ADDITIONAL MATHEMATICS

Paper 4037/12
Paper 12

Key Messages

Candidates should be encouraged to check that they have read each question carefully so that they give the answer in the correct form. If a specific form in not requested they should remember that non-exact answers should be given correct to three significant figures and angles in degrees to one decimal place, as stated on the question paper.

General Comments

Well prepared candidates were able to show their skills in applying their knowledge of the syllabus content. For these candidates, there appeared to be no timing issues. There were, however, candidates who were ill-prepared for the examination. This resulted in many questions that were poorly done or not attempted at all.

Many candidates are now setting out their solutions in an ordered and tidy fashion, thus making the marking process more straightforward. It appears that Centres are now only giving extra paper when absolutely necessary, a practice which is more than welcome and will hopefully continue.

Comments on Specific Questions

Question 1

This was intended to be a relatively straightforward question for the start of the paper, but very few candidates were able to get full marks. There was no real pattern of errors in a type of question that should be familiar to all candidates.

Answer: $a = 3, b = 2, c = 1$

Question 2

Most candidates adopted the approach of considering the discriminant of the quadratic equation. The correct critical values were obtained by most of those who did not make any algebraic slips in the consideration of the discriminant. Full marks were extremely rare as the most common final answer given was $k > \frac{1}{2}, k < -\frac{5}{2}$. Candidates did not consider the fact that, to lie below the *x*-axis, $k < -\frac{5}{2}$ is the only correct condition.

Answer: $k < -\frac{5}{2}$

Question 3

Most candidates realised that they needed to write the left hand side of the given expression in terms of a single fraction. This was usually done correctly apart from the occasional error in the expansion of $(1 + \sin\theta)^2$. Use of the correct trigonometric identity was common. However, some candidates did not show a correct factorisation of the numerator which would then result in a common factor in both numerator and denominator leading to the required result.

Question 4

Some candidates are still unfamiliar with the set notation used in the first two parts of this question.

(i) The most common error made was to list the elements in set *A* and then not to state the number of elements in set *A*.

(ii) Listing of the elements in set *B* and then not stating the number of elements in set *B* was the most common error made.

(iii) Follow through marks were available for those candidates who had incorrectly listed sets *A* and *B* and then correctly found the union of their sets *A* and *B.*

(iv) Follow through marks were available for those candidates who had incorrectly listed sets *A* and *B* and then correctly found the intersection of their sets *A* and *B.*

Answer: **(i)** 3 **(ii)** 4 **(iii)** 60°, 240°, 300°, 420°, 600° **(iv)** 60°, 420°

Question 5

(i) Most candidates were able to obtain some marks for attempting to integrate. The most common errors involved the sign and magnitude of the $\cos 3x$ term.

(ii) Candidates should be reminded that the word 'Hence' implies that candidates need to make use of work just completed. Many candidates did not see the connection between parts **(i)** and **(ii)** and started the question again. Candidates were required to show that the answer could be written in the form $a\pi + b$. It had been intended that this would be a relatively straightforward exercise of substituting limits into the result from part **(i)**. However, many candidates made use of their calculators and gave decimal answers for the terms obtained from $9x$. Many errors were also made when attempting to evaluate the terms obtained from $-\frac{1}{3}\cos 3x$, due to calculators being in the incorrect mode.

Answer: **(i)** $9x - \frac{1}{3}\cos 3x \ (+c)$ **(ii)** $8\pi + \frac{1}{2}$

Question 6

This question was 'misread' by many candidates, hence the need for candidates to read a question carefully. Many ignored the first statement that $2x-1$ was a factor of $f(x)$. Those candidates that did obtain an equation using this fact then often did not make use of it subsequently. Many chose to equate both $f(2)$ and $f(-1)$ to zero and solve the resulting equations simultaneously, never using the information concerning the remainders. Very few correct solutions were seen.

Answer: $a = -2,\ b = \frac{5}{2}$

Question 7

Provided candidates realised that part **(a)** involved permutations and part **(b)** involved combinations, most were able to gain marks. Part **(b)(iii)** was the most problematic with candidates not realising that different cases needed to be considered

Answer: **(a)(i)** 360 **(ii)** 120 **(b)(i)** 924 **(ii)** 28 **(iii)** 672

Question 8

(i) Completely correct sketches were common, showing that candidates had a good understa[nding of] the modulus function. Common errors, however, included placing the maximum point on the y[-axis] itself rather than in its correct position in the second quadrant.

