CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the May/June 2014 series

4037 ADDITIONAL MATHEMATICS

4037/11 Paper 1, maximum raw mark 80

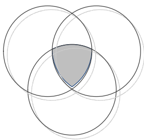
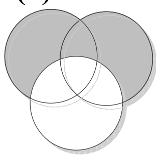
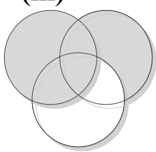
This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

1	$\text{LHS} = \dfrac{\sin\ \theta}{\cos\ \theta} + \dfrac{\cos\ \theta}{1+\sin\ \theta}$	**B1**	**B1** for use of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
	$= \dfrac{\sin\theta(1+\sin\theta)+\cos^2\theta}{\cos\theta(1+\sin\theta)}$	**M1**	**M1** for attempt to obtain a single fraction
	$= \dfrac{1+\sin\theta}{\cos\theta(1+\sin\theta)}$	**DM1**	**DM1** for use of $\sin^2\theta+\cos^2\theta=1$
	$= \dfrac{1}{\cos\theta}$ leading to $\sec\theta$	**A1**	**A1** for 'finishing off'
	Alternative solution: $\text{LHS} = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta\,(1\text{-}\sin\theta)}{(1+\sin\theta)(1\text{-}\sin\theta)}$	**B1**	**B1** for use of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
	$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta(1-\sin\theta)}{\cos^2\theta}$	**M1**	**M1** for multiplication by $(1-\sin\theta)$
	$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{(1-\sin\theta)}{\cos\theta}$	**DM1**	**DM1** for use of $\sin^2\theta+\cos^2\theta=1$
	$= \dfrac{1}{\cos\theta}$ leading to $\sec\theta$	**A1**	**A1** for 'finishing off'
	Alternative solution: $\text{LHS} = \dfrac{\tan\theta(1+\sin\theta)+\cos\theta}{1+\sin\theta}$	**M1**	**M1** for attempt to obtain a single fraction
	$= \dfrac{\dfrac{\sin\theta}{\cos\theta} + \dfrac{\sin^2}{\cos\theta} + \cos\theta}{1+\sin\theta}$	**B1**	**B1** for use of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
	$= \dfrac{\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}$		
	$= \dfrac{1+\sin\theta}{\cos\theta(1+\sin\theta)}$	**DM1**	**DM1** for use of $\sin^2\theta+\cos^2\theta=1$
	$= \dfrac{1}{\cos\theta}$ leading to $\sec\theta$	**A1**	**A1** for 'finishing off'

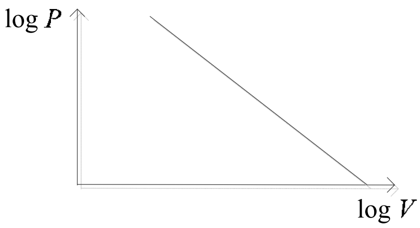
2	(i)	$\lvert\mathbf{a}\rvert = \sqrt{4^2 + 3^2} = 5$ $\lvert\mathbf{b}+\mathbf{c}\rvert = \sqrt{(-3)^2 + 4^2} = 5$	M1 A1	**M1** for finding the modul… **a** or **b** + **c** **A1** for completion
	(ii)	$\lambda\begin{pmatrix}4\\3\end{pmatrix} + \mu\begin{pmatrix}2\\2\end{pmatrix} = 7\begin{pmatrix}-5\\2\end{pmatrix}$ $4\lambda + 2\mu = -35$ and $3\lambda + 2\mu = 14$ leading to $\lambda = -49$, $\mu = 80.5$	M1 DM1 A1	**M1** for equating like vectors and obtaining 2 linear equations **DM1** for solution of simultaneous equations **A1** for both
3	(a)	(i)  (ii)  (iii) 	B1 B1 B1	**B1** for each
	(b) (i)	2	B1	
	(ii)	0	B1	
4		$k(4x-3) = 4x^2 + 8x - 8$ $4x^2 + x(8-4k) + 3k - 8 = 0$ $b^2 - 4ac = (8-4k)^2 - 16(3k-8)$ $= 16k^2 - 112k + 192$ $b^2 - 4ac < 0$, $k^2 - 7k + 12 < 0$ critical values $k = 3, 4$ $\therefore 3 < k < 4$	M1 DM1 DM1 A1 A1	**M1** for equating the line and the curve and attempt to obtain a quadratic equation in *k* **DM1** for use of $b^2 - 4ac$ with *k* **DM1** for solution of a 3 term quadratic equation, dependent on both previous M marks **A1** for both critical values **A1** for the range
5	(i)	$\frac{dy}{dx} = 2xe^{x^2}$	B1B1	**B1** for e^{x^2}, **B1** for $2xe^{x^2}$
	(ii)	$\frac{1}{2}e^{x^2}$	M1A1	**M1** for ke^{x^2} **A1** for $\frac{1}{2}e^{x^2}$
	(iii)	$\left(\frac{1}{2}e^4\right) - \left(\frac{1}{2}\right) = 26.8$	DM1 A1	**DM1** for correct use of limits **A1** for 26.8, allow exact value

