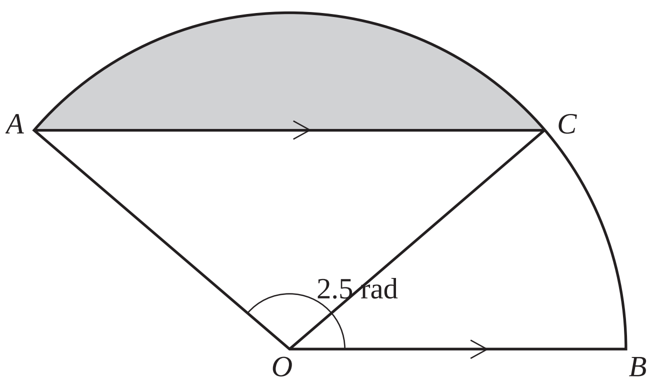
10 **(a)** Given that $a = \sec x + \operatorname{cosec} x$ and $b = \sec x - \operatorname{cosec} x$, show that

$$a^2 + b^2 \equiv 2\sec^2 x \operatorname{cosec}^2 x.$$ [4]

(b) Find, correct to 2 decimal places, the values of y between 0 and 6 radians which satisfy the equation

$$2\cot y = 3\sin y.$$ [5]

11



The diagram shows a sector $OACB$ of a circle, centre O, in which angle $AOB = 2.5$ radians. The line AC is parallel to OB.

(i) Show that angle $AOC = (5 - \pi)$ radians. [3]

Given that the radius of the circle is 12 cm, find

(ii) the area of the shaded region, [3]

(iii) the perimeter of the shaded region. [3]

12 Answer only **one** of the following two alternatives.

EITHER

(i) Express $2x^2 - 8x + 3$ in the form $a(x + b)^2 + c$, where a, b and c are integers. [

A function f is defined by $\mathrm{f} : x \mapsto 2x^2 - 8x + 3,\ x \in \mathbb{R}$.

(ii) Find the coordinates of the stationary point on the graph of $y = \mathrm{f}(x)$. [2]

(iii) Find the value of $\mathrm{f}^2(0)$. [2]

A function g is defined by $\mathrm{g} : x \mapsto 2x^2 - 8x + 3,\ x \in \mathbb{R}$, where $x \leqslant N$.

(iv) State the greatest value of N for which g has an inverse. [1]

(v) Using the result obtained in part **(i)**, find an expression for g^{-1}. [3]

OR

The equation of a curve is $y = 10 - x^2 + 6x$.

(i) Find the set of values of x for which $y \geqslant 15$. [3]

(ii) Express y in the form $a - (x + b)^2$, where a and b are integers. [2]

(iii) Hence, or otherwise, find the coordinates of the stationary point on the curve. [2]

Functions f and g are defined, for $x \in \mathbb{R}$, by

$\mathrm{f} : x \mapsto 10 - x^2 + 6x$,

$\mathrm{g} : x \mapsto 2x - k$, where k is a constant.

(iv) Find the value of k for which the equation $\mathrm{gf}(x) = 0$ has two equal roots. [3]

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