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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS **4037/13**

Paper 1 **October/November 2010**

2 hours

Additional Materials: Answer Booklet/Paper
Graph Paper (1 sheet)

1581608333

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Booklet/Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **5** printed pages and **3** blank pages.

UNIVERSITY *of* CAMBRIDGE
International Examinations

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 Show that $\sec x - \cos x = \sin x \tan x$.

2 A 4-digit number is formed by using four of the seven digits 2, 3, 4, 5, 6, 7 and 8. No digit can be used more than once in any one number. Find how many different 4-digit numbers can be formed if

(i) there are no restrictions, [2]

(ii) the number is even. [2]

3 The line $y = mx + 2$ is a tangent to the curve $y = x^2 + 12x + 18$. Find the possible values of m. [4]

4 The remainder when the expression $x^3 + kx^2 - 5x - 3$ is divided by $x - 2$ is 5 times the remainder when the expression is divided by $x + 1$. Find the value of k. [4]

5 Solve the simultaneous equations

$$\log_3 a = 2 \log_3 b,$$

$$\log_3 (2a - b) = 1.$$ [5]

6 Solve the equation $3x^3 + 7x^2 - 22x - 8 = 0$. [6]

7 **(i)** Sketch the graph of $y = |3x - 5|$, for $-2 \leqslant x \leqslant 3$, showing the coordinates of the points where the graph meets the axes. [3]

(ii) On the same diagram, sketch the graph of $y = 8x$. [1]

(iii) Solve the equation $8x = |3x - 5|$. [3]

8 **(a)** A function f is defined, for $x \in \mathbb{R}$, by

$$f(x) = x^2 + 4x - 6.$$

(i) Find the least value of $f(x)$ and the value of x for which it occurs. [2]

(ii) Hence write down a suitable domain for $f(x)$ in order that $f^{-1}(x)$ exists. [1]

(b) Functions g and h are defined, for $x \in \mathbb{R}$, by

$$g(x) = \frac{x}{2} - 1,$$

$$h(x) = x^2 - x.$$

(i) Find $g^{-1}(x)$. [2]

(ii) Solve $gh(x) = g^{-1}(x)$. [3]

9 **(a)** Find $\int \left(x^{\frac{1}{3}} - 3\right)^2 \mathrm{d}x$.

(b) **(i)** Given that $y = x\sqrt{x^2 + 6}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(ii) Hence find $\int \frac{x^2 + 3}{\sqrt{x^2 + 6}} \mathrm{d}x$. [2]

10 A particle travels in a straight line so that, t s after passing through a fixed point O, its displacement s m from O is given by $s = \ln(t^2 + 1)$.

(i) Find the value of t when $s = 5$. [2]

(ii) Find the distance travelled by the particle during the third second. [2]

(iii) Show that, when $t = 2$, the velocity of the particle is $0.8\,\mathrm{ms}^{-1}$. [2]

(iv) Find the acceleration of the particle when $t = 2$. [3]

11 Solve the equation

(i) $3 \sin x - 4 \cos x = 0$, for $0° \leqslant x \leqslant 360°$, [3]

(ii) $11 \sin y + 1 = 4 \cos^2 y$, for $0° \leqslant y \leqslant 360°$, [4]

(iii) $\sec\left(2z + \frac{\pi}{3}\right) = -2$, for $0 \leqslant z \leqslant \pi$ radians. [4]

12 Answer only **one** of the following two alternatives.

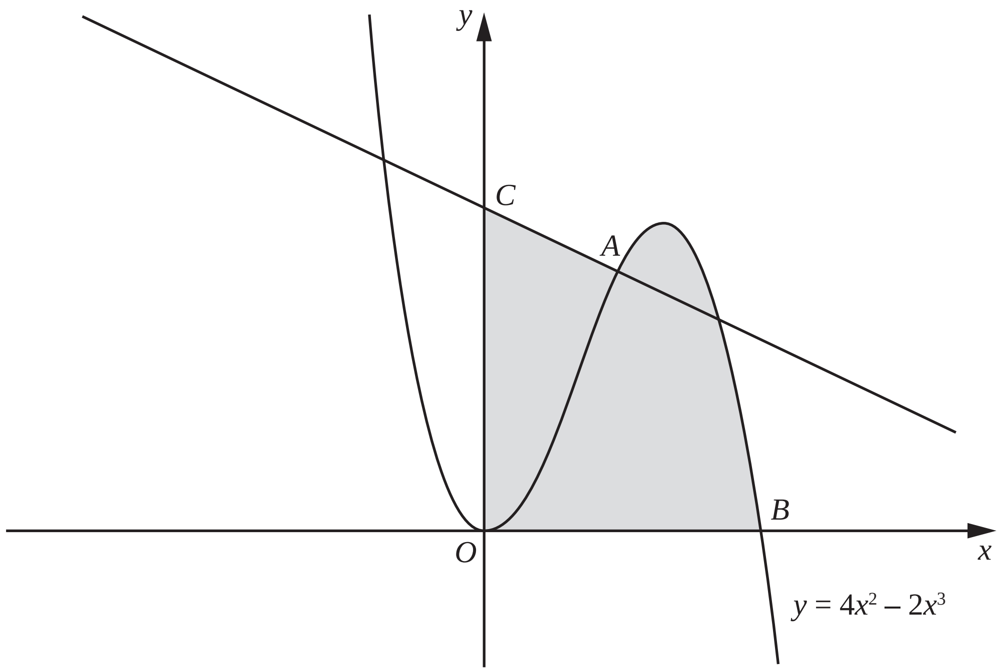
EITHER

A curve has the equation $y = A\sin 2x + B\cos 3x$. The curve passes through the point with coordinates $\left(\frac{\pi}{12}, 3\right)$ and has a gradient of -4 when $x = \frac{\pi}{3}$.

(i) Show that $A = 4$ and find the value of B. [6]

(ii) Given that, for $0 \leqslant x \leqslant \frac{\pi}{3}$, the curve lies above the x-axis, find the area of the region enclosed by the curve, the y-axis and the line $x = \frac{\pi}{3}$. [5]

OR



The diagram shows the curve $y = 4x^2 - 2x^3$. The point A lies on the curve and the x-coordinate of A is 1. The curve crosses the x-axis at the point B. The normal to the curve at the point A crosses the y-axis at the point C.

(i) Show that the coordinates of C are (0, 2.5). [5]

(ii) Find the area of the shaded region. [6]

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