UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the May/June 2008 question paper

4037 ADDITIONAL MATHEMATICS

4037/01 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the May/June 2008 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW -1,2 This is deducted from A or B marks when essential working is omitted.

PA -1 This is deducted from A or B marks in the case of premature approximation.

S -1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX -1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1	$\frac{8-3\sqrt{2}}{4+3\sqrt{2}}\frac{\left(4-3\sqrt{2}\right)}{\left(4-3\sqrt{2}\right)}$ $\frac{32-12\sqrt{2}-24\sqrt{2}+18}{16-18}$ $\frac{50-36\sqrt{2}}{-2}$ $a = -25,\ b = 18$	M1 DM1 A1 [3]	M1 for attempt to rationalise DM1 for attempt to expand out and simplify Allow A1 at this stage
2	**(i)** ${}^{10}C_5 = 252$	B1 [1]	
	(ii) 4 women, 1 man: 6 3 women, 2 men: ${}^4C_3 \times {}^6C_2$ $= 60$ Total = 66	M1 B1 B1 A1 [4]	M1 for a plan B1 for 6 B1 for 60 A1 for total Allow marks for other valid methods
3	**(i)** $4x^2 + kx + 16 = 0$ $\left(4y^2 - 5ky + \left(k^2 + 144\right) = 0\right)$ $b^2 = 4ac,\ \ k^2 = 256,\ \ k = \pm 16$	M1 DM1, A1 [3]	M1 for attempt to get a quadratic in terms of one variable DM1 for use of $b^2 - 4ac$ A1 for both
	(ii) using $x = -\frac{b}{2a}$, or equivalent When $k = -16$, $(2, -10)$ When $k = 16$, $(-2, 10)$	B1 B1 [2]	B1 for each pair Allow B1 for x values only
4	**(i)** gradient = 2, equation of line of form $Y = mX + c$, where $c = 0.6$ $\therefore e^y = 0.6$	M1 A1 [2]	M1 for attempt to get equation of straight line
	(ii) $e^y = 2x^2 + 0.6$	A1	A1 for correct form (allow if seen in **(i)**)
	$\therefore y = \ln(2x^2 + 0.6)$	M1 A1 [3]	M1 for attempt to take ln

5	$\frac{dy}{dx} = \frac{\tan x - x\sec^2 x}{\tan^2 x}$ When $x = \frac{\pi}{4}, \frac{dy}{dx} = 1 - \frac{\pi}{2}$ Using $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$, $\frac{dy}{dt} = 2 - \pi$ (-1.14)	M1 A1 M1 M1 A1 [5]	M1 for correct attempt to differentiate a quotient A1 all correct M1 for attempt to sub $x = \frac{\pi}{4}$ in to their $\frac{dy}{dx}$ M1 for attempt to use rates of change
6	$2x^3 + 3x^2 - 17x + 12 = 0$ f(1) = 0, $(x - 1)$ is a factor $(x-1)(2x^2 + 5x - 12) = 0$ $(x-1)(2x-3)(x+34) = 0$ $x = 1, \frac{3}{2}, -4$	M1 M1 M1 DM1 B1,A1 [6]	M1 for simplification M1 for attempt to find a root M1 for attempt to get quadratic factor DM1 for factorising on all previous M marks B1 for solution from first root A1 for the other pair
7	**(i)** $\frac{1}{2}4^2\theta = 10$, leading to $\theta = 1.25$ rads **(ii)** $AB = 5$ $AC = 4\tan 1.25$, $AC = 12.038$ $BC = \frac{4}{\cos 1.25} - 4$, $BC = 8.685$ Perimeter = 25.7, allow 25.8	M1 A1 [2] B1 M1 M1 A1 [4]	M1 for use of $\frac{1}{2}r^2\theta$ M1 for attempt to get AC M1 for attempt to get BC
8	**(i)** $a = \frac{1}{2}$ **(ii)** $b = \frac{1}{3}$ (allow 0.33 or better) **(iii)** $3\log_3 x + \log_3 y = 8$ $\log_3 x + \log_3 y = 2$ $\log_3 x = 3$, $x = 27$ $\log_3 y = -1$, $y = \frac{1}{3}$ Allow solutions using index notation	B1 [1] B1 [1] M1 DM1 A1 A1 [4]	 M1 for reducing equations to terms of base 3 logs DM1 for dealing with simultaneous equations and logs to get final answers A1 for each

