STATISTICS

Paper 4040/12

Paper 12

Key Messages

If a question specifies a certain degree of accuracy for numerical answers, full marks will not be obtained if the instruction is not followed.

Candidates need to develop the skill of holding the intermediate values of a calculation in the calculator to obtain maximum accuracy in the final answer.

Candidates should try to relate their knowledge to the specific requirements of a question rather than simply repeat memorised knowledge.

A valuable skill is to have a rough idea of the magnitude of the answer to be expected in a problem to see that an answer obtained is reasonable.

General comments

The overall standard of work was comparable to that of last year. Some high marks were obtained, and there were few exceptionally low marks. As is noted regularly in these reports, there were again many instances of marks being needlessly lost due to final answers not being given to the required accuracy where this was stated in the question. Too often it seems as though an instruction printed in bold at the start of a question (see **Question 8** below) is totally ignored. This year some good attempts were seen from candidates trying to answer those parts of questions requiring comment related to results calculated for the situation in the question. However, in one particular case (see **Question 7** below) answers tended to be mathematical rather than contextual.

Any candidate of statistics ought to be able to observe whether or not the result of a calculation is reasonable in a given practical situation. If it is clearly unreasonable, the work can be checked to find the error. If the bags heavier than 20 kg on an aeroplane are classified as overweight (see **Question 9** below) it should be obvious that the median weight of the overweight bags cannot be less than 20 kg.

It sometimes seems as though candidates try to write down as much as they know when giving an explanatory answer, with the result that the words overflow from the answer space. Careful thought is given in the preparation of examinations to ensure that plenty of space is available for the construction of good answers. If candidates find themselves writing much more than they can fit into the answer space, they should pause to consider if they are properly focused on the specific requirements of the question.

Comments on specific questions

Section A

Question 1

In part **(ii)** many criticisms were offered, but the difficulty of quantifying a half face, for example, is relatively trivial compared with the overwhelming fault of the pictogram in conveying a false visual impression of the relative numbers of satisfied and dissatisfied customers. In part **(iii)** almost all said (correctly) that the results would be biased, but tended to focus on the bias of the interviewers, rather than the fact that the survey was done on one particular weekday afternoon. Good answers took account of this given information, which was quite specific and required no imaginative speculation, and referred to the fact that, for example, people working during the day were excluded, or people who shopped on another day were excluded.

Answers: **(i)(a)** 300 **(b)** 39 **(ii)** because different scales are used the false impression is given that more customers were dissatisfied than satisfied **(iii)** biased

Question 2

The responses to part **(i)** were mixed, with few candidates giving four correct answers. There were many correct answers to part **(ii)**, but rather more answers than there should have been which contained the values 1.5 cars and 4.5 cars.

Answers: **(i)(a)** true **(b)** true **(c)** false **(d)** false **(ii)** 1, 5, 3 or 2, 4, 3

Question 3

There were few fully correct answers to this question. As was observed last year, many candidates do not understand clearly what the regions of the different parts of a Venn diagram represent. For example, a common answer to part **(ii)** was 6, revealing lack of understanding of the difference between bass and drums, and bass and drums but not keyboards.

Answers: **(i)** 22 **(ii)** 8 **(iii)** 40 **(iv)** 13 **(v)** 14

Question 4

Many correct answers were seen for the values of mode and median in parts **(i)** and **(ii)**. Candidates have clearly learned the disadvantages in general of the mode as a measure of central tendency and these were usually recited in part **(i)**. However, the question specifically asks why it is a poor measure “in this case”. Thus the best answers stated that here, since the mode is zero, and is the smallest value of the variable, it is simply not “central” at all for this distribution.

There were good attempts at part **(iii)**, with many adopting an algebraic approach, setting up an equation containing an unknown, and then solving. Unfortunately it was not always clear to Examiners what the variable represented, and apparently it was not always clear to the candidates either. So, for example, a common answer given was 8.5. Because the unknown had not been defined at the outset, it was not realised that this is the mid-class value of the open class, not its upper limit, so that at that stage the solution had not been completed.

Answers: **(i)** 0, smallest value **(ii)** 1 **(iii)** 12

Question 5

Many answers to part **(i)** only gave the values 1 and 2, totally ignoring the possibility that no chocolate biscuits might be obtained. There was huge variety in the quality of answers to part **(ii)**. Stronger candidates were able to consider the different possibilities for obtaining x = 0, 1 or 2, but the best answers analysed these possibilities in terms of chocolate/not chocolate, rather than chocolate/ginger/plain. For all candidates a mark was available for presenting their answers (even if incorrect) in the form of a probability distribution table.

