CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the October/November 2014 series

4037 ADDITIONAL MATHEMATICS

4037/23 Paper 2, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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1	**(i)**	$f(2)=0 \rightarrow 3(2)^3+8(2)^2-33(2)+p=0$ correct working to $p = 10$ AG method for quadratic factor $f(x)=(x-2)(3x^2+14x-5)$	**M1** **A1** **M1** **A1**	
	(ii)	$f(x)=(x-2)(3x-1)(x+5)$ $f(x)=0 \rightarrow x=2,\ -5,\ \frac{1}{3}$	**M1** **A1**	factorise or solve quadratic factor = 0
2	**(i)**	${}^{12}C_4=495$	**B1**	
	(ii)	${}^7C_2\times{}^5C_2=21\times10$ $=210$	**M1** **A1**	
	(iii)	not K and B = ${}^6C_2\times{}^4C_1=15\times4=60$ K and not B = ${}^6C_1\times{}^4C_2=6\times6=36$ 60 + 36 96 OR K and B = ${}^6C_1\times{}^4C_1=6\times4=24$ not K and not B = ${}^6C_2\times{}^4C_2=15\times6=90$ 210 – 90 – 24 96	**B1** **B1** **M1** **A1** **B1** **B1** **M1** **A1**	
3	**(i)**	C is (1, 6) D is $(1,6)+(12,9)$ = (13, 15)	**B1** **M1** **A1ft**	
	(ii)	gradient of $CD=\frac{15-6}{13-1}\left(=\frac{3}{4}\right)$ gradient of $AB=\frac{10-2}{-2-4}\left(=\frac{8}{-6}=\frac{-4}{3}\right)$ $\frac{3}{4}\times\frac{-4}{3}=-1$ lines are perpendicular	**B1ft** **B1** **B1**	 correct completion www
	(iii)	area $=\frac{1}{2}\times AB\times CD=\frac{1}{2}\times10\times15$ =75 or array method	**M1** **A1**	good attempt at two relevant lengths for $\frac{1}{2}$ base × height method

4	**(i)**	$2000 = 1000e^{a+b} \rightarrow a+b = \ln 2$	**B1**	
	(ii)	$3297 = 1000e^{2a-b} \rightarrow 2a+b$ $= \ln 3.297$ oe	**M1** **A1**	substitution of 2, 3297 and rearrange
	(iii)	Solve for one value $a = 0.5$ and $b = 0.193$ or 0.19	**M1** **A1**	
	(iv)	$n = 10 \quad P = 1000e^{5.193}$ $= \$180\,000$.	**M1** **A1**	
5	**(i)**	$\overrightarrow{OX} = \mu(a+b)$	**B1**	
	(ii)	$\overrightarrow{RP} = b-3a$ or $\overrightarrow{RX} = \lambda(b-3a)$ oe $\overrightarrow{OX} = 3a+\lambda(b-3a)$	**B1** **B1**	
	(iii)	$\overrightarrow{OX} = \overrightarrow{OX}$ and equate both coefficients $\mu = 3-3\lambda \quad \mu = \lambda$ $\mu = \lambda = 0.75$ $\frac{RX}{XP} = 3$ or 3:1	**M1** **A1** **A1ft**	$\frac{\lambda}{1-\lambda}$
6	**(i)**	$m = 4$ equation of line is $\frac{\ln y - 39}{3^x - 9} = \frac{39-19}{9-4}$ $\ln y = 4(3^x)+3$	**B1** **M1** **A1ft**	forms equation of line ft only on their gradient
	(ii)	$x = 0.5 \rightarrow \ln y = 4\sqrt{3}+3 = 9.928$ $y = 20\,500$	**M1** **A1**	correct expression for ln$y$
	(iii)	Substitutes y and rearrange for 3^x Solve $3^x = 1.150$ $x = 0.127$	**M1** **M1** **A1**	

