CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the May/June 2014 series

4037 ADDITIONAL MATHEMATICS

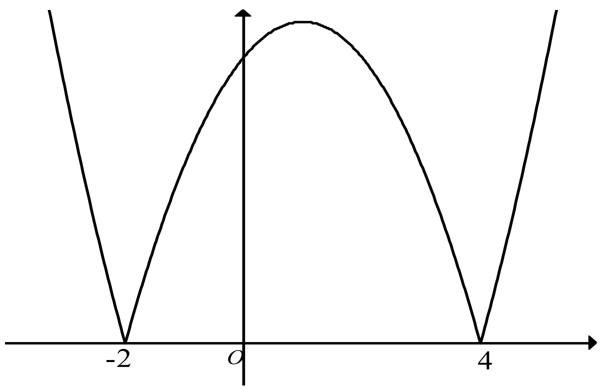
4037/21 Paper 2, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

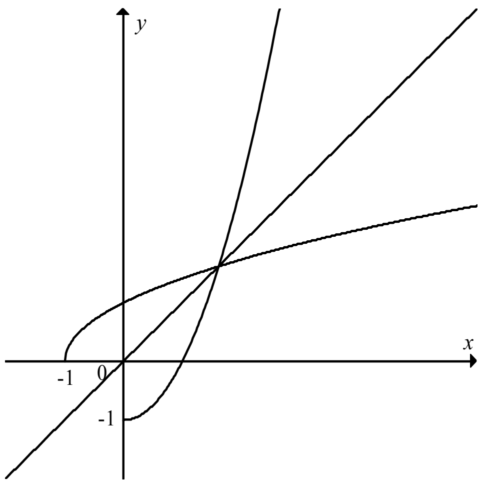
Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

1	$x^2 + x\,[> 0]$	**M1**	expands and rearranges
	critical values 0 and -1 soi	**A1**	
	$-1 < x < 0$	**A1**	condone space, comma, "and" but not "or" Mark final answer.
2	$\dfrac{6}{\left(1+\sqrt{3}\right)^2}$ or $6 = \left(a + b\sqrt{3}\right)\left(1+\sqrt{3}\right)^2$	**M1**	for dealing with the negative index (condone treating 6 as have negative index at this stage)
	$\dfrac{6}{4+2\sqrt{3}}$ or $6 = \left(a + b\sqrt{3}\right)\left(4+2\sqrt{3}\right)$	**M1**	for squaring
	$\dfrac{6}{4+2\sqrt{3}} \times \dfrac{4-2\sqrt{3}}{4-2\sqrt{3}}$ AND attempting to multiply out	**M1**	for rationalising or for obtaining a pair of simultaneous equations $4a + 6b = 6$ and $2a + 4b = 0$
	$6 - 3\sqrt{3}$ isw	**A1**	
3 (i)		**B1** **B1**	correct shape *x* intercepts marked or implied by tick marks, for example or seen nearby; condone *y* intercept omitted
(ii)	$x = 1$ (only) soi	**B1**	can be implied by second **B1**
	$y = \pm 9$ (only)	**B1**	or $k = \pm 9, +9$ or -9 or both;
	$0 < k < 9$	**B1**	must be strict inequality in *k*; condone space, comma, "and", "or"
4	Attempt to find f(4) or f(1) or division to a remainder	**M1**	condone one error
	$128 + 16a + 4b + 12 = 0$ or better $(16a + 4b = -140)$	**A1**	
	$2 + a + b + 12 = -12$ or better $(a + b = -26)$	**A1**	
	Solves linear equations in *a* and *b*	**M1**	
	$a = -3$, $b = -23$	**A1**	both

5 **(i)**	$2\left(x-\frac{1}{4}\right)^2+\frac{47}{8}$ (5.875) isw	**B3,2,1,0**	one mark for each of p, q, r; allow correct equivalent values. **B0**, then **SC2** for $2\left(x-\frac{1}{4}\right)+\frac{47}{8}$, or **SC1** for correct values but incorrect format
(ii)	$\frac{47}{8}$ is min value when $x=\frac{1}{4}$	**B1ft +** **B1ft**	strict **ft** *their* $\frac{47}{8}$ and *their* $\frac{1}{4}$; each value must be correctly attributed; condone $y=\frac{47}{8}$ for **B1**, or $\left(\frac{1}{4},\frac{47}{8}\right)$ for **B1B1**
6 **(a)**	${}^8C_3\times 3^3\times(\pm 2)^5$ or $3^8\left[{}^8C_3\left(\pm\frac{2}{3}\right)^5\right]$	**M1**	condone ${}^8C_5, -2x^5$
	–48384	**A1**	can be in expansion
(b) **(i)**	$1 + 12x + 60x^2$	**B2,1,0**	ignore additional terms. If **B0**, allow **M1** for 3 correct unsimplified terms
(ii)	Coefficient of x correct or correct **ft** $(12+a)$ soi Coefficient of x^2 correct or correct **ft** $(60+12a)$ soi	**B1ft** **B1ft**	**ft** *their* $1+12x+60x^2$ **ft** *their* $1+12x+60x^2$
	$1.5\times \textit{their}(12+a)=\textit{their}(60+12a)$ -4	**M1** **A1**	no x or x^2
7 **(i)**	$-\frac{1}{x^2}+\frac{1}{x^{1/2}}$	**B1 + B1**	or equivalent with negative indices
(ii)	$\frac{2}{x^3}-\frac{1}{2x^{3/2}}$	**B1ft +** **B1ft**	or equivalent with negative indices. Strict **ft**
(iii)	Attempting to solve *their* $\frac{dy}{dx}=0$	**M1**	must achieve x =… (allow slips)
	$x = 1$ $y = 3$	**A1**	**SC2** for (1, 3) stated, nfww
	Substitute *their* $x = 1$ into *their* $\frac{d^2y}{dx^2}$; or examines $\frac{dy}{dx}$ or y on both sides of *their* $x = 1$	**M1**	for using *their* value from $\frac{dy}{dx}=0$
	Complete and correct determination of nature. If correct, minimum.	**A1**	must be from correct work