Answers: **(i)** 0, 1, 2 **(ii)** $P(x = 0) = 1/7$, $P(x = 1) = 4/7$, $P(x = 2) = 2/7$

Question 6

This question was designed to test the ability of candidates to carry out calculations to a high degree of accuracy in a context where high accuracy is absolutely essential. Some obtained the answer to part **(i)** exactly, but many lost the required accuracy immediately by rounding or truncating values in their working to fewer than 5 decimal places. For such a problem candidates need to have the ability to retain intermediate values of maximum accuracy within the calculator, by making use of the memory. Good answers clearly displayed this ability. Candidates should also be advised that the method for standard deviation which uses Σx and Σx^2 is generally better for computational purposes than that which uses $\Sigma(x - \text{mean})^2$.

Good answers to part **(ii)** made the necessary comparisons with clear inequalities and arrived at appropriate conclusions. Some credit here was allowed where the answer to part **(i)** was incorrect, but only where sufficient decimal place accuracy was maintained in the working.

In part **(iii)** good answers displayed clear understanding that a small amount of variation was to be desired in this situation. Many revealed a misunderstanding of what the standard deviation measures, confusing it with the quantity of the dosage.

Answers: **(i)** 0.01031 **(ii)** (range = 0.027) satisfies both conditions **(iii)** disagree, as small standard deviation is a positive feature, indicating precision

Section B

Question 7

Many candidates showed good skills in reading information from the pie charts in parts **(i)** and **(ii)**, and from the histogram in parts **(iii)**, **(iv)**, **(v)** and **(vi)**. The errors which did occur were mainly in the non-use of the squares of the radii to find the total number of graduates at University *B* in part **(ii)**, and in giving column heights as answers in parts **(v)** and **(vi)**. Part **(vii)** was less well done, though a reasonable number of candidates did realise that the fraction of all graduates finding employment within 6 months had to be applied to the science graduates. The weaker answers to part **(viii)** simply expressed in words the mathematics of how the calculations in part **(vii)** had been carried out, whilst the better answers gave a contextual explanation of the assumption behind the use of the same fraction for the science graduates as for all graduates.

Answers: **(i)** 325 **(ii)** 254 **(iii)** 1 month – (under) 2 months **(iv)** 470 **(v)** 120 **(vi)** 60 **(vii)** 148 **(viii)** time taken to find employment does not depend on subject of study

Question 8

On the subject of crude and standardised rates, there have been many questions in the past on death rates. This question aimed to test if the knowledge accrued in that context could be applied to a different context. Overwhelmingly it was found that it could, and many good answers were seen to the first four parts of the question. However, as was mentioned in general comments above, this was one of the questions where marks tended to be lost through not following the accuracy instruction given in bold at the start of the question. In part **(v)** some seemed to think, incorrectly, that the sum of the group fertility rates had to be used, or the difference between the standardised fertility rates, rather than just the second and third columns in the second table to find the births in each group. Many good answers were seen to part **(vi)**.

Answers: **(i)** 91 **(ii)** 35, 204, 160, 19 **(iii)** 79 **(iv)** 86 **(v)** 283 **(vi)** Redville, because it has a higher standardised fertility rate than Bluedorf

Question 9

This was another question where an instruction in bold was given at the start of the question s followed, candidates might be allowed method marks if a given answer was incorrect. It has to be st yet again that an incorrect answer with no indication of method cannot be awarded marks. Many did sh lines on the graph, and where such lines were drawn Examiners were able to inspect them for possible credit.

Parts **(i)(a)** and **(b)** were very well done. Marks were often dropped in part **(i)(c)** where the value 24 was frequently used in the calculation instead of the cumulative frequency corresponding to 24 kg. There were also many correct answers to part **(ii)**. Part **(iii)** was less well done, with a substantial number providing just a part of the solution, using the 36 and/or the 8, but omitting to consider that the charge was only made for the amount by which the weight exceeded 20 kg.

A fair number of candidates thought that the intercept in part **(iv)** indicated that there were no bags weighing 5 kg.

Answers: **(i)(a)** 18 **(b)** 4 **(c)** 85 **(ii)(a)** 36 **(b)** 24.5 **(iii)** 1296 **(iv)** there were no bags less than 5 kg in weight

Question 10

Some good graphical work was seen in this question, but candidates should be advised to make plotted points very clear. If Examiners cannot actually see them they cannot be credited.