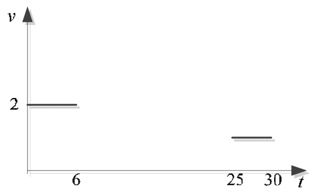
6	(i)	$\mathbf{AB} = \begin{pmatrix} 10 & 19 \\ 32 & 37 \\ 14 & 14 \end{pmatrix}$	M1 A1	**M1** for at least 3 correct [illegible] a 3×2 matrix **A1** for all correct
	(ii)	$\mathbf{B}^{-1} = \frac{1}{7}\begin{pmatrix} 5 & -1 \\ -3 & 2 \end{pmatrix}$	B1 B1	**B1** for $\frac{1}{7}$, **B1** for $\begin{pmatrix} 5 & -1 \\ -3 & 2 \end{pmatrix}$
	(iii)	$2\begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3 \\ -22 \end{pmatrix}$	M1	**M1** for obtaining in matrix form
		$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 5 & -1 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} -1.5 \\ -11 \end{pmatrix} = \frac{1}{7}\begin{pmatrix} 3.5 \\ -17.5 \end{pmatrix}$	M1	**M1** for pre-multiplying by $\mathbf{B}^{-1}$
		$x = 0.5,\ y = -2.5$	A1	**A1** for both
7	(i)	$y = 2x^2 - \frac{1}{x+1}(+c)$	B1 B1	**B1** for each correct term
		when $x = \frac{1}{2},\ y = \frac{5}{6}$ so $\frac{5}{6} = \frac{1}{2} - \frac{2}{3} + c$	M1	**M1** for attempt to find $+c$, must have at least 1 of the previous B marks
		leading to $c = 1$ $\left(y = 2x^2 - \frac{1}{x+1} + 1\right)$	A1	Allow **A1** for $c = 1$
	(ii)	When $x = 1,\ y = \frac{5}{2}$	M1	**M1** for using $x = 1$ in their **(i)** to find y
		$\frac{dy}{dx} = \frac{17}{4}$ so gradient of normal $= -\frac{4}{17}$	B1	**B1** for gradient of normal
		Equation of normal $y - \frac{5}{2} = -\frac{4}{17}(x-1)$	DM1	**DM1** for attempt at **normal** equation
		$(8x + 34y - 93 = 0)$	A1	**A1** – allow unsimplified (fractions must not contain decimals)

Question	Answer	Marks	Guidance
8 (i)	$\log p = n\log V + \log k$	B1	**B1** for statement, but may [illegible]y later work.