(ii) Many candidates stated the coordinates of the cusps as stationary points, but were not penalised for this. Sign errors and misunderstanding of the modulus function meant that many candidates were not able to find the correct coordinates.

(iii) Very few correct solutions were seen in what was intended to be a relatively straightforward question leading on from part **(ii)**. Many candidates did not attempt this part.

Answer: **(ii)** $\left(-\frac{1}{2},\frac{25}{4}\right)$ **(iii)** $k > \frac{25}{4}$

Question 9

(a) Provided it was recognised that differentiation of a product was involved, many candidates were able to obtain a fully correct solution. There were occasional errors in signs and the omission of a factor of 2 in the differentiation of $\ln(2x+1)$.

(b) (i) Most candidates realised that they needed to differentiate a quotient. Problems occurred with the algebraic manipulation of often correct results to obtain the given answer.

(ii) Again, many candidates did not recognise the significance of the word 'Hence' and tried to integrate without making use of the result from part **(i)**. Few correct solutions were seen.

(iii) Yet again, the word 'Hence' was not appreciated with many candidates attempting to start the question again. All that was required was a substitution of limits into the response for part **(ii)**.

Answer: **(a)** $\frac{8x^3}{2x+1}+12x^2\ln(2x+1)$ **(b)(ii)** $\frac{10x}{\sqrt{x+2}}\ (+c)$ **(iii)** $\frac{40}{3}$

Question 10

(i) Most candidates were able to use Pythagoras' theorem and obtain a correct length for the line *AB*.

(ii) Many correct responses were seen from candidates who were well prepared in coordinate geometry.

(iii) This was intended to be a discriminating question, and many candidates were unable to make an attempt at this part.

The most straightforward approach was to find the displacement vector $\overrightarrow{AB}=\begin{pmatrix}4\\2\end{pmatrix}$ and then use the fact that $\overrightarrow{OC}=\begin{pmatrix}1\\4\end{pmatrix}+\begin{pmatrix}-2\\4\end{pmatrix}$ or $\overrightarrow{OC}=\begin{pmatrix}1\\4\end{pmatrix}+\begin{pmatrix}2\\-4\end{pmatrix}$. Very few candidates used this approach. Most who attempted this part usually made use of Pythagoras' theorem together with their answer to part **(ii)** and were usually able to gain most, if not all, marks. Some candidates attempted a method using the area of the triangle *ABC* and their answer to part **(ii)** with varying degrees of success.

Answer: **(i)** $\sqrt{20}$ **(ii)** $y=-2x+6$ **(iii)** $(3, 0)$ and $(-1, 8)$

Question 11

(a) (i) Provided candidates were aware of the identity matrix, most gave the matrix required corre

(ii) Many correct solutions were seen. Some candidates squared each element rather than carry out correct matrix multiplication.

(iii) Many candidates were able to obtain a correct inverse matrix for $\mathbf{A}^2$, but then carried on and performed another unnecessary matrix multiplication (often the identity matrix). This frequently resulted in an incorrect matrix.

(b) Most candidates were able to attempt to find the determinant of the given matrix, often with sign errors. Few were able to complete the question correctly. Having obtained a quadratic equation that had no real roots, many contrived to find real solutions for this equation, not realising that they had to show that the equation did not have real roots.

Answer: **(a)(i)** $\begin{pmatrix} 4 & 3 \\ 4 & 3 \end{pmatrix}$ **(ii)** $\begin{pmatrix} 16 & 9 \\ 12 & 13 \end{pmatrix}$ **(iii)** $\frac{1}{100}\begin{pmatrix} 13 & -9 \\ -12 & 16 \end{pmatrix}$

Question 12

(a) (i) Few candidates found the correct range with many candidates finding the values of $f(-10)$ and $f(8)$. The most common response was $191 \le y \le 299$, with candidates not taking into account that there was a minimum value when $x = 0$.

(ii) Very few correct solutions were seen. Most candidates did not realise that they needed to give values of *x* which made the given function a one-one function.

(b) (i) Many correct solutions were seen showing a good understanding of inverse functions.

(ii) Most candidates who attempted this part of the question were able to apply the correct order of operations to solve the given equation. Problems arose when it came to solving the resulting equation, with many incorrect methods seen, many of which led to a fortuitously correct answer, the most common error of this type being $\ln 5x = \ln 5,\ x = \frac{\ln 5}{\ln 5} = 1$.