Very good marks were generally earned on the first four parts, with good understanding shown of the need to order data to find the semi-averages. The most accurate answers in part **(iv)** were those which used the given averages to find the gradient. Less accurate answers used points chosen from the line. The use of data points to find a gradient is extremely risky in such a situation since, if they are not on the line which passes through the averages, the method becomes invalid.

It was not often that full marks were obtained on the last three parts. Some candidates were unable to make the connection between part **(iv)** and part **(v)**. In part **(vi)** the straight line for calls longer than 2 minutes was frequently correct, but the line for calls less than 2 minutes incorrect or omitted. In part **(vii)** another accuracy instruction in the question was often ignored.

Answers: **(ii)** (25+28+34+39)/4 **(iii)** (10, 83.75) **(iv)** m = 6.3 to 6.4, c = 20 to 21 **(v)** value of c, value of m (from previous part) **(vi)** straight line from (0, 10) to (2, 10), straight line from (2, 10) to (14, 46) **(vii)** 9

Question 11

Less success was achieved on the probability question this year. Marks were earned most readily in part **(a)**, though even here many did not consider the two ways of making the choice in part **(a)(ii)**.

Many attempts at part **(b)** broke down immediately as a result of the “otherwise” alternative clearly stated in the question being ignored. Consequently too many incorrect answers of 0.6 were seen for part **(b)(i)**, which did not allow for the fact that if Kwame does not know the correct answer, he guesses. Similarly in part **(b)(ii)(b)** both parts of what was happening in the situation (the not knowing, and the answering correctly) were not considered.

Marks were occasionally earned at the start of part **(c)** from the different ways of obtaining a score of 5, but it was only the strongest candidates who made any progress with the final conditional element.

Answers: **(a)(i)** 22/35 **(ii)** 12/35 **(b)(i)** 7/10 **(ii)(a)** 343/1000 **(b)** 1/1000 **(c)** 2/15

STATISTICS

Paper 4040/13

Paper 13

Key messages

If a question specifies a certain degree of accuracy for numerical answers, full marks will not be obtained if the instruction is not followed.

If words are emphasised in a question they should be noted carefully by the candidate so that unnecessary errors are avoided.

A valuable skill is to have a rough idea of the magnitude of the answer to be expected to a problem to see that an answer obtained is reasonable.

General comments

The overall standard of work was comparable to that of last year, with a wide range of marks being obtained. As is noted regularly in these reports, there were again instances of marks being needlessly lost when final answers were not given to the required accuracy, where this was stated in the question. Too often it seems as though accuracy instructions (see **Questions 9**, **10(b)** and **11(b)** below) are totally ignored.

Any candidate of statistics ought to be able to observe whether or not the result of a calculation is reasonable in a given practical situation. If it is clearly unreasonable, the work can be checked to find the error. If the values of the variables in a distribution vary between 18 and 22 (see **Question 10(b)** below), it should be obvious that the mean has to be somewhere between these values. Similarly it should be recognised that a death rate, per thousand, cannot have a value greater than 1000.

Comments on specific questions

Section A

Question 1

Random and stratified, and to a lesser extent systematic, seemed to be best recognised of the different methods of sampling. Most incorrect answers appeared in part **(vi)**.

Answers: **(i)** quota **(ii)** systematic **(iii)** stratified **(iv)** systematic
(v) (simple) random or systematic **(vi)** quota

Question 2

Good general understanding was shown of the most appropriate choice of independent and dependent variables in this situation. A common misconception in part **(ii)** was that the children whose ages fell below the average age should be the ones chosen to find the lower semi-average. These answers, incorrectly, resulted in only four children being chosen. Some credit was given in part **(iii)** for knowing the form of a semi-average, even if the choice made in part **(ii)** was incorrect.

Answers: **(i)** age, independent, x, test score, dependent, y, score more likely to depend on age than age depend on test score **(ii)** A C F G I, these children have the five lowest values of x when the data are arranged in ascending order of x **(iii)** (135.6, 41.2)

Question 3

Candidates showed good knowledge of how to find the equation of the line of best fit from t averages, and there were many fully correct answers to parts **(i)** and **(ii)**. A good number of answers t **(iii)** lacked clarity and did not identify the relevant idea. Some answers effectively restated the ques without giving a reason.

