MARK SCHEME for the October/November 2014 series

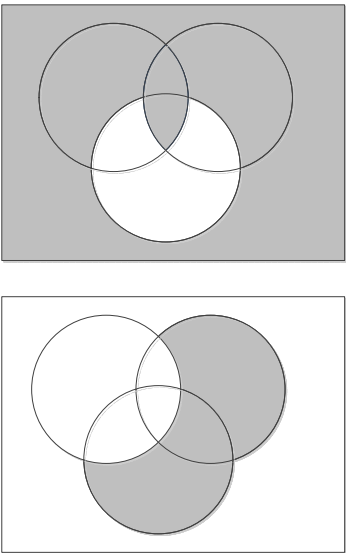## 4037 ADDITIONAL MATHEMATICS

4037/22   Paper 2, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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1	(a)		B1	
			B1	
	(b)	No.in H only $= 50 - x$; No in F only $= 60 - x$ Sum: $50 - x + 60 - x + x + 30 - 2x = 98$ $x = 14$	B1 M1 A1	Both written or on diagram Add at least 3 terms each with x involved and equate to 98 soi
2		$9x^2 + 2x - 1 < (x+1)^2$ $8x^2 < 2$ oe isw $-\dfrac{1}{2} < x < \dfrac{1}{2}$	M1 A1 A1	Expand and collect terms
3		$\log_2(x+3) = \log_2 y + 2 \quad \rightarrow \quad x+3 = 4y$ $\log_2(x+y) = 3 \quad \rightarrow \quad x+y = 8$ $x + 3 = 4(8 - x)$ $5x = 29 \quad \rightarrow \quad x = 5.8, \quad$ oe $y = 2.2$ oe	B1 B1 M1 A1 A1	Eliminate y or x from two linear three term equations

4	(i)	$f(37) = 3$ or $gf(x) = \dfrac{\sqrt{x-1}-3-2}{2(\sqrt{x-1}-3)-3}$	B1	
		$gf(37) = \dfrac{3-2}{6-3} = \dfrac{1}{3}$	B1	
	(ii)	$y = \sqrt{x-1} - 3 \;\rightarrow\; (y+3)^2 = x-1$	M1	Rearrange and square in any order
		$(x+3)^2 + 1 = f^{-1}(x)$ oe isw	A1	Interchange x and y and complete
	(iii)	$y = \dfrac{x-2}{2x-3}$ $2xy - 3y = x - 2 \;\rightarrow\; 2xy - x = 3y - 2$	M1	Multiply and collect like terms
		$\dfrac{3x-2}{2x-1} = g^{-1}(x)$ oe	A1	Interchange and complete Mark final answer
5	(i)	$B = 900$	B1	
	(ii)	$B = 500 + 400e^2 = 3455$ or 3456 or 3460	B1	3455.6 scores **B0**
	(iii)	$\left(\dfrac{dB}{dt} =\right) 80e^{0.2t}$	B1	
		$t = 10 \;\rightarrow\; \dfrac{dB}{dt} = 80e^2 = 591 \,(/\text{day})$	B1	awrt
	(iv)	$10\,000 = 500 + 400e^{0.2t} \;\rightarrow\; e^{0.2t} = (23.75)$ $0.2t = \ln 23.75$ $t = 15.8 \,(\text{days})$	M1 DM1 A1	$e^{0.2t} = k$ take logs: $0.2t = \ln k$ awrt

6	(i)	$(x+2)^2 + x^2 = 10$	B1	
		$x^2 + 2x - 3 = 0 \rightarrow (x+3)(x-1) = 0$	M1	3 term quadratic with attempt to solve
		Points $(1, 3)$, $(-3, -1)$ isw	A1	both x or a pair
			A1	both y or second pair
		or elimination of x leads to $y^2 - 2y - 3 = 0$, then as above		
	(ii)	$m^2 x^2 + 10mx + 25 + x^2 = 10$	B1	
		$(m^2 + 1)x^2 + 10mx + 15 = 0$		
		$b^2 - 4ac = (0) \rightarrow 100m^2 - 60(m^2 + 1) = 0$	M1 A1	attempt to use discriminant on three term quadratic. Allow unsimplified
		$m = \pm\sqrt{\dfrac{3}{2}}$ oe isw	A1	cao $\pm$ is required
		Alternative solution:		
		$\dfrac{dy}{dx} = \dfrac{-x}{\sqrt{10-x^2}}$ or $\dfrac{dy}{dx} = -\dfrac{x}{y}$	B1	allow unsimplified
		Result:		
		$y^2 = x^2 + 5y$ after inserted in $y = mx + 5$		
		Attempt to solve with $x^2 + y^2 = 10$	M1	Eliminate x or y
		$y = 2, x = \pm\sqrt{6}$	A1	both
		$m = \pm\dfrac{3}{\sqrt{6}}$ oe	A1	
7	(i)	$v = 2\cos t + 1$	B1	mark final answer
	(ii)	$2\cos t + 1 = 0$	M1	equate their v to zero (must be a differential) and attempt to solve to find an **angle**
		$t = \dfrac{2\pi}{3}$ or 2.09	A1	awrt
	(iii)	$t = \dfrac{2\pi}{3} \rightarrow x = 2\sin\left(\dfrac{2\pi}{3}\right) + \dfrac{2\pi}{3} = 3.83\,\text{m}$	B1	awrt
		$a = -2\sin t$	B1ft	ft *their v* (2nd differential)
		$t = \dfrac{2\pi}{3}\ a = -\sqrt{3} = -\dfrac{1.73}{4}\,\text{ms}^{-2}$	DB1ft	ft using *their* **angle** t in correct a awrt
8	(i)	$\dfrac{dy}{dx} = \dfrac{(2+x^2) \times 2x - x^2 \times 2x}{(2+x^2)^2} = \dfrac{4x}{(2+x^2)^2}$	M1 A1	apply quotient or product rule unsimplified
		$k = 4$	A1	$k=4$ does not need to be specifically identified
	(ii)	$\displaystyle\int \dfrac{x}{(2+x^2)^2}\,dx = \dfrac{1}{4} \times \dfrac{x^2}{2+x^2} + (c)$ isw	B1 B1	$\dfrac{1}{their\ k} \times$ original function