Answer: **(a)(i)** $-1 \le y \le 299$ **(ii)** $x \ge 0$ or equivalent **(b)(i)** $g^{-1}(x) = \ln\left(\frac{x+2}{4}\right)$ **(ii)** $x = 1$

ADDITIONAL MATHEMATICS

Paper 4037/13
Paper 13

Key Messages

Candidates should remember that the rubric requires that non-exact answers should be given correct to three significant figures with the exception of angles in degrees which should be given correct to one decimal place. Candidates must ensure that when an answer is given, particular care is taken to show all working leading to that answer. Candidates should be aware that the identities $\tan A = \frac{\sin A}{\cos A}$ and $\sec A = \frac{1}{\cos A}$ are not included in the list of formulae on the question paper.

General Comments

Most candidates were able to attempt a high proportion of the questions and the standard of presentation was very good. However, candidates should not work in pencil and then go over it in pen as this can lead to problems with legibility. Although most candidates were careful to show working, further care would be appropriate for questions with answers given and for the working in the solution of quadratic equations. Candidates would benefit from practice in questions concerning transforming relationships into straight line form.

Comments on Specific Questions

Question 1

(i) Most candidates recognised this as an application of the binomial theorem, identified the relevant term and associated it with the correct binomial coefficient and power of 2. Not all realised that in this term px was squared to obtain p^2x^2, which resulted in $240p = 60$ being used by a significant number of candidates, rather than $240p^2 = 60$.

(ii) Nearly all candidates appreciated that the sum of two terms was required and a good number obtained one term by subtracting 192 times their p. However, a significant number did not use 3 times the given coefficient of 60 for their other term.

Answers: **(i)** $\frac{1}{2}$ **(ii)** 84

Question 2

Many good solutions were seen, with candidates generally showing a good understanding of the laws of logarithms. Most candidates knew that $2\lg y = \lg y^2$ and that $1 = \lg 10$, but some candidates would benefit from practice in obtaining a correct expression when logarithms are subtracted or added to form a single logarithm. A significant number of candidates did not realise that –10 was not a valid solution.

Answer: 60

Question 3

Many good and ingenious proofs were given, with a majority of candidates being careful to show ev... in their proof. The majority of successful proofs started with a use of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$. Candidates who u... $\tan^2\theta = \sec^2\theta - 1$ and $\sin^2\theta = 1 - \cos^2\theta$ tended to make sign errors or produce steps that led back to the left hand side. Candidates should be aware that, in trigonometric proofs, common denominators and common factors must be retained throughout. Candidates should also be aware that, although the identity $\sin^2 A + \cos^2 A = 1$ is provided in the formula list, the other relationships required in this question, i.e. $\tan A = \dfrac{\sin A}{\cos A}$ and $\sec A = \dfrac{1}{\cos A}$, should be known.

Question 4

(i) Candidates need to be aware that in questions where an answer is given, or partially given as in this case, they need to show in full the steps taken to arrive at the answer and that they should ensure that those steps are accurate**.** Nearly all candidates knew that they had to use the quotient rule and a very high number used it correctly. Candidates who chose to use the product rule also did well. Some candidates missed out steps in their working leading to the given answer. Some had difficulty in expressing the answer in the given form and needed to take more care when cancelling powers of $(x + 3)$.

(ii) Candidates should remember that when an expression of this form is equated to zero, the solution comes from $x + 2 = 0$ and not from the denominator $(x + 3)^3$ or from e^{2x}. An exact answer was requested and candidates should be aware that, in order to achieve this, the y-coordinate has to be given as a power of e. This question was left out by some candidates who did not realise that it could be answered without a successful outcome to the previous part.

Answers: **(i)** $A = 2$ **(ii)** $(-2, e^{-4})$

Question 5

(i) Candidates need to be aware that a clear understanding of function notation is necessary and that they should appreciate the difference between $\mathrm{ff}(x)$ and $(\mathrm{f}(x))^2$. Many candidates obtained $2(2x^3)^3$ but many had difficulty with dealing with the powers when trying to simplify that expression before substituting. Candidates who correctly obtained $\dfrac{1}{32}$ often had difficulty expressing this as a power of 2.

(ii) This question required careful reading and candidates need to be aware that they are required to associate rate of change with differentiation and not just equate the two functions. Candidates who equated two derivatives nearly always went on to obtain a quadratic equation and a correct solution.

