UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS General Certificate of Education Ordinary Level

CANDIDATE NAME CENTRE
NUMBER


CANDIDATE NUMBER


## ADDITIONAL MATHEMATICS

4037/11
Paper 1
May/June 2013
2 hours
Candidates answer on the Question Paper.
No additional materials are required.

## READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80 .

## Mathematical Formulae

## 1. ALGEBRA

## Quadratic Equation

For the equation $a x^{2}+b x+c=0$,

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} .
$$

Binomial Theorem

$$
(a+b)^{n}=a^{n}+\binom{n}{1} a^{n-1} b+\binom{n}{2} a^{n-2} b^{2}+\ldots+\binom{n}{r} a^{n-r} b^{r}+\ldots+b^{n},
$$

where $n$ is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}$.

## 2. TRIGONOMETRY

Identities

$$
\begin{gathered}
\sin ^{2} A+\cos ^{2} A=1 \\
\sec ^{2} A=1+\tan ^{2} A \\
\operatorname{cosec}^{2} A=1+\cot ^{2} A
\end{gathered}
$$

Formulae for $\triangle A B C$

$$
\begin{gathered}
\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
a^{2}=b^{2}+c^{2}-2 b c \cos A \\
\Delta=\frac{1}{2} b c \sin A
\end{gathered}
$$

1 On the axes below sketch, for $0 \leqslant x \leqslant 2 \pi$, the graph of
(i) $y=\cos x-1$,
(ii) $y=\sin 2 x$.

(iii) State the number of solutions of the equation $\cos x-\sin 2 x=1$, for $0 \leqslant x \leqslant 2 \pi$.

2 Variables $x$ and $y$ are such that $y=A b^{x}$, where $A$ and $b$ are constants. The diagram shows graph of $\ln y$ against $x$, passing through the points $(2,4)$ and $(8,10)$.


Find the value of $A$ and of $b$.

3 A committee of 6 members is to be selected from 5 men and 9 women. Find the number different committees that could be selected if
(i) there are no restrictions,
(ii) there are exactly 3 men and 3 women on the committee,
(iii) there is at least 1 man on the committee.

4 (i) Given that $\log _{4} x=\frac{1}{2}$, find the value of $x$.
(ii) Solve $2 \log _{4} y-\log _{4}(5 y-12)=\frac{1}{2}$.

5 (i) Find $\int\left(1-\frac{6}{x^{2}}\right) \mathrm{d} x$.
(ii) Hence find the value of the positive constant $k$ for which $\int_{k}^{3 k}\left(1-\frac{6}{x^{2}}\right) \mathrm{d} x=2$.

6 (i) Given that $\mathbf{A}=\left(\begin{array}{rr}2 & -1 \\ 3 & 5\end{array}\right)$, find $\mathbf{A}^{-1}$.
(ii) Using your answer from part (i), or otherwise, find the values of $a, b, c$ and $d$ such that

$$
\mathbf{A}\left(\begin{array}{rr}
a & b  \tag{5}\\
c & -1
\end{array}\right)=\left(\begin{array}{rr}
7 & 5 \\
17 & d
\end{array}\right) .
$$

7 Calculators must not be used in this question.


The diagram shows a triangle $A B C$ in which angle $A=90^{\circ}$. Sides $A B$ and $A C$ are $\sqrt{5}-2$ and $\sqrt{5}+1$ respectively. Find
(i) $\tan B$ in the form $a+b \sqrt{5}$, where $a$ and $b$ are integers,
(ii) $\sec ^{2} B$ in the form $c+d \sqrt{5}$, where $c$ and $d$ are integers.


The diagram shows a square $A B C D$ of side $16 \mathrm{~cm} . M$ is the mid-point of $A B$. The points $E$ and $F$ are on $A D$ and $B C$ respectively such that $A E=B F=6 \mathrm{~cm} . E F$ is an arc of the circle centre $M$, such that angle $E M F$ is $\theta$ radians.
(i) Show that $\theta=1.855$ radians, correct to 3 decimal places.
(ii) Calculate the perimeter of the shaded region.
(iii) Calculate the area of the shaded region.


The figure shows points $A, B$ and $C$ with position vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ respectively, relative to an origin $O$. The point $P$ lies on $A B$ such that $A P: A B=3: 4$. The point $Q$ lies on $O C$ such that $O Q: Q C=2: 3$.
(i) Express $\overrightarrow{A P}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ and hence show that $\overrightarrow{O P}=\frac{1}{4}(\mathbf{a}+3 \mathbf{b})$.
(ii) Find $\overrightarrow{P Q}$ in terms of $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$.
(iii) Given that $5 \overrightarrow{P Q}=6 \overrightarrow{B C}$, find $\mathbf{c}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

10 The point $A$, whose $x$-coordinate is 2 , lies on the curve with equation $y=x^{3}-4 x^{2}+x+$ (i) Find the equation of the tangent to the curve at $A$.

This tangent meets the curve again at the point $B$.
(ii) Find the coordinates of $B$.
(iii) Find the equation of the perpendicular bisector of the line $A B$.

Question 11 is printed on the next page.

11 (a) Solve $2 \sin \left(x+\frac{\pi}{3}\right)=-1$ for $0 \leqslant x \leqslant 2 \pi$ radians.
(b) Solve $\tan y-2=\cot y$ for $0^{\circ} \leqslant y \leqslant 180^{\circ}$.

