UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS General Certificate of Education Ordinary Level

CANDIDATE NAME CENTRE
NUMBER


CANDIDATE NUMBER


## ADDITIONAL MATHEMATICS

4037/21
Paper 2
May/June 2013
2 hours
Candidates answer on the Question Paper.
No Additional Materials are required.

## READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80 .

## Mathematical Formulae

## 1. ALGEBRA

## Quadratic Equation

For the equation $a x^{2}+b x+c=0$,

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} .
$$

Binomial Theorem

$$
(a+b)^{n}=a^{n}+\binom{n}{1} a^{n-1} b+\binom{n}{2} a^{n-2} b^{2}+\ldots+\binom{n}{r} a^{n-r} b^{r}+\ldots+b^{n},
$$

where $n$ is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}$.

## 2. TRIGONOMETRY

Identities

$$
\begin{gathered}
\sin ^{2} A+\cos ^{2} A=1 \\
\sec ^{2} A=1+\tan ^{2} A \\
\operatorname{cosec}^{2} A=1+\cot ^{2} A
\end{gathered}
$$

Formulae for $\triangle A B C$

$$
\begin{gathered}
\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
a^{2}=b^{2}+c^{2}-2 b c \cos A \\
\Delta=\frac{1}{2} b c \sin A
\end{gathered}
$$

1 Prove that $\left(\frac{1+\sin \theta}{\cos \theta}\right)^{2}+\left(\frac{1-\sin \theta}{\cos \theta}\right)^{2}=2+4 \tan ^{2} \theta$.


The velocity-time graph represents the motion of a particle moving in a straight line.
(i) Find the acceleration during the first 5 seconds.
(ii) Find the length of time for which the particle is travelling with constant velocity.
(iii) Find the total distance travelled by the particle.

3 Variables $x$ and $y$ are related by the equation $y=10-4 \sin ^{2} x$, where $0 \leqslant x \leqslant \frac{\pi}{2}$. Giv $x$ is increasing at a rate of 0.2 radians per second, find the corresponding rate of change when $y=8$.

4 (i) Sketch the graph of $y=|4 x-2|$ on the axes below, showing the coordinates of the where the graph meets the axes.

(ii) Solve the equation $|4 x-2|=x$.


A piece of wire of length 96 cm is formed into the rectangular shape $P Q R S T U$ shown in the diagram. It is given that $P Q=T U=S R=x \mathrm{~cm}$. It may be assumed that $P Q$ and $T U$ coincide and that $T S$ and $Q R$ have the same length.
(i) Show that the area, $A \mathrm{~cm}^{2}$, enclosed by the wire is given by $A=\frac{96 x-3 x^{2}}{2}$.
(ii) Given that $x$ can vary, find the stationary value of $A$ and determine the nature of this stationary value.

6 Find the equation of the normal to the curve $y=\frac{x^{2}+8}{x-2}$ at the point on the curve where

7 (i) Find the first four terms in the expansion of $(2+x)^{6}$ in ascending powers of $x$.
(ii) Hence find the coefficient of $x^{3}$ in the expansion of $(1+3 x)(1-x)(2+x)^{6}$.

8 The line $y=2 x-8$ cuts the curve $2 x^{2}+y^{2}-5 x y+32=0$ at the points $A$ and $B$. P length of the line $A B$.

9 It is given that $x \in \mathbb{R}$ and that $\mathscr{E}=\{x:-5<x<12\}$,

$$
S=\left\{x: 5 x+24>x^{2}\right\},
$$

$$
T=\{x: 2 x+7>15\} .
$$

Find the values of $x$ such that
(i) $x \in S$,
(ii) $x \in S \cup T$,
(iii) $\quad x \in(S \cap T)^{\prime}$.

10 A plane, whose speed in still air is $240 \mathrm{kmh}^{-1}$, flies directly from $A$ to $B$, where $B$ is 500 $A$ on a bearing of $032^{\circ}$. There is a constant wind of $50 \mathrm{kmh}^{-1}$ blowing from the west.
(i) Find the bearing on which the plane is steered.
(ii) Find, to the nearest minute, the time taken for the flight.

11 A one-one function f is defined by $\mathrm{f}(x)=(x-1)^{2}-5$ for $x \geqslant k$.
(i) State the least value that $k$ can take.

For this least value of $k$
(ii) write down the range of f,
(iii) find $\mathrm{f}^{-1}(x)$,
(iv) sketch and label, on the axes below, the graph of $y=\mathrm{f}(x)$ and of $y=\mathrm{f}^{-1}(x)$,

(v) find the value of $x$ for which $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$.

12 The function $\mathrm{f}(x)=x^{3}+x^{2}+a x+b$ is divisible by $x-3$ and leaves a remainder of 20 when divided by $x+1$.
(i) Show that $b=6$ and find the value of $a$.
(ii) Using your value of $a$ and taking $b$ as 6 , find the non-integer roots of the equation $\mathrm{f}(x)=0$ in the form $p \pm \sqrt{q}$, where $p$ and $q$ are integers.