Answers: **(i)** 2^{-5} **(ii)** $\dfrac{1}{3}, -2$

Question 6

There were many good solutions to this question with the majority of candidates working in terms of required. There were some candidates who obtained the area under the curve by integration and went further and there were some candidates who found the area under the curve between 0 and 4 or between $\sqrt{2}$ and 4 without an appropriate subtraction. Those who formed a plan involving the subtraction of the area of the trapezium from the area under the curve between 0 and $\sqrt{2}$ were usually successful. Candidates who subtracted two equations and integrated tended to be less successful as the equation of the straight line used was sometimes that of a tangent rather than a chord. There was also sometimes confusion with the signs of the terms. Some candidates tried to form the equation of a chord but used (0, 4) and($\sqrt{2}$, 0) rather than (0, 4) and ($\sqrt{2}$, 2).

Answers: $\frac{\sqrt{2}}{3}$

Question 7

(i) Candidates should be familiar with matrix terminology and need to be aware that in this question the determinant of **A** is required rather than $\frac{1}{\text{determinant } \mathbf{A}}$. Candidates would benefit from practice in simplifying expressions such as $2t^2 - 2(t^2 - t + 1)$ as care is required to multiply each term in the bracket by –2 to obtain an expression with each term multiplied by 2 and with correct signs.

(ii) Most candidates correctly substituted for *t* and found an inverse for the matrix **A**. Candidates should be aware that the use of 'hence' meant that they were expected to use $\mathbf{A}^{-1}$ in a matrix method of solution of the simultaneous equations and that an algebraic solution should not be used. Candidates should realise that the first equation is equivalent to 6x + 2y = 10 and that a solution can then be obtained by pre-multiplying $\begin{pmatrix} 10 \\ 11 \end{pmatrix}$ by $\mathbf{A}^{-1}$. Candidates who used a matrix method of solution showed a good understanding of this method and of the matrix multiplication involved.

Answers: **(i)** $\frac{3}{2}$ **(ii)** $x = 2$, $y = -1$

Question 8

(i) Most candidates correctly equated the area of the triangle, using $\frac{1}{2} \times 4^2 \sin COD$, to 7.5. For this question, merely concluding that $\sin^{-1}\frac{15}{16} = 1.215$ was insufficient. Candidates should be aware that as the given answer is stated as being to 3 decimal places, they are required to show an answer to their calculation with at least 4 decimal places. This answer would then be rounded to the given answer. Candidates are advised that when an answer can be directly obtained in radians they should not risk introducing inaccuracy by first obtaining an answer in degrees and converting.

(ii) Candidates should be aware that, in this question, they should use arc length $s = r\theta$, which is not a given formula and should be learned. Many candidates formed a correct plan for this question but it was common to make the assumption that *OCD* was an equilateral triangle rather than calculate the length *CD*. Candidates should be aware that the cosine rule is given in the list of formulae and would benefit from practice in the order of operations involved in its application. Candidates using right angled triangles to find *CD* need to remember that the angle sum of a triangle is π when angles are measured in radians. Candidates who used an angle measured in degrees often introduced unnecessary errors and inaccuracy.

(iii) This part was well answered with most candidates identifying a correct plan. Som... appeared to forget that the area of the triangle had been given in part **(i)** and did unnece... work. Candidates need to learn the formula $A = \frac{1}{2}r^2\theta$ as it is not given on the question pap...

Answers: **(ii)** 15.9 **(iii)** 14.4

Question 9

(a) (i) This question was well answered with most candidates obtaining and solving a quadratic equation in $\cos x$. Candidates should remember that working for quadratic equations should be shown. Finding an angle from the positive value of $\cos x$ was well done but candidates would benefit from practice in finding angles from negative values of $\cos x$ in a given range.

(ii) This question was well answered, with most candidates preferring a fresh start to the problem rather than equating their values obtained for $\cos x$ in the previous part to $\sin y$. Most candidates found solutions from $\sin y = \frac{1}{3}$ correctly, but some candidates found extra values within the given range from $\sin y = -\frac{1}{2}$.

(b) A good number of candidates obtained $\tan z = \frac{4}{3}$ but not always from $\cot z(4\cot z - 3) = 0$. Therefore, very few candidates realised that there could be a further solution coming from $\cot z = 0$, and even those who obtained $\cot z = 0$ assumed that this solution was zero. Candidates should be aware that $\tan z = \frac{4}{3}$ has only one solution in the given range. It is also necessary to remember that solutions for angles in radians should be given to 3 significant figures.