$\ln V$	2.30	3.91	4.61	5.30
$\ln p$	4.55	2.14	1.10	0.10

$\lg V$	1	1.70	2	2.30
$\lg p$	1.98	0.93	0.48	0.04



Question	Answer	Marks	Guidance
		M1 A2,1,0	**M1** for plotting a suitable graph –1 for each error in points plotted
(ii)	Use of gradient = n $n = -1.5$ (allow –1.4 to –1.6)	DM1 A1	**DM1** for equating numerical gradient to n
(iii)	Allow 13 to 16	DM1 A1	**DM1** for use of *their* graph or substitution into *their* equation.
9 (a)	Distance travelled = area under graph $= \frac{1}{2}(60+20)\times 12 = 480$	M1 A1	**M1** for realising that area represents distance travelled and attempt to find area
(b)		B1 B1 B1	**B1** for velocity of 2 ms^{-1} for $0 \leqslant t \leqslant 6$ **B1** for velocity of zero for *their* '6' to *their* '25' **B1** for velocity of 1 ms^{-1} for $25 \leqslant t \leqslant 30$
(c) (i)	$v = 4 - \frac{16}{t+1}$ When $v = 0$, $t = 3$	M1 DM1 A1	**M1** for attempt at differentiation **DM1** for equating velocity to zero and attempt to solve
(ii)	$a = \frac{16}{(t+1)^2}$ $0.25(t+1)^2 = 16$ $t = 7$	M1 A1	**M1** for attempt at differentiation and equating to 0.25 with attempt to solve

10 (a)	1 digit even numbers 2	**B1**	
	2 digit even numbers $4 \times 2 = 8$	**B1**	
	3 digit even numbers $3 \times 3 \times 2 = 18$	**B1**	
	Total = 28	**B1**	
(b) (i)	3M 5W = 35 4M 4W = 175 5M 3W = 210	**B1** **B1** **B1**	
	Total = 420	**B1**	**B1** for addition to obtain final answer, must be evaluated.
	or ${}^{12}C_8$ – 6M 2W – 7M 1W 495 – 70 – 5 = 420		**or**: as above, final **B1** for subtraction to get final answer
(ii)	Oldest man in, oldest woman out and vice–versa		
	${}^{10}C_7 \times 2 = 240$	**B1, B1**	**B1** for ${}^{10}C_7$, B1 for realising there are 2 identical cases
	Alternative: 1 man out 1 woman in 6 men 4 women		
	6M 1W : ${}^{6}C_6 \times {}^{4}C_1 = 4$ 5M 2W : ${}^{6}C_5 \times {}^{4}C_2 = 36$ 4M 3W : ${}^{6}C_4 \times {}^{4}C_3 = 60$ 3M 4W : ${}^{6}C_3 \times {}^{4}C_4 = 20$ Total = 120	**B1**	All separate cases correct for **B1**
	There are 2 identical cases to consider, so 240 ways in all.	**B1**	**B1** for realising there are 2 identical cases, which have integer values

11 (a)			**In each case the last A [illegible] a second correct solution an[illegible] solutions within the range**
	$5\sin 2x + 3\cos 2x = 0$	M1	**M1** for use of tan
	$\tan 2x = -0.6$	DM1	**DM1** for dealing with $2x$ correctly
	$2x = 149°, 329°$		
	$x = 74.5°, 164.5°$	A1,A1	**A1** for each
	Alternatives: $\sin(2x + 31°) = 0$ or $\cos(2x - 59°) = 0$	M1	**M1** for either, then mark as above
(b)	$2\cot^2 y + 3\operatorname{cosec} y = 0$		
	$2(\operatorname{cosec}^2 y - 1) + 3\operatorname{cosec} y = 0$	M1	**M1** for use of correct identity
	$2\operatorname{cosec}^2 y + 3\operatorname{cosec} y - 2 = 0$		
	$(2\operatorname{cosec} y - 1)(\operatorname{cosec} y + 2) = 0$	M1	**M1** for attempt to factorise a 3 term quadratic equation
	One valid solution $\operatorname{cosec} y = -2,\ \sin y = -\frac{1}{2}$		
	$y = 210°,\ 330°$	A1,A1	**A1** for each
	Alternative: $2\frac{\cos^2 y}{\sin^2 y} + \frac{3}{\sin y} = 0$	M1	**M1** for use of $\cot y = \frac{\cos y}{\sin y}$ and $\operatorname{cosec} y = \frac{1}{\sin y}$
	leads to $2\sin^2 y - 3\sin y - 2 = 0$ and $\sin y = -\frac{1}{2}$ only	M1	**M1** for attempt to factorise a 3 term quadratic equation
	$y = 210°, 330°$	A1A1	
(c)	$3\cos(z + 1.2) = 2$		
	$\cos(z + 1.2) = \frac{2}{3}$		
	$(z + 1.2) = 0.8411, 5.442, 7.124$	M1	**M1** for correct order of operations to end up with 0.8411 radians or better
	$z = 4.24, 5.92$	A1 A1A1	**A1** for one of 5.441 or 7.124 (or better) **A1** for each valid solution