UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the October/November 2011 question paper for the guidance of teachers

4037 ADDITIONAL MATHEMATICS

4037/13 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1,2 This is deducted from A or B marks when essential working is omitted.

PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1	$648x^{\frac{11}{2}}y^{\frac{21}{5}}$	B1 B1 B1 [3]	B1 for each correct
2	$\sqrt{\dfrac{\sin^2\theta}{4\tan^2\theta}}$	B1 B1	B1 for numerator B1 for denominator
	$= \dfrac{\sin\theta}{2\tan\theta}$	M1	M1 for rearrangement
	$= \dfrac{\cos\theta}{2}, k = 0.5$	A1 [4]	
	OR		
	$\sqrt{\dfrac{1-\cos^2\theta}{\dfrac{4}{\cos^2\theta}-4}}$	B1	
	$= \sqrt{\dfrac{1-\cos^2\theta}{\dfrac{4-4\cos^2\theta}{\cos^2\theta}}}$	B1	
	$= \dfrac{\cos\theta}{2}$	M1 A1	
3	**(i)** $\mathbf{A}^{-1} = \dfrac{1}{16}\begin{pmatrix}-2 & -3\\ 8 & 4\end{pmatrix}$	B2, 1, 0	–1 each error
	(ii) $\mathrm{M} = \dfrac{1}{16}\begin{pmatrix}-2 & -2\\ 8 & 4\end{pmatrix}\begin{pmatrix}1 & 4\\ 2 & 3\end{pmatrix}$	M1 DM1	M1 for pre-multiplication DM1 for attempt to multiply matrices, at least one element correct
	$= \dfrac{1}{16}\begin{pmatrix}-8 & -17\\ 16 & 44\end{pmatrix}$	A1 [5]	A1 all correct
4	**(a)** $11 - x + x + 13 - x = 18$ $x = 6$	M1 A1	M1 for a valid method
	(b) **(i)** $X: \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$	B1	B1 for both
	(ii) $Y: \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$	B1, B1	B1 for each pair
	(iii) $X \subset Y$, $X \cap Y = X$ or $X \cup Y = Y$	B1 [6]	

5	**(i)** $\lg p^3 + \lg q = 10a$ or $\lg p - \lg q^2 = a$ $3\lg p + \lg q = 10a$ $\lg p - 2\lg q = a$ $\lg p = 3\lg q$ leading to $\log p = 3a$ and $\log q = a$	M1 A1 A1 M1 A1	M1 for attempt to simplify logs a A1 for each of any 2 correct M1 for attempt to solve simultaneous equations A1 for both
	(ii) $\frac{\log q}{\log p} = \frac{1}{3}$	√B1 [6]	ft on their $\log p$ and $\log q$, both need to be simple functions of a
6	**(i)** $\frac{dy}{dx} = -3\sin\frac{x}{2} + 2\cos\frac{x}{2}$	M1 A1, A1	M1 for attempt to differentiate A1 for each correct term
	When $\frac{dy}{dx} = 0$, $\tan\frac{x}{2} = \frac{2}{3}$	M1	M1 for their $\frac{dy}{dx} = 0$
	$x = 1.18$	A1	A1 correct solution only
	(ii) $\frac{d^2y}{dx^2} = -\frac{3}{2}\cos\frac{x}{2} - \sin\frac{x}{2}$	M1	M1 for a valid method – needs to be seen,
	When $x = 1.18$ $\frac{d^2y}{dx^2}$ is –ve (–1.8) Maximum	 A1 [7]	
7	B (6, 4)	B1	
	grad $AM = \frac{1}{5}$ ∴ grad $BC = -5$	M1	M1 for attempt at gradient of BC
	BC equation: $y - 4 = -5(x - 6)$	M1 A1	M1 for attempt at straight line equation A1 for correct equation in any form
	When $y = 0$, $x = 6.8$	√B1	Ft on their BC equation
	Area = 20.8	M1,A1 [7]	M1 for a correct method for area of triangle