Answers: **(i) (a)** 70.5°, 120° **(b)** 19.5°, 160.5° **(ii)** 0.927, 1.57

Question 10

(i) Candidates should be aware that the given relationship can be expressed as $\lg t = n \lg s + \lg k$.

(ii) Candidates who were aware that $\lg s$ had to be used for the horizontal axis often went on to produce well drawn and accurate graphs. However, some candidates found the vertical scale difficult to work with and some chose a difficult horizontal scale.

(iii) Candidates need to understand that they are required to obtain values of n and k from their graph and that this has to be a graph of $\lg t$ against $\lg s$. The majority of successful candidates found the gradient using two points on their line to obtain n and used the $\lg t$ intercept to obtain $\lg k$ and hence k. The minority of candidates who used points on their line to form simultaneous equations were also usually successful. Some candidates knew to find a gradient but either lost the negative sign or did not equate it to n. Candidates would benefit from practice in this type of question, both in identifying the relationship required for a correct straight line graph and in using points from that graph. They should be aware that points from their line and not points from their table should be used.

(iv) The majority of successful candidates used their straight line graph to find $\lg s$ and hence s. Candidates who used their values of k and n in either $t = ks^n$ or $\lg t = n\lg s + \lg k$ tended to confuse the order of operations or not deal with logarithms correctly.

Answers: **(i)** $\lg s$ **(iii)** k =100, n = –2 **(iv)** 4.9

Question 11

(i) Some candidates seemed to be confused by the notation in this question and althou[illegible] candidates attempted to substitute limits not all integrated before doing so. Candidates sho[illegible] aware that in this question they need to show fully the steps taken to arrive at the given answer a[illegible] that they should ensure that those steps are completely accurate.

(ii) This question was well answered and the majority of candidates realised that the suggested substitution led to a quadratic equation. Candidates should be aware that if a decimal value is given as an answer it should be given correct to 3 significant figures.

Answer: **(ii)** 0.458, –0.347

ADDITIONAL MATHEMATICS

Paper 4037/22
Paper 22

Key Messages

In order to do well in this paper, candidates need to have a sound knowledge of all the subject areas on the syllabus. Candidates should remember that in order to be accurate to 3 s.f. they should work to 4 s.f. up to the end of the calculation and only then give the final answer to 3 s.f. Too often candidates either rounded too early or did not work to sufficient significant figures throughout, thus losing marks.

General Comments

There were the usual wide spread of marks with many candidates demonstrating a thorough grasp of all subject areas. However, there were a number of candidates who should perhaps not have been entered for this syllabus as they only achieved single figure scores on this paper.

Candidates answered the following questions well: **Question 1** (inequalities), **Question 2** (surds), **Question 5** (indices) and **Question 6(a)** (binomial expansion).

Candidates had most difficulty on the following questions: **Question 4(iii)** (logs), **Question 6(b)** (more demanding binomial), **Question 8** (graph), **Question 9** (relative velocity), **Question 11** (calculus involving exponentials).

Candidates should be dissuaded from answering in pencil first and writing over this attempt in ink as this can lead to legibility problems.

Comments on Specific Questions

Question 1

Almost all candidates were able to successfully factorise the correct quadratic equation. Inevitably a few got the signs reversed, a few chose 2 and 3 as the relevant factors of 6 and a number chose to use 5 and 1. Those who factorised correctly usually went on to identify 1 and – 6 as the critical values and those who used the formula were also successful in this respect. The final step of putting the critical values into the correct inequality proved to be more difficult. Many produced two separate inequalities without linking them, with $x < -6$, $x < 1$ being the most popular incorrect answer. Quite a number produced diagrams with all the potential inequalities displayed but did not clearly identify the correct answer.

Answer: $-6 < x < 1$

Question 2

Most candidates knew that they needed to square the numerator before rationalising but many were let down by errors in evaluation, particularly sign errors. Some candidates made things more difficult by not simplifying the numerator and $(80 - 16\sqrt{5} + 4)$ being multiplied by $(\sqrt{5} + 1)$ was often seen. Even those who arrived at the correct expression of $\dfrac{68\sqrt{5} + 4}{4}$ often divided only one of the terms in the numerator by 4.

Answer : $17\sqrt{5} + 1$

Question 3

(i) This part was generally well done with the vast majority of candidates clearly aware of how to use the chain rule. However, a number of candidates corrupted a correct expression by taking a factor of 2 inside the bracket.