8	**(i)**	$2r + r\theta = 30$ giving $\theta = \dfrac{30 - 2r}{r}$	**M1**	correct arc formula + (2) rearranged
		Substitute *their* expression for θ into $A = \frac{1}{2}r^2\theta$	**M1**	
		Correct simplification to $A = 15r - r^2$ AG	**A1**	
	(ii)	$15 - 2r = 0$ $r = 7.5$ 56.25	**M1** **A1** **A1**	*their* $\dfrac{dA}{dr} = 0$ 56.3 is **A0** unless 56.25 seen; if **M0**, then **SC2** for A = 56.25 with no working; or **SC1** for r = 7.5 with no working
9	**(i)**	(3, 5)	**B1B1**	column vector **B0B1**
	(ii)	$m_{BD}\left(= \dfrac{6-4}{1-5}\right) = -\dfrac{1}{2}$	**M1**	can be implied by second M1
		$m_{AC}\left(= -1 \div -\dfrac{1}{2}\right)$ seen or used	**M1**	
		$y - 5 = 2(x - 3)$ or $y = 2x + c,\ c = -1$ or better	**A1**	
	(iii)	$p = 1$ $q = 7$ [A(1, 1) C(4, 7)] Method for finding area numerically	**M1** **M1**	could be in **(ii)** e.g. $24 - \left(\frac{1}{2}\times 1\times 3 + \frac{1}{2}\times 1\times 3 + \frac{1}{2}\times 4\right)$ or shoelace method
		15	**A1**	**SC2** for 15 with no working
10	**(i)**	$-2\sin 2x$ and $\dfrac{1}{3}\cos\left(\dfrac{x}{3}\right)$	**B1+B1**	each trig function correctly differentiated
		Attempt at product rule	**M1**	
		$\dfrac{1}{3}\cos 2x \cos\left(\dfrac{x}{3}\right) - 2\sin 2x \sin\left(\dfrac{x}{3}\right)$ isw	**A1ft**	**ft** $k_1 \sin 2x$ and $k_2 \cos\left(\dfrac{x}{3}\right)$ provided $k_1,\ k_2$ are non-zero
	(ii)	$\sec^2 x$ and $\dfrac{1}{x}$	**B1 + B1**	
		Attempt at quotient rule (with given quotient)	**M1**	or rearrangement to correct product and attempt at product rule
		$\dfrac{(\sec^2 x)(1 + \ln x) - \frac{1}{x}(\tan x)}{(1 + \ln x)^2}$ isw	**A1**	penalise poor bracketing if not recovered

11 (a)	$2^{x^2-5x}=2^{-6}$	M1	Or $(x^2-5x)\ln 2=\ln\left(\frac{1}{64}\right)$
	$x^2-5x+6=0$	M1	*their* "6"
	Correct method of solution of their 3 term quadratic	M1	
	$x=2$ or $x=3$	A1	
(b)	Correct change of base to $\frac{\log_a 4}{\log_a 2a}$	B1	base *a* only at this stage but can recover at end
	$\frac{\log_a 4}{\log_a 2+\log_a a}$	M1	for $\log 2a=\log 2+\log a$
	$\log_a a=1$ used soi	M1	
	simplification to $\log_a 4$	A1	

12 (i)	f(3)	**M1**	or $\mathrm{fg}(x)=\dfrac{2\sqrt{(x+1)}}{\sqrt{(x+1)}+1}$
	$\dfrac{6}{4}$ oe	**A1**	
(ii)	$\dfrac{2\left(\dfrac{2x}{x+1}\right)}{\dfrac{2x}{x+1}+1}$	**M1**	allow omission of 2(…..) in numerator or (…..) + 1 in denominator, but not both.
	A correct and valid step in simplification	**dM1**	e.g. multiplying numerator and denominator by $x+1$, , or simplifying $\dfrac{2x}{x+1}+1$ to $\dfrac{2x+x+1}{x+1}$
	Correctly simplified to $\dfrac{4x}{3x+1}$	**A1**	
(iii)	Putting $y=\mathrm{g}(x)$, changing subject to x and swopping x and y or vice versa	**M1**	condone $x=y^2-1$; reasonable attempt at correct method
	$\mathrm{g}^{-1}(x)=x^2-1$	**A1**	condone $y=$….. , $\mathrm{f}^{-1}=$……
	(Domain) $x>0$ (Range) $\mathrm{g}^{-1}(x)>-1$	**B1** **B1**	 condone $y>-1$ $\mathrm{f}^{-1}>-1$
(iv)		**B1 + B1** **B1**	correct graphs; –1 need not be labelled but could be implied by 'one square' idea of reflection or symmetry in line $y=x$ must be stated.