CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

# MARK SCHEME for the May/June 2013 series

## 4037 ADDITIONAL MATHEMATICS

**4037/12** Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

**Mark Scheme Notes**

Marks are of the following three types:

M  Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A  Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B  Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1,2 This is deducted from A or B marks when essential working is omitted.

PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

| | | | |
|---|---|---|---|
| 1 (i) | $n(A \cap B) = 5$ | B1 | |
| (ii) | $n(A) = 16$ | B1 | |
| (iii) | $n(B' \cap A)$ | B1 | |
| 2 (i) | $6 \times 5 \times 4 \times 3 = 360$ or ${}^6P_4 = 360$ | B1 | B1 unsimplified/evaluated |
| (ii) | Position: 1, 2, 3, 4; Number of ways: 5, 4, 3, 1 | | |
| | or $\frac{1}{6}$ **(i)** or ${}^5P_3$ or ${}^5C_3 \times {}^6C_1$ | M1 | M1 for a correct attempt unsimplified |
| | Number of 4 digit numbers = 60 | A1 | |
| (iii) | Position: 1, 2, 3, 4; Number of ways: 3, 4, 3, 1 | | |
| | or ${}^3P_1 \times {}^4P_2$ | M1 | M1 for a correct attempt unsimplified |
| | Number of 4 digit numbers = 36 | A1 | |
| 3 | **EITHER** | | |
| | $1 - 2\sin\theta - 2\cos\theta + \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$ | B1 | **B1** for correct expansion of $(1 - \cos\theta - \sin\theta)^2$ |
| | Use of $\sin^2\theta + \cos^2\theta = 1$ in simplification = 0 | M1 | **M1** for use of $\sin^2\theta + \cos^2\theta = 1$ in this form |
| | | A1 | **A1** must be convinced as AG |
| | **OR** $(1 - \cos\theta - \sin\theta)^2 =$ $1 - 2\sin\theta - 2\cos\theta + \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$ | [B1 | **B1** for correct expansion of $(1 - \cos\theta - \sin\theta)^2$ |
| | $= 2 - 2\sin\theta - 2\cos\theta + 2\sin\theta\cos\theta$ | M1 | **M1** for use of $\sin^2\theta + \cos^2\theta = 1$ in this form |
| | $= 2(1 - \sin\theta)(1 - \cos\theta)$ | A1] | **A1** for simplification and factorising |

Table for 2 (ii):

| Position | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Number of ways | 5 | 4 | 3 | 1 |

Table for 2 (iii):

| Position | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Number of ways | 3 | 4 | 3 | 1 |

| | | | |
|---|---|---|---|
| 4 | **EITHER**<br>$2x^2 + kx + 2k - 6 = 0$ has no real roots<br>$k^2 - 16k + 48 < 0$<br>$(k - 4)(k - 12) < 0$ | **M1**<br>**DM1** | **M1** for attempted use of $b^2$ –<br>**DM1** for attempt to obtain critic<br>values from a 3 term quadratic |
| | Critical values 4 and 12<br>$4 < k < 12$ or $k > 4$ **and** $k < 12$ | **A1**<br>**A1** | **A1** for both critical values<br>**A1** for correct final answer |
| | **OR** $\left(x + \frac{k}{4}\right)^2 - \frac{k^2}{16} + k - 3 = 0$ | **[M1]** | **M1** for attempting to complete the square and obtain a 3 term quadratic |
| | $-\frac{k^2}{16} + k - 3 > 0$ so $k^2 - 16k + 48 < 0$ | | Then as **EITHER** |
| | **OR** $\frac{dy}{dx} = 4x + k$ | **[M1** | **M1** for differentiation, equating to zero and obtaining a quadratic equation in $x$ |
| | When $\frac{dy}{dx} = 0$, $k = -4x$<br>By substitution $x^2 + 4x + 3 < 0$<br>leading to $x = -1$, $k = 4$ | **DM1** | **DM1** for attempt to obtain critical values of $k$ from a 3 term quadratic in $x$ followed by substitution to obtain a value for $k$ |
| | and $x = -3$, $k = 12$<br>$4 < k < 12$ or $k > 4$ **and** $k < 12$ | **A1**<br>**A1]** | **A1** for both critical values<br>**A1** for correct final answer |
| | **OR** $\frac{dy}{dx} = 4x + k$ | **[M1]** | **M1** for differentiation, equating to zero and obtaining a quadratic equation in $k$ |
| | When $\frac{dy}{dx} = 0$, $x = -\frac{k}{4}$<br>leading to $k^2 - 16k + 48 < 0$ | | Then as **EITHER** |