(ii) Most candidates made some attempt to use part **(i)** but only the more capable were successful. Many did not realise that they needed to use both $x = 12$ and $\delta x = p$ with their expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. An error that was often seen was the use of 12 + p for δx rather than just p. Some candidates seemed unaware of this application of calculus and made no attempt at this part of the question.

Answers: **(i)** $2\left(\frac{1}{4}x - 5\right)^7$ **(ii)** $-256p$

Question 4

This question prompted a variety of responses and, in many cases, indicated considerable uncertainty about logarithms.

(i) A common incorrect answer was 25.

(ii) Answers of $\frac{1}{5}$ and 1 – 5 = – 4 were often encountered, whilst many candidates left their answer as $\log_p 1 - 5$.

(iii) Candidates found this part challenging and were not able to cope with the change of base. The misconceptions often led to the incorrect answers of $\frac{1}{5}\times\frac{1}{2}$ and $\frac{1}{5}+\frac{1}{2}$.

Answers: **(i)** 10 **(ii)** –5 **(iii)** $\frac{1}{7}$

Question 5

This question was answered well by many candidates, including some who struggled elsewhere. Manipulation to produce and solve two linear equations was generally handled accurately although 9^y became 3^{3y} more often than it should. Those candidates who attempted the problem by using logarithms usually made no progress, despite considerable determination by the candidates, leading to a plethora of complicated equations.

Answers: $x = 3$, $y = -0.5$

Question 6

(a) (i) This was generally well done although some candidates lost the minus sign, some did not cube the two and a few had problems evaluating the binomial coefficient.

(ii) Candidates seemed to be familiar with this type of question and it was tackled well by many. Most errors were due to mistakes with the signs but most candidates were adept at selecting the relevant terms.

(b) This question was not dealt with very well and many candidates left their unsimplified expansion as their final answer. Dealing with products of powers of $\sqrt{x}$ was found difficult by most which often resulted in just the first and last terms being correctly evaluated.

Answers: **(a)(i)** –160 **(a)(ii)** –130 **(b)** $16x^2 + 32x + 24 + \frac{8}{x} + \frac{1}{x^2}$

Question 7

(i) Many candidates did not appreciate the rigour required in a ‘show that’ question and did not the detail required to obtain all the marks. Most were able to establish that the length of the was $\frac{3500}{x^2}$ even though many used L for this length which caused confusion with the L given in the question. Many were not able to explain clearly that there were 14 lengths of x required.

(ii) This part was done well on the whole even by those candidates who did not score in part **(i)**. Some candidates had difficulty dealing with differentiating negative powers of x but most knew how to tackle the problem. Quite a few forgot to find the value of L.

Answers: **(i)** $x = 10$, $L = 210$. Minimum

Question 8

This was a challenging question for many candidates.

(i) Many used x on the horizontal axis and as a result were not able to score any further marks.

(ii) Those candidates who did identify x^2 as required generally plotted the graph accurately and drew a straight line as requested.

(iii) Those with a correct graph or a correct table of values were usually successful in establishing values of a and b. There were some who calculated the inverse of the gradient and also some whose y intercept was a little inaccurate, but most got appropriate values.

(iv) Few knew how to tackle this and those who did manage to use their graph often quoted their result as x when it was, in fact, x^2.

Answers: **(i)** x^2 **(iii)** $a = 0.4$, $b = 3.2$ **(iv)** 4.8

Question 9

Completely correct solutions were rare and the vast majority of candidates had little or no idea about what was involved in this topic. Many candidates were able to evaluate the speed required to get from P to Q and the angle that PQ made with the bank. A number of these candidates then stopped, presumably under the impression that they had answered the question.

Some thought that the boat should be steered at right angles to the bank. Others seemed to be happy dealing with a sine or cosine rule calculation in which mixed measures of distance and speed occurred.

The few candidates who set up an appropriate triangle to find the required speed and angle usually did not work to sufficient accuracy to earn all the marks.

Answers: Speed 5.23 ms^{-1}; angle 39.6°

Question 10

(i) Many candidates got *AB* correct using a variety of methods usually $24\sin 0.7$ and, less fre... the cosine rule. Most knew to use $r\theta$ but many did not correctly identify θ as $2\pi - 1.4$ often ... just 1.4 or $\pi - 1.4$.

There were a variety of other values which were considered to be equivalent to the required angle. Some candidates also included the lengths *OA* and *OB* in the perimeter calculation. A number also combined lengths and areas in their calculation.