9	$\left(a+3\sqrt{5}\right)^2 = a^2 + 3\sqrt{5}a + 3\sqrt{5}a + 45$ oe	B1	anywhere
	Equate: $a^2 + a + 45 = 51$ and $6a - b = 0$	B1 B1	
	$(a+3)(a-2)=0$	M1	Attempt to solve three term quadratic with integer coefficients obtained by equating coeffs
	$a = -3, 2$ $b = -18, 12$	A1 A1	Both as correct or one correct pair Both bs correct
10 (i)	$\sec x \operatorname{cosec} x = \dfrac{1}{\cos x \sin x}$	B1	anywhere
	$\cot x = \dfrac{\cos x}{\sin x}$	B1	anywhere
	LHS $= \dfrac{1 - \cos^2 x}{\cos x \sin x}$ oe	B1ft	correct addition of *their* terms
	$= \dfrac{\sin^2 x}{\cos x \sin x} = \tan x$ AG	B1	use of identity and cancel
(ii)	$3\cot x - \cot x = \tan x \;\rightarrow\; 2\cot x = \tan x$	M1	equate and collect like terms, allow sign errors
	$\tan^2 x = 2$ oe $x = 54.7, 125.3, 234.7, 305.3$	A1 A1 A1	 2 values only 2 more values. awrt
11 (i)	Area of sector $= \dfrac{1}{2} \times x^2 \times 0.8 \left(= 0.4x^2 \,\text{cm}^2\right)$	B1	anywhere
	$SR = 5\sin 0.8 \,(= 3.59)$ or $OR = 5\cos 0.8 \,(= 3.48)$	B1	SR may be seen in stated $\dfrac{1}{2}ab\sin C$
	Area of triangle $=$ $\dfrac{1}{2}5\cos 0.8 \times 5\sin 0.8 = 6.247\,\text{cm}^2$ $0.08x^2 = 6.247$ $x = 8.837\,\text{cm}$ AG	M1 A1 A1	insert correct terms into correct area formulae
(ii)	$SQ = 8.84 - 5 \,(= 3.84\,\text{cm})$ $PR = 8.84 - 5\cos 0.8 \,(= 5.35 \text{ or } 5.36\,\text{cm})$ $PQ = 8.84 \times 0.8 \,(= 7.07\,\text{cm})$ Perimeter $= 19.84$ to $19.86\,\text{cm}$ or rounded to 19.8 or 19.9	 B1 B1 B1	 two lengths from SQ, PR, PQ awrt third length awrt sum
(iii)	Area $PQSR = 4 \times 6.247$ $= 25\,\text{cm}^2$	M1 A1	 24.95 to 25

12	(i)	$f(2) = 3(2^3) - 14(2^2) + 32 = 0$ Or complete long division	B1	
	(ii)	$f(x) = (x-2)(3x^2 - 8x - 16)$ $f(x) = (x-2)(x-4)(3x+4)$	M1 A1 M1 A1	$3x^2$ and 16 $8x$ and correct signs Factorise three term quadratic
	(iii)	$x = 2, 4$	B1	
	(iv)	$\int 3x - 14 + \dfrac{32}{x^2}\, dx = 1.5x^2 - 14x - \dfrac{32}{x}(+\,c)$ Area $= \left[1.5x^2 - 14x - \dfrac{32}{x} \right]_2^4$ $= (-)\,2$	B1 B1 M1 A1	first 2 terms third term correct unsimplified Limits of 2 and 4 and subtract