



## ADDITIONAL MATHEMATICS

4037/01
Paper 1
For Examination from 2013
SPECIMEN PAPER

Candidates answer on the Question Paper.
No Additional Materials are required.

## READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.
Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80 .

This document consists of 15 printed pages and 1 blank page.

## Mathematical Formulae

## 1. ALGEBRA

## Quadratic Equation

For the equation $a x^{2}+b x+c=0$,

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} .
$$

Binomial Theorem

$$
(a+b)^{n}=a^{n}+\binom{n}{1} a^{n-1} b+\binom{n}{2} a^{n-2} b^{2}+\ldots+\binom{n}{r} a^{n-r} b^{r}+\ldots+b^{n},
$$

where $n$ is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}$.

## 2. TRIGONOMETRY

Identities

$$
\begin{gathered}
\sin ^{2} A+\cos ^{2} A=1 \\
\sec ^{2} A=1+\tan ^{2} A \\
\operatorname{cosec}^{2} A=1+\cot ^{2} A
\end{gathered}
$$

Formulae for $\triangle A B C$

$$
\begin{gathered}
\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} . \\
a^{2}=b^{2}+c^{2}-2 b c \cos A . \\
\Delta=\frac{1}{2} b c \sin A .
\end{gathered}
$$

1 Shade the region corresponding to the set given below each Venn diagram.

$A \cup(B \cap C)$

$A \cap(B \cup C)$

$(A \cup B \cup C)^{\prime}$

2 Find the set of values of $x$ for which $(2 x+1)^{2}>8 x+9$.

3 Prove that $\frac{\sin A}{1+\cos A}+\frac{1+\cos A}{\sin A} \equiv 2 \operatorname{cosec} A$.

4 A function f is such that $\mathrm{f}(x)=a x^{3}+b x^{2}+3 x+4$. When $\mathrm{f}(x)$ is divided by $x-1$, the re is 3 . When $\mathrm{f}(x)$ is divided by $2 x+1$, the remainder is 6 . Find the value of $a$ and of $b$.

5 (i) Solve the equation $2 t=9+\frac{5}{t}$.
(ii) Hence, or otherwise, solve the equation $2 x^{\frac{1}{2}}=9+5 x^{-\frac{1}{2}}$.

6 Given that $\mathbf{a}=5 \mathbf{i}-12 \mathbf{j}$ and that $\mathbf{b}=p \mathbf{i}+\mathbf{j}$, find
(i) the unit vector in the direction of $\mathbf{a}$,
(ii) the values of the constants $p$ and $q$ such that $q \mathbf{a}+\mathbf{b}=19 \mathbf{i}-23 \mathbf{j}$.

7 (i) Express $4 x^{2}-12 x+3$ in the form $(a x+b)^{2}+c$, where $a, b$ and $c$ are constants and
(ii) Hence, or otherwise, find the coordinates of the stationary point of the curve $y=4 x^{2}-12 x+3$.
(iii) Given that $\mathrm{f}(x)=4 x^{2}-12 x+3$, write down the range of f .

8 A curve is such that $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}=4 \mathrm{e}^{-2 x}$. Given that $\frac{\mathrm{d} y}{\mathrm{~d} x}=3$ when $x=0$ and that the curve passes thi the point $\left(2, \mathrm{e}^{-4}\right)$, find the equation of the curve.

9 (i) Find, in ascending powers of $x$, the first 3 terms in the expansion of $(2-3 x)^{5}$.

The first 3 terms in the expansion of $(a+b x)(2-3 x)^{5}$ in ascending powers of $x$ are $64-192 x+c x^{2}$.
(ii) Find the value of $a$, of $b$ and of $c$.

10 (a) Functions f and g are defined, for $x \in \mathbb{R}$, by

$$
\begin{aligned}
& \mathrm{f}(x)=3-x, \\
& \mathrm{~g}(x)=\frac{x}{x+2}, \text { where } x \neq 2 .
\end{aligned}
$$

(i) Find $\mathrm{fg}(x)$.
(ii) Hence find the value of $x$ for which $\operatorname{fg}(x)=10$.
(b) A function h is defined, for $x \in \mathbb{R}$, by $\mathrm{h}(x)=4+\ln x$, where $x>1$.
(i) Find the range of $h$.
(ii) Find the value of $h^{-1}(9)$.
(iii) On the same axes, sketch the graphs of $y=\mathrm{h}(x)$ and $y=\mathrm{h}^{-1}(x)$.

11 Solve the following equations.
(i) $\tan 2 x=3 \cot 2 x$, for $0^{\circ}<x<180^{\circ}$
(ii) $\operatorname{cosec} y=1-2 \cot ^{2} y$, for $0^{\circ} \leqslant y \leqslant 360^{\circ}$
(iii) $\sec \left(z+\frac{\pi}{2}\right)=-2$, for $0<z<\pi$ radians.

12 A curve has equation $y=\frac{x^{2}}{x+1}$.
(i) Find the coordinates of the stationary points of the curve.

The normal to the curve at the point where $x=1$ meets the $x$-axis at $M$. The tangent to the the point where $x=-2$ meets the $y$-axis at $N$.
(ii) Find the area of the triangle $M N O$, where $O$ is the origin.

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