7	**(i)**	$x = \frac{2}{y} + 1 \quad \rightarrow \quad y = \frac{2}{x-1}$	**M1**	any valid method
		$f^{-1}(x) = \frac{2}{x-1}$	**A1**	
	(ii)	$gf(x) = \left(\frac{2}{x} + 1\right)^2 + 2$	**B2/1/0**	–1 each error
	(iii)	$fg(x) = \frac{2}{x^2+2} + 1$	**B2/1/0**	–1 each error
	(iv)	$ff(x) = \frac{2}{\frac{2}{x}+1} + 1 = \frac{2x}{x+2} + 1$	**M1**	correct starting expression
		$= \frac{3x+2}{x+2}$	**A1**	correct algebra to given answer
		$\frac{3x+2}{x+2} = x \quad \rightarrow \quad x^2 - x - 2 = 0$	**M1**	form and solve 3 term quadratic
		$(x-2)(x+1) = 0$ $x = 2$ only	**A1**	
8	**(i)**	$v = C + K\sin 2t \qquad C \neq 0$	**M1**	
		$v = 5 + 6\sin 2t$	**A1**	
		$a = 12\cos 2t$	**A1ft**	
	(ii)	$a = 0 \quad \rightarrow \quad \cos 2t = 0$ and solve	**M1**	set $a = 0$ and solve for t
		$t = \frac{\pi}{4}$ or 0.785 or 0.79	**A1**	
		$v = 5 + 6\sin\frac{\pi}{2} = 11$	**A1ft**	ft only on K
	(iii)	$v = 2 \quad \rightarrow \quad \sin 2t = -\frac{1}{2}$ and solve	**M1**	set $v = 2$ and solve for t
		$t = \frac{7\pi}{12}$ or $1.83 - 1.84$	**A1**	
		$a = 12\cos\frac{7\pi}{6} = -6\sqrt{3}$ or -10.4	**A1**	

9 **(i)**	$\frac{dy}{dx} = 4 - \frac{1}{(x-2)^2}$	**B1**	
	$\frac{dy}{dx} = 0 \rightarrow (x-2)^2 = \frac{1}{4}$ $(4x^2 - 16x + 15 = 0)$	**M1**	solve 3 term quadratic from $\frac{dy}{dx} = 0$
	$x = 2.5$ or 1.5	**A1**	x values or 1 pair
	$y = 12$ or 4	**A1**	y values or 1 pair
	$\frac{d^2y}{dx^2} = 2(x-2)^{-3}$	**M1**	use $\frac{d^2y}{dx^2}$ with solution from $\frac{dy}{dx} = 0$
	$x = 2.5 \rightarrow \frac{d^2y}{dx^2} > 0 \rightarrow$ minimum $x = 1.5 \rightarrow \frac{d^2y}{dx^2} < 0 \rightarrow$ maximum	**A1**	both identified www
(ii)	$x = 3 \rightarrow \frac{dy}{dx} = 3$	**B1**	
	Use $m_1 m_2 = -1$ for gradient normal from gradient tangent	**M1**	must use numerical values
	Eqn of normal : $\frac{y-13}{x-3} = -\frac{1}{3}$	**A1ft**	
	Intersection of norm and curve		
	$14 - \frac{x}{3} = 4x + \frac{1}{x-2}$	**M1**	equation and attempt to simplify
	$13x^2 - 68x + 87 = 0$	**DM1**	attempt to solve 3 term quadratic
	$x = \frac{29}{13}$ or 2.23	**A1**	
10 **(i)**	$\text{LHS} = \frac{1+\cos x + 1 - \cos x}{(1-\cos x)(1+\cos x)}$	**B1**	correct fraction
	$= \frac{2}{1-\cos^2 x}$	**B1**	correct evaluation
	$= \frac{2}{\sin^2 x} = \text{RHS}$	**B1**	use of $1 - \cos^2 x = \sin^2 x$ and completion of fully correct proof
(ii)	$2\text{cosec}^2 x = 8$	**M1**	identity used
	$\sin^2 x = \frac{1}{4}$	**A1**	
	$\sin x = \pm\frac{1}{2}$	**A1**	
	$x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$	**A1**	