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| 5 | $2\left(\frac{15-4y}{3}\right)y = 9$ or $2x\left(\frac{15-3x}{4}\right) = 9$ | M1 | **M1** for attempt to obtain equation in one variable |
| | $8y^2 - 30y + 27 = 0$ or $3x^2 - 15x + 18 = 0$<br>$(4y - 9)(2y - 3) = 0$ or $(x - 3)(x - 2) = 0$ | DM1 | **DM1** for attempt to solve a 3 term quadratic in that variable |
| | $x = 2, y = \frac{9}{4}$ and $x = 3, y = \frac{3}{2}$ | A1, A1 | **A1** for each 'pair', $x$ values must be simplified to single integer form |
| | $AB^2 = 1^2 + (0.75)^2$, $AB = 1.25$ | M1, A1 | **M1** for a correct attempt to find $AB$, must have non zero differences and be using points calculated previously. |
| 6 | $\frac{dy}{dx} = 3\sec^2 x$ | B1 | **B1** for $3\sec^2 x$ |
| | When $x = \frac{3\pi}{4}$, $\frac{dy}{dx} = 6$ | B1 | **B1** for $\frac{dy}{dx} = 6$, may be implied by later work |
| | $y = 5$ | B1 | **B1** for $y$ |
| | Perpendicular gradient $= -\frac{1}{6}$ | M1 | **M1** for perpendicular gradient from $\frac{dy}{dx}$ |
| | Equation of normal $y + 5 = -\frac{1}{6}\left(x - \frac{3\pi}{4}\right)$ | M1 | **M1** for attempt at the normal using *their* $y$ value correctly and $x = \frac{3\pi}{4}$ and substitution of $x = 0$ |
| | When $x = 0$, $y = \frac{\pi}{8} - 5$ o.e.<br>or –4.61 or –4.6 but not –4.60 | A1 | **A1** for obtaining $y$ value |

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| 7 (i) | f (–2) leads to $68 = b - 2a$ | M1 | attempt at f (–2) = 0<br>allow unsimplified |
| | f(1) leads to $26 = a + b$ | M1 | attempt at f (1) = 27<br>allow unsimplified |
| | $a = -14$, $b = 40$ | A1, B1 | **A1** for $b = 40$, **B1** for $a = -14$ |
| (ii) | $f(x) = (x + 2)(6x^2 - 17x + 20)$ | B2, 1, 0 | –1 each error |
| (iii) | $6x^2 - 17x + 20 = 0$ has no real roots | B1 | **B1** for dealing with quadratic factor either by use of formula, completing the square or use of $b^2 - 4ac$ to show that there are no real solutions |
| | $x = -2$ | B1 | |
| 8 (a) (i) | $\begin{pmatrix} 22 & -2 \\ -3 & 31 \end{pmatrix}$ | B2, 1, 0 | –1 each element error |
| (ii) | $\begin{pmatrix} 16 & 6 \\ 9 & -11 \end{pmatrix}$ | B2, 1, 0 | –1 each element error |
| (b) (i) | $\frac{1}{18+9}\begin{pmatrix} 3 & -1 \\ 9 & 6 \end{pmatrix}$ | B1, B1 | **B1** for $\frac{1}{\text{determinant}}$ (allow unsimplified),<br>**B1** for matrix |
| (ii) | $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{27}\begin{pmatrix} 3 & -1 \\ 9 & 6 \end{pmatrix}\begin{pmatrix} 5 \\ 1.5 \end{pmatrix}$,<br>$= \frac{1}{27}\begin{pmatrix} 13.5 \\ 54 \end{pmatrix}$ | M1 | **M1** for correct use of inverse matrix, including correct multiplication to solve equation |
| | $x = 0.5$, $y = 2$ | A1, A1 | **A1** for each |