(ii) The problems regarding the value of θ continued to hinder progress here. Incorrect application of either $\frac{1}{2}ab\sin C$ or $\frac{1}{2}\text{base} \times \text{height}$ for the area of the triangle also caused difficulty.

Answers: **(i)** 74.1 cm **(ii)** 422 to 423 cm^2

Question 11

(i) Differentiation of $e^{\frac{x}{3}}$ often proved a challenge with many candidates including an *x* or changing the power. Many did not substitute $x = 9$ into their gradient function to get a numerical value and proceeded to use their general gradient function in attempting to find the equation of the tangent. This gained no credit.

(ii) Integration of the function was not always successful and the correct limits for the area under the curve were not always in evidence, with 6 often being used. Many tried to find the area under the line by integration rather than use the easier method of finding the area of a right angled triangle. This attempt was sometimes combined with the area under the curve in one integral disregarding the fact that the limits were not the same.

Answers: **(i)** (6, 0) **(ii)** $1.5e^3 - 3$

Question 12

(a) Most candidates took the correct step of replacing $\operatorname{cosec} x$ by $\frac{1}{\sin x}$ but many then had difficulty in manipulating the expression to arrive at an equation in a single trigonometric function. A number got the function inverted along the way. Those who obtained the correct expression for $\tan x$ usually obtained the two correct answers.

(b) Candidates seemed confident working in radians and only a few resorted to degrees. Accuracy of answers proved to be a stumbling block in this question together with the fact that many candidates were not able to get four answers. A number of candidates just got 0.79 leading to 0.898 and did not proceed any further. A sizeable number had no grasp of the concepts and started off by expanding $7\sin(2y - 1) = 14\sin y - 7$ or something similar.

Answers: **(a)** 164.1°, 344.1° **(b)** 0.898, 1.67, 4.04, 4.81

ADDITIONAL MATHEMATICS

Paper 4037/23
Paper 23

Key Messages

Each question should be read carefully by the candidate before answering, in order to be absolutely clear on what is required.

The work leading to an answer should always be shown so that marks for method can be awarded, even if the answer is incorrect.

When a question refers to a specific method of solution, whether it is a method to be used, or a method not to be used, enough work must be shown by the candidate to demonstrate that the instruction is being followed.

General comments

There was great variation in the quality of work seen. Some good marks were obtained by candidates who presented very clear answers which displayed an impressive range of knowledge and skills; the standard of algebra especially was generally very good. But there were also some very low marks, where candidates obviously needed more practice in all topics before entering for the examination.

It might seem superfluous to remark that method must always be shown, as well as the answer, when solving a mathematical problem. This becomes even more essential when a question makes reference to a specific method of solution (see **Question 8** below). In such a situation an answer unsupported by working, even a correct answer, cannot be credited if the Examiner cannot see that the instruction has been followed.

Candidates should be advised not to overwrite a first attempt, presumably in pencil, with a second, presumably in pen. This makes the script extremely difficult for the Examiner to read and if an answer cannot be read and understood, credit for the work cannot be given.

Comments on specific questions

Question 1

This was generally well answered, although some candidates seemed to think that stationary points are those where $y = 0$. A few wasted time by determining the nature of the points, when careful reading of the question would have made clear that this was not required.

Answers: (–2, 56), (6, –200)

Question 2

There was a very mixed response to this question. Candidates achieved more success with parts **(i)** and **(ii)** than with part **(iii)**. Throughout, there often seemed to be a tendency to use permutations and/or combinations formulae, or factorial expressions, without trying to analyse the possibilities for the four spaces in the four-digit numbers. Amongst those who did follow the latter approach in part **(iii)**, a common error was to include the impossible situation of the number beginning with 1 and ending with 1.

Answers: **(i)** 840 **(ii)** 480 **(iii)** 140

Question 3

The response to this question was very good. Commonly however, the final mark was lost beca inequalities presented were incorrect. It was also sometimes the case that the correct inequalities mig presented, then incorrect inequalities given as an answer, or vice versa. Candidates need to be aware of t importance of not making contradictory statements at the conclusion to this type of question.

Answer: $k < -2$ or $k > 8$

Question 4

Of the three values to be found in part **(a)**, *A* was most often found correctly. Much fruitless work was often seen in attempts to find *B* and *C*. Performance in part **(b)** was better, although common errors were to give the period as 3 and the amplitude as –5.