9	**(i)** $y = \sin\left(2x - \frac{\pi}{2}\right) + c$	M1 A1	M1 for $\sin\left(2x - \frac{\pi}{2}\right)$ A1 correct
	$c = 2$	M1, A1 [4]	M1 for attempt to get c Allow A1 for $c = 2$
	(ii) at $x = \frac{3\pi}{4}, \frac{dy}{dx} = -2$ Grad of normal = $\frac{1}{2}$	M1	M1 for attempt to get $\frac{dy}{dx}$ and for $\perp$ gradient
	When $x = \frac{3\pi}{4}, y = 2$	M1	M1 for attempt to obtain y using $x = \frac{3\pi}{4}$ in answer to **(i)**
	normal $y - 2 = \frac{1}{2}\left(x - \frac{3\pi}{4}\right)$	M1, A1 [4]	M1 for attempt to obtain normal, must be using $\perp$ gradient – allow unsimplified
10	**(i)** $\mathbf{v} = 15\sqrt{2}\frac{(\mathbf{i} + \mathbf{j})}{\sqrt{2}}$	M1	M1 for attempt to get a direction vector
	$\mathbf{v} = 15\mathbf{i} + 15\mathbf{j}$	A1 [2]	
	(ii) $(2\mathbf{i} + 3\mathbf{j}) + (15\mathbf{i} + 15\mathbf{j})1.5$ $24.5\mathbf{i} + 25.5\mathbf{j}$	B1 [1]	Answer given
	(iii) $(2 + 15t)\mathbf{i} + (3 + 15t)\mathbf{j}$ Allow $(2\mathbf{i} + 3\mathbf{j}) + (15\mathbf{i} + 15\mathbf{j})t$	M1, √A1 [2]	M1 for use of their velocity vector with $2\mathbf{i} + 3\mathbf{j}$. Follow through on their velocity vector
	(iv) relative velocity $(15\mathbf{i} + 15\mathbf{j}) - 25\mathbf{j} = 15\mathbf{i} - 10\mathbf{j}$	M1, A1 [2]	M1 for a difference of velocities
	(v) relative displacement $(47\mathbf{i} - 27\mathbf{j}) - (2\mathbf{i} + 3\mathbf{j}) = 45\mathbf{i} - 30\mathbf{j}$ Time taken = 3 hours Position vector at interception $47\mathbf{i} + 48\mathbf{j}$	M1 A1 [2]	M1 for attempt to get relative displacement or other valid method.
	or $2\mathbf{i} + 3\mathbf{j} + (15\mathbf{i} + 15\mathbf{j})t = (47\mathbf{i} - 27\mathbf{j}) + 25t$ or equivalent Allow solutions to **(v)** by drawing		M1 for equating like vectors and attempt to get t

11 **(i)** $\tan x = -\frac{5}{3}$	M1	M1 for use of tan and attempt at one solution
$x = 121.0^\circ, 301.0^\circ$	A1, √A1 [3]	A1 for each, √ on first solution for x
(ii) $3\sec^2 y - \sec y - 4 = 0$	M1	M1 for use of correct identity and formation of a 3 term quadratic in one variable.
$(3\sec y - 4)(\sec y + 1) = 0$	M1	M1 for factorising a 3 term quadratic
$\cos y = \frac{3}{4}, -1$	M1	M1 for all terms in terms of cos
$y = 41.4^\circ, 318.6^\circ, 180^\circ$	B1, A1 [5]	B1 for 180^0, A1 for the other pair
(iii) $2z - 0.6 = 0.9273, 2.2143$	M1 M1	M1 for correct order of operations M1 for a valid attempt at a second solution
z = 0.764, 1.407 (allow 1.41)	A1, A1 [4]	A1 for each
12 **EITHER**		
(i) $(\pm\sqrt{3}, 0)$ allow	B1, B1 [2]	
(ii) $\frac{dy}{dx} = -(x^2 - 3)e^{-x} + e^{-x}2x$ $= e^{-x}(2x - x^2 + 3)$	M1, A1	M1 for a correct attempt to differentiate a product or a quotient A1 allow unsimplified
$\frac{dy}{dx} = 0$, $x^2 - 2x - 3 = 0$ leading to x = 3, –1 and $y = 6e^{-3}$(0.299), –2e (5.44)	M1 A1 A1 [5]	M1 for attempting to solve $\frac{dy}{dx} = 0$ A1 for each pair
(iii) $\frac{d^2y}{dx^2} = e^{-x}(2 - 2x) - e^{-x}(2x - x^2 + 3)$	M1	M1 for attempt at second differential or use of gradient method
When x = 3, $\frac{d^2y}{dx^2}$ is –ve, max	B1	B1 for each
When x = –1, $\frac{d^2y}{dx^2}$ is +ve, min	B1 [3]	

12 OR		
(i) $v = \dfrac{1}{t+1}, v_0 = 1$	M1, A1 [2]	M1 for attempt to differentiate
(ii) $v = \dfrac{1}{2(t-2)} - \dfrac{1}{t+1}$	M1	M1 for attempt to differentiate
$v_4 = \dfrac{1}{4} - \dfrac{1}{5}$; $v_4 = \dfrac{1}{20}$ (0.05)	A1 [2]	
(iii) $a = -\dfrac{1}{2(t-2)^2} + \dfrac{1}{(t+1)^2}$; $a_4 = -\dfrac{17}{200}$ (–0.085)	M1, A1 [2]	M1 for attempt to differentiate
(iv) $\dfrac{1}{2(t-2)} - \dfrac{1}{t+1} = 0$, $t = 5$	DM1, A1 [2]	DM1 for equating v to zero
(v) $s_3 = \ln 4$ (1.386)	M1	M1 for attempt to find s_3 and s_4
$s_4 = \ln \dfrac{16\sqrt{2}}{5}$ (1.509)		
In 4th sec, $s = \ln \dfrac{4\sqrt{2}}{5}$ (0.123) (allow 0.124)	A1 [2]	