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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS **4037/02**

Paper 2 **October/November 2007**

2 hours

Additional Materials: Answer Paper

2146585536

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **6** printed pages and **2** blank pages.

SPA (KN) T28444/3

UNIVERSITY *of* CAMBRIDGE
International Examinations

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1} b + \binom{n}{2}a^{n-2} b^2 + \dots + \binom{n}{r}a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A.$$

1 The two variables x and y are related by the equation $yx^2 = 800$.

(i) Obtain an expression for $\frac{dy}{dx}$ in terms of x.

(ii) Hence find the approximate change in y as x increases from 10 to $10 + p$, where p is small. [

2 Solve the equation $3\sin\left(\frac{x}{2} - 1\right) = 1$ for $0 < x < 6\pi$ radians. [5]

3 **(i)** Express 9^{x+1} as a power of 3. [1]

(ii) Express $\sqrt[3]{27^{2x}}$ as a power of 3. [1]

(iii) Express $\dfrac{54 \times \sqrt[3]{27^{2x}}}{9^{x+1} + 216(3^{2x-1})}$ as a fraction in its simplest form. [3]

4 A cycle shop sells three models of racing cycles, A, B and C. The table below shows the numbers of each model sold over a four-week period and the cost of each model in $.

Week \ Model	A	B	C
1	8	12	4
2	7	10	2
3	10	12	0
4	6	8	4
Cost ($)	300	500	800

In the first two weeks the shop banked 30% of all money received, but in the last two weeks the shop only banked 20% of all money received.

(i) Write down three matrices such that matrix multiplication will give the total amount of money banked over the four-week period. [2]

(ii) Hence evaluate this total amount. [4]

5 **(i)** Expand $(1 + x)^5$. [1]

(ii) Hence express $(1 + \sqrt{2})^5$ in the form $a + b\sqrt{2}$, where a and b are integers. [3]

(iii) Obtain the corresponding result for $(1 - \sqrt{2})^5$ and **hence** evaluate $(1 + \sqrt{2})^5 + (1 - \sqrt{2})^5$. [2]

6 Two circular flower beds have a combined area of $\frac{29\pi}{2}$ m^2. The sum of the circumferences of the flower beds is 10π m. Determine the radius of each flower bed.

7 The position vectors of points A and B, relative to an origin O, are $2\mathbf{i} + 4\mathbf{j}$ and $6\mathbf{i} + 10\mathbf{j}$ respectively. The position vector of C, relative to O, is $k\mathbf{i} + 25\mathbf{j}$, where k is a positive constant.

(i) Find the value of k for which the length of BC is 25 units. [3]

(ii) Find the value of k for which ABC is a straight line. [3]

8 Given that $x \in \mathbb{R}$ and that

$$\mathscr{E} = \{x : 2 < x < 10\},$$
$$A = \{x : 3x + 2 < 20\}$$
and $$B = \{x : x^2 < 11x - 28\},$$

find the set of values of x which define

(i) $A \cap B$,

(ii) $(A \cup B)'$.

[7]

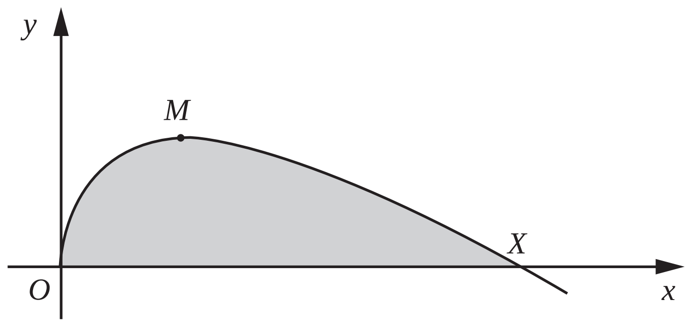
9 A particle travels in a straight line so that, t s after passing through a fixed point O, its speed, v ms^{-1}, is given by $v = 8\cos\left(\frac{t}{2}\right)$.

(i) Find the acceleration of the particle when $t = 1$. [3]

The particle first comes to instantaneous rest at the point P.

(ii) Find the distance OP. [4]

10

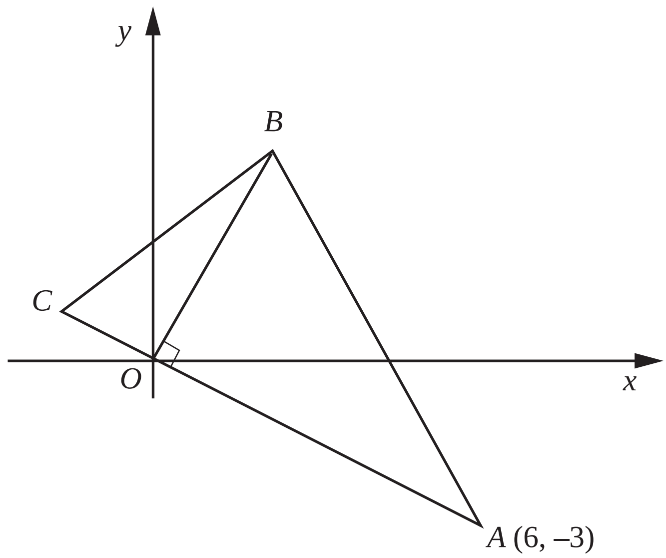


The diagram shows part of the curve $y = 4\sqrt{x} - x$. The origin O lies on the curve and the curve intersects the positive x-axis at X. The maximum point of the curve is at M. Find

(i) the coordinates of X and of M, [5]

(ii) the area of the shaded region. [4]

11 **Solutions to this question by accurate drawing will not be accepted.**



The diagram shows a triangle ABC in which A is the point (6, –3). The line AC passes through the origin O. The line OB is perpendicular to AC.

(i) Find the equation of OB. [2]

The area of triangle AOB is 15 units^2.

(ii) Find the coordinates of B. [3]

The length of AO is 3 times the length of OC.

(iii) Find the coordinates of C. [2]

The point D is such that the quadrilateral $ABCD$ is a kite.

(iv) Find the area of $ABCD$. [2]

12 Answer only **one** of the following two alternatives.

EITHER

The function f is defined, for $x > 0$, by $\mathrm{f} : x \mapsto \ln x$.

(i) State the range of f. [1]

(ii) State the range of f^{-1}. [1]

(iii) On the same diagram, sketch and label the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$. [2]

The function g is defined, for $x > 0$, by $\mathrm{g} : x \mapsto 3x + 2$.

(iv) Solve the equation $\mathrm{fg}(x) = 3$. [2]

(v) Solve the equation $\mathrm{f}^{-1}\mathrm{g}^{-1}(x) = 7$. [4]

OR

(i) Find the values of k for which $y = kx + 2$ is a tangent to the curve $y = 4x^2 + 2x + 3$. [4]

(ii) Express $4x^2 + 2x + 3$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(iii) Determine, with explanation, whether or not the curve $y = 4x^2 + 2x + 3$ meets the x-axis. [2]

The function f is defined by $\mathrm{f} : x \mapsto 4x^2 + 2x + 3$ where $x \geqslant p$.

(iv) Determine the smallest value of p for which f has an inverse. [1]

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