Answers: **(a)(i)** $A = 3$, $B = 2$ **(ii)** $C = 4$ **(b)** 120°, 5

Question 5

A very good general understanding of Venn diagrams was shown in this question, and there were many fully correct answers to parts **(a)** and **(b)**. Where errors occurred in part **(c)** it was usually a consequence of marking 18 and 14 on the diagram, instead of $18 - x$ and $14 - x$, for the regions representing *V* only and *W* only respectively.

Answers: **(b)** $S \cap T'$ **(c)** 4

Question 6

Good knowledge of the factor and remainder theorems was shown in this question, and many fully correct answers were seen. Where marks were lost it was usually a consequence of algebraic slips made in solving the equations formed in part **(i)**. The correct value of *x* was almost always substituted in the expression in part **(ii)**.

Answers: **(i)** 5, 4 **(ii)** 20

Question 7

Most candidates were successful in finding the coordinates of *A* and *B*, displaying sound algebraic skills in doing so. But relatively few were then able to proceed successfully to the coordinates of *C*. A great deal of fruitless work was often seen involving expressions for the lengths of the line segments *AB*, *AC* and *CB*, which could not be solved. Amongst successful attempts not using the ratio theorem on the coordinates of *A* and *B*, vector methods were popular.

Answer: (4, 0)

Question 8

Some very good, well explained, answers to part **(i)** were seen. The best answers made clear at each step of the solution which lines were being considered, and whether or not their gradients or equations were being calculated. When answers were presented with such clarity Examiners were able to award marks for method even if calculation errors were made. Weaker answers often contained many unidentified expressions and equations; when Examiners were unable to follow the logic of an answer, especially one containing errors, little credit for the work could be given. Because of the instruction given in bold at the start of the question, credit was also not given when a correct answer was produced with no supporting working.

Part **(ii)** was generally well answered, with the array method being the most common, and most efficient, method employed.

Answers: **(i)** (8, 10) **(ii)** 100 square units

Question 9

Most candidates were able to obtain some marks on this question, although full marks were relative Reasonable understanding was shown of the need to differentiate in part **(ii)**, and to integrate in par Marks were repeatedly lost in all parts through the use of angles in degrees rather than radians. Marks we also commonly lost in part **(iii)** because the bottom limit of the definite integral was completely ignored.

Answers: **(i)** 9 ms^{-1} **(ii)** –7.84 ms^{-2} **(iii)** 11.1 m

Question 10

In part **(i)**, good answers immediately established the relationship between the radius of the cone of water and its height, using the 30 cm and 120 cm, and the proof was very quickly completed. There were, however, many attempts where this was not done, and a page of mathematics was produced, to no avail.

In parts **(ii)** and **(iii)**, it was vital to work with relevant differentials, and to connect them together properly with the chain rule. More success was generally achieved in part **(ii)**, where the given formula for V could be used, even if it had not been proved, and $\frac{dV}{dh}$ readily connected to the required rate of change through the 20π given. In part **(iii)** many candidates struggled as a result of working with the formula for the circular area in terms of its radius, rather than h; it was through the latter that the required rate of change could be readily linked to the rate of change already found in part **(ii)**.

Candidates need to be aware that, in this type of question, it is not possible to differentiate an expression which contains two (or more) variables. Furthermore they need to know that it is completely incorrect to try to reduce the expression to one containing just one of the variables by substituting a specific numerical value for the other. These errors were commonly seen in part **(iii)**.

Answers: **(ii)** 0.128 cms^{-1} **(iii)** 2.51 $cm^2 s^{-1}$

Question 11

Part **(i)** clearly asked for the position vectors of the boats in terms of t. Yet many answers were seen containing no t whatsoever. Not only could no credit be given to such answers, but candidates starting this way handicapped themselves for the remainder of the question, where it was necessary to work with, or find, specific values of t.

Those who had followed the instruction in part **(i)** were often successful in part **(ii)**. Good answers to part **(iii)** found an expression for the length of the vector **AB** in terms of t, set this equal to 25, and solved the resulting equation. Some weaker attempts set the position vectors equal to each other as though the boats were meeting, which they do not. Many candidates made no attempt at all at this final part.

Answers: **(i)** $(2\mathbf{i} + 4\mathbf{j})t$, $(-21\mathbf{i} + 22\mathbf{j}) + (5\mathbf{i} + 3\mathbf{j})t$ **(iii)** 13 hours