UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS **4037/02**

Paper 2 **May/June 2009**

2 hours

Additional Materials: Answer Paper
Graph paper (1 sheet)

9762991949

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **6** printed pages and **2** blank pages.

SP (SC) T70863/4

UNIVERSITY *of* CAMBRIDGE
International Examinations

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

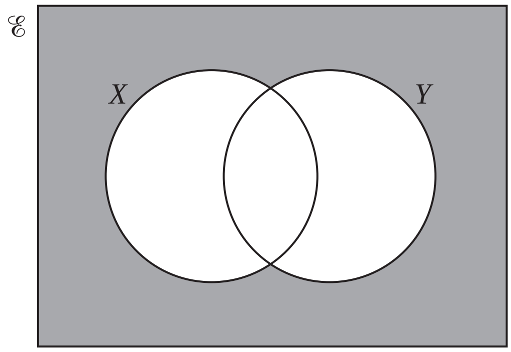
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} .$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

1 **(a)**



Express, in set notation, the set represented by the shaded region. [1]

(b) In a class of 30 students, 17 are studying politics, 14 are studying economics and 10 are studying both of these subjects.

(i) Illustrate this information using a Venn diagram. [1]

Find the number of students studying

(ii) neither of these subjects, [1]

(iii) exactly one of these subjects. [1]

2 Given that $\mathbf{A} = \begin{pmatrix} 7 & 6 \\ 3 & 4 \end{pmatrix}$, find $\mathbf{A}^{-1}$ and hence solve the simultaneous equations

$$7x + 6y = 17,$$
$$3x + 4y = 3.$$

[4]

3 Sketch the graph of $y = |x^2 - 8x + 12|$. [4]

4 Find the coefficient of x^4 in the expansion of

(i) $(1 + 2x)^6$, [2]

(ii) $\left(1 - \frac{x}{4}\right)(1 + 2x)^6$. [3]

5 Two variables, x and y, are related by the equation

$$y = 6x^2 + \frac{32}{x^3}.$$

(i) Obtain an expression for $\frac{dy}{dx}$. [2]

(ii) Use your expression to find the approximate change in the value of y when x increases from 2 to 2.04. [3]

6 The function f is defined by $f(x) = 2 + \sqrt{x - 3}$ for $x \geqslant 3$. Find

(i) the range of f, [1]

(ii) an expression for $f^{-1}(x)$. [2]

The function g is defined by $g(x) = \frac{12}{x} + 2$ for $x > 0$. Find

(iii) gf(12). [2]

7 Given that $\log_p X = 9$ and $\log_p Y = 6$, find

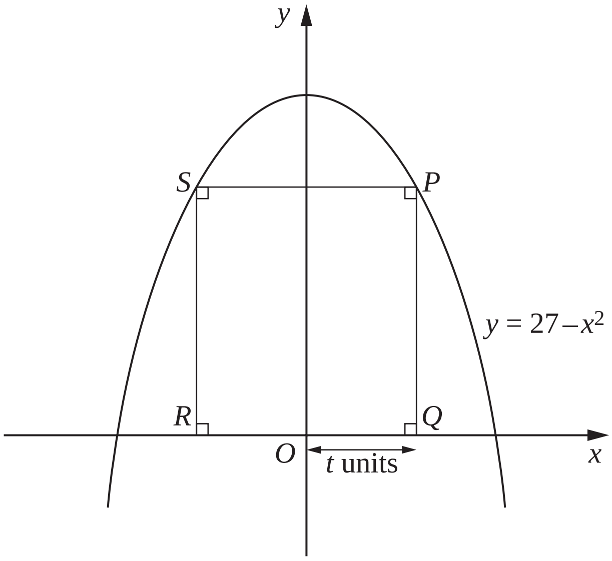
(i) $\log_p \sqrt{X}$, [1]

(ii) $\log_p \left(\frac{1}{X}\right)$, [1]

(iii) $\log_p (XY)$, [2]

(iv) $\log_Y X$. [2]

8



The diagram shows part of the curve $y = 27 - x^2$. The points P and S lie on this curve. The points Q and R lie on the x-axis and $PQRS$ is a rectangle. The length of OQ is t units.

(i) Find the length of PQ in terms of t and hence show that the area, A square units, of $PQRS$ is given by

$$A = 54t - 2t^3.$$ [2]

(ii) Given that t can vary, find the value of t for which A has a stationary value. [3]

(iii) Find this stationary value of A and determine its nature. [3]

9 A musician has to play 4 pieces from a list of 9. Of these 9 pieces 4 were written by Beethoven, 3 by Handel and 2 by Sibelius. Calculate the number of ways the 4 pieces can be chosen if

(i) there are no restrictions, [2]

(ii) there must be 2 pieces by Beethoven, 1 by Handel and 1 by Sibelius, [3]

(iii) there must be at least one piece by each composer. [4]

10 The line $2x + y = 12$ intersects the curve $x^2 + 3xy + y^2 = 176$ at the points A and B. Find the equation of the perpendicular bisector of AB. [9]

11 **(a)** Find all the angles between 0° and 360° which satisfy

(i) $2\sin x - 3\cos x = 0$, [3]

(ii) $2\sin^2 y - 3\cos y = 0$. [5]

(b) Given that $0 \leqslant z \leqslant 3$ radians, find, correct to 2 decimal places, all the values of z for which $\sin(2z + 1) = 0.9$. [3]

12 Answer only **one** of the following two alternatives.

EITHER

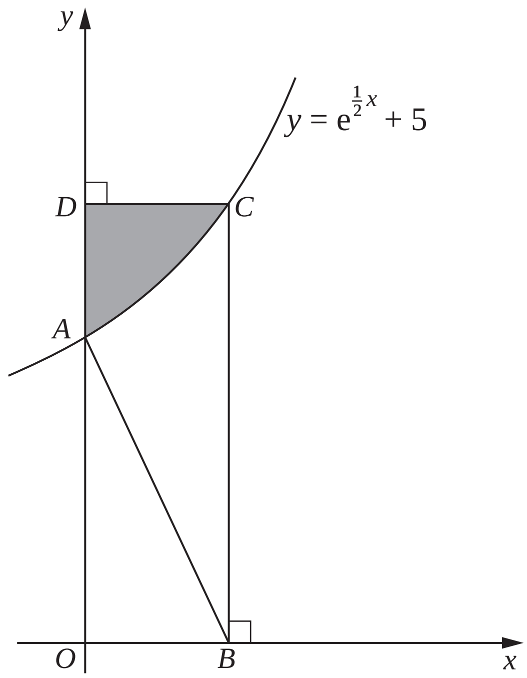
The point $P(0, 5)$ lies on the curve for which $\frac{dy}{dx} = e^{\frac{1}{2}x}$. The point Q, with x-coordinate 2, also lies on the curve.

(i) Find, in terms of e, the y-coordinate of Q. [5]

The tangents to the curve at the points P and Q intersect at the point R.

(ii) Find, in terms of e, the x-coordinate of R. [5]

OR



The diagram shows part of the curve $y = e^{\frac{1}{2}x} + 5$ crossing the y-axis at A. The normal to the curve at A meets the x-axis at B.

(i) Find the coordinates of B. [4]

The line through B, parallel to the y-axis, meets the curve at C. The line through C, parallel to the x-axis, meets the y-axis at D.

(ii) Find the area of the shaded region. [6]

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