8	**(i)** $12 + 3m = 3n$ $30 + 3m = 6n$ leading to $m = 2$, $n = 6$	M1 A1 M1, A1	M1 for equating like vectors A1 for both correct equations M1 for solution of equations
	(ii) $\mathbf{b} = \begin{pmatrix} -8 \\ 16 \end{pmatrix}$, $\lvert\mathbf{b}\rvert = 8\sqrt{5}$	M1, A1	M1 for attempt at magnitude
	(iii) $\frac{1}{8\sqrt{5}}\begin{pmatrix} -8 \\ 16 \end{pmatrix}$	√B1 [7]	Ft on their **b** and its magnitude Allow decimals
9	**(i)** Amplitude = 2 Period = 120º	B1 B1	
	(ii) Max Value = 1 Occurs when $\sin 3x = 1$ $x = 30º, 150º, 270º$	B1 B1,B1	
	(iii) Sketch	B1 B1 [7]	B1 one cycle correct – ignore x values B1 all correct
10	**(a)** $3\sec^2(3x + 2)$	B1, B1	B1 for 3, B1 for $3\sec^2(3x + 2)$
	(b) $\frac{2}{3} \times \frac{1}{2} x^{-\frac{1}{2}} \left(x^{\frac{1}{2}} + 1 \right)^{-\frac{1}{3}}$	B1 B1 B1	B1 for $\frac{2}{3}$, B1 for $\frac{1}{2} x^{-\frac{1}{2}}$ B1 for $\left(x^{\frac{1}{2}} + 1 \right)^{-\frac{1}{3}}$
	(c) $\dfrac{(2x-3)\dfrac{3x^2}{(x^3-1)} - 2\ln(x^3-1)}{(2x+3)^2}$	M1, B1 A1 [8]	M1 for differentiation of a quotient B1 for differentiation of ln term A1 for everything else correct

11 (i) 3	B1	
(ii) $a = 6e^{2t} + 4$	M1, A1	M1 for attempt at differentiation
When $t = 0$, $a = 10$	A1	
(iii) $x = \frac{3}{2}e^{2t} + 2t^2 (+ c)$	M1, A1	M1 for attempt at integration
$x_3 = 623.14 + c$, or 621.64 $\quad x_2 = 89.90 + c$ or 88.40	M1	M1 for attempt to find difference between x_3 and x_2 – condone omission of or incorrect c
Distance travelled = 533	A1 [8]	
12 EITHER		
(i) $\mathbb{R}$ or equivalent	B1	
(ii) $e^y = 5x - 10$, $\frac{e^y + 10}{5} = x$	M1	M1 rearrangement to x in terms of y
$f^{-1}(x) = \frac{e^x + 10}{5}$	DM1 A1	DM1 for interchange of x and y A1 for correct form
(iii) $f^{-1}(x) > 2$ or $y > 2$	B1	
(iv) $1 = 5x - 10$	B1	
$x = 2.2$	B1	
(v) $g(\ln(5x - 10)) = \ln(5x^2 - 10)$	M1	M1 for correct order gf
$2\ln(5x - 10) - \ln 2 = \ln(5x^2 - 10)$	M1	M1 for dealing with x^2 correctly
$25x^2 - 100x - 100 = 10x^2 - 20$	A1	A1 correct quadratic– allow unsimplified
$3x^2 - 20x + 24 = 0$, leading to	M1	M1 for correct attempt at solution of a 3 term quadratic
$x = 5.10$ only	A1 [12]	A1 for valid solution only

12 OR		
(i) $f(x) > 2$	B1	
(ii) $26 = 4e^{-x} + 2$	B1	
$6 = e^{-x}$ so $x = -\ln 6$, $\ln\frac{1}{6}$ or -1.79	B1	
(iii) $\frac{(y-2)}{4} = e^{-x}$, $\ln\frac{(y-2)}{4} = -x$	M1	M1 rearrangement to x in terms of y
$f^{-1}(x) = \ln\frac{4}{x-2}$ or $-\ln\frac{x-2}{4}$	M1 A1	M1 for interchange of x and y A1 for correct form
(iv) $f^{-1}(x)$ or $y > 2$	B1	
(v) $2e^{x} - 4 = 4e^{-x} + 2$ $(2t - 4 = 4t^{-1} + 2)$	M1 A1	M1 for attempt to deal with t^{-1} or e^{-x} A1 for correct quadratic equation
$e^{2x} - 3e^{x} - 2 = 0$ $(t^2 - 3t - 2 = 0)$ $e^{x} = 3.56$ so $x = 1.27$	M1 M1 A1 [12]	M1 for solution of quadratic M1 for correct attempt to obtain x A1 for 1 solution only