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| 9 (i) | $\left(1+\frac{1}{2}x\right)^n = 1 + n\left(\frac{x}{2}\right) + \frac{n(n-1)}{2}\left(\frac{x}{2}\right)^2$ | B1, B1 | **B1** for 1 + second term, **B1** for 3rd term<br>Allow unsimplified |
| (ii) | $(1-x)\left(1 + n\left(\frac{x}{2}\right) + \frac{n(n-1)}{2}\left(\frac{x}{2}\right)^2\right)$ | M1 | dealing with 2 terms involving $x^2$ |
| | Multiply $x$ and $\frac{n}{2}x$ to get $\frac{n}{2}(x^2)$ | DM1 | attempt to obtain one term |
| | Multiply 1 and $\frac{n(n-1)x^2}{8}$ or $\frac{n(n-1)x^2}{4}$ | DM1 | attempt to obtain a second term |
| | $\frac{n^2-n}{8} - \frac{n}{2} = \frac{25}{4}$ | | |
| | $n^2 - 5n - 50 = 0$ | A1 | correct quadratic equation |
| | $n = 10$ | A1 | **A1** for $n = 10$ only |
| 10 (a) (i) | $\frac{1}{3}(2x-5)^{\frac{3}{2}}$ | B1, B1 | **B1** for $k(2x-5)^{\frac{3}{2}}$, **B1** for $\frac{1}{3}(2x-5)^{\frac{3}{2}}$ |
| (ii) | $\frac{125}{3} - \frac{1}{3} = \frac{124}{3}$<br>Allow awrt 41.3 | M1, A1 | **M1** for correct use of limits |
| (b) (i) | $x^3\frac{1}{x} + 3x^2\ln x$ | B1, B1 | **B1** for each term, allow unsimplified |
| (ii) | $\int 3x^2 \ln x\,dx = x^3\ln x - \int x^2 dx$ o.e. | M1 | for a use of answer to **(i)** |
| | $\int x^2 dx = \frac{x^3}{3}$ **or** | A1 | **A1** for intergrating $x^2$ or dividing by 3 |
| | $\int x^2 \ln x\,dx = \frac{1}{3}\left(x^3\ln x - \int x^2 dx\right)$ o.e. | | |
| | $\int x^2 \ln x\,dx = \frac{1}{3}\left(x^3\ln x - \frac{x^3}{3}\right)$ $(+c)$ | A1 | |

| | | | |
|---|---|---|---|
| **11 (a)** | $\cos 2x + \dfrac{2}{\cos 2x} + 3 = 0$ | **M1** | dealing with sec or cos |
| | leading to $\cos^2 2x + 3\cos 2x + 2 = 0$<br>$2\sec^2 2x + 3\sec 2x + 1 = 0$ | **A1** | simplification to correct 3 term quadratic in $\sec 2x$ or $\cos 2x$ (does not have to be equated to zero) |
| | $(\cos 2x + 2)(\cos 2x + 1) = 0$<br>or $(2\sec 2x + 1)(\sec 2x + 1) = 0$ | **M1** | attempt to solve a 3 term quadratic, must obtain solutions in terms of $\cos 2x$ |
| | leading to $\cos 2x = -1$ or $\sec 2x = -1$ only<br>$2x = 180°, 540°$<br>$x = 90°, 270°$ | **A1, A1** | |
| **(b)** | $\sin^2\left(y - \dfrac{\pi}{6}\right) = \dfrac{1}{2}$ so | | |
| | $\sin\left(y - \dfrac{\pi}{6}\right) = \dfrac{1}{\sqrt{2}}$ | **M1** | division by 2 and square root |
| | $\left(y - \dfrac{\pi}{6}\right) = \dfrac{\pi}{4}, \dfrac{3\pi}{4}$ | **DM1** | correct order of operation and attempt to solve |
| | $y = \dfrac{5\pi}{12}, \dfrac{11\pi}{12}$<br>Allow awrt 1.31, 2.88 | **A1, A1** | |
| **12 (i)** | $\dfrac{dy}{dt} = 36 - 6t$ | **M1** | attempt to differentiate and equate to zero |
| | When $\dfrac{dy}{dt} = 0$, $t = 6$ | **A1** | |
| **(ii)** | When $v = 0$, $t = 12$ | **M1, A1** | **M1** for equating $v$ to zero and attempt to solve |
| **(iii)** | $s = 18t^2 - t^3\ (+c)$ | **M1, A1** | **M1** for a correct attempt to integrate at least one term, allow unsimplified<br>**A1** for all correct |
| | When $t = 12$, $s = 864$ | | **A1** for $s = 864$ |
| **(iv)** | When $s = 0$, $t = 18$ | **M1** | **M1** for substitution of $s = 0$ into *their s* equation |
| | | **√A1** | **√A1** on *their s* |
| | $v = -324$ | **DM1** | **DM1** for substitution of *their t* back into $v$ equation |
| | So speed is 324 | | **A1** for 324 only |