Answers: **(i)** 2 **(ii)** $y = 2x + 2.95$ **(iii)** because there is only one straight line which passes through all the averages

Question 4

There were very few completely correct answers to this question because of the graphs presented in part **(ii)**. Candidates do not seem to have considered why the word "appropriate" was emphasised in the question, because almost all produced a totally inappropriate graph. As the variable in this situation is discrete, a continuous curve is inappropriate. Full credit could only be given where a step polygon was drawn.

Answers: **(i)** 51, 85, 95, 99, 100

Question 5

There was a wide range of responses to part **(i)** with some candidates producing a table with apparent ease, whilst others presented a table much like the original, persisting with boys and girls rather than combining them to find the number of children. Responses to part **(ii)** were better, but the values given by many did not total 15, the number of girls in the first table.

Answers: **(i)** values for number of children per family: 0, 1, 2, 3, 4, 5 corresponding values for number of families: 1, 2, 2, 2, 4, 1 **(ii)** 1, 4, 9, 1

Question 6

Basic features of the measures required in parts **(i)** and **(ii)** seem to be generally well known, though there were instances of a measure of dispersion being written in the central tendency space and vice versa. There was much less success with the other parts. In part **(iii)** some candidates did not write a unit at all, and in part **(iv)** many non-symmetrical sketches were given.

Answers: **(i)** mean, range or standard deviation or variance **(ii)** median, interquartile range or semi-interquartile range **(iii)** cm^2 **(iv)** a symmetrical multi-modal distribution with no mode at the centre

Section B

Question 7

In part **(a)(i)** there was another instance of the significance of an emphasised word in the question not being recognised by candidates. The question does not ask for the different sequences of events which may occur, only for what is recorded. Unfortunately in most instances, sequences were given, and the probabilities associated with the sequences, so marks were lost.

There were generally better responses to part **(b)**. In part **(b)(ii)** it is necessary to include the 6 arrangements of thrush, starling, robin. Many answers stopped with the product of the three probabilities. In part **(b)(iii)** too many continued using the original fractions from the start of the problem.

Part **(c)** was generally well done. Only a few answers presented probabilities instead of arrangements.

Answers: **(a)(i)** T, P(T) = 3/4, H, P(H) = 1/4 **(ii)** 9/16 **(b)(i)** 1/27 **(ii)** 5/24 **(iii)** 1/12 **(c)(i)** 2 **(ii)** 6 **(iii)** 24

Question 8

In this question an instruction in bold was given at the start of the question so that, if followed, ca might be allowed method marks if a given answer was incorrect. It has to be stressed that an inc answer with no indication of method cannot be awarded marks. Many did show lines on the graph, a where such lines were drawn they were inspected for possible credit when answers were wrong.

The methods for answering parts **(i)** and **(ii)** were well known, but because there were two curves given on the graph, readings were sometimes taken from the wrong curve. Candidates should be advised to take extra care when two curves are given.

Many candidates struggled with the remainder of the question, especially parts **(iii)**, **(iv)** and **(v)**, where a reason had to be given to explain choice. Here the best answers were those which used specific values from the graph by way of justification. For example, for part **(iii)**, the fact that the marks of the strongest candidates go up to 90 on Paper 1, whilst they only reach 70 on Paper 2.

For part **(vii)** the curves intersect at about (47, 62). It was not necessary to find this point to answer the question, but many candidates found it easier to do so in order to make an answer. Where many answers were incorrect was in deducing that there were 62 candidates who scored 47 marks on both papers. This totally overlooked the cumulative nature of the curves. The good answer using these values said that it was only the 62nd candidate who scored 47 marks on both papers.

Answers: **(i)** 45 to 46 **(ii)** 34 to 34.5, 52, 17.5 to 18 **(iii)** because marks achieved continue to a higher value, Paper 1 **(iv)** because marks achieved start at a higher value, Paper 2
(v) because marks achieved have a larger range, Paper 1 **(vi)** 68 to 69, 59
(vii) the mark above which (or below which) the same number of candidates achieved marks in both papers

Question 9

The calculation of crude and standardised death rates is well known by most, and there were many good answers to the first three parts. However, some marks were lost through not following the accuracy instruction at the start of the question. Part **(iv)** was usually answered correctly, but scarcely ever were the results used to answer part **(v)**, which was the purpose of calculating the percentages. When entered into the table, these percentages can be seen side by side with the percentages for the standard population. Too often it seemed as though memorised answers from other problems were being reproduced in answer to this question. Most made the correct choice in part **(vi)**.

Answers: **(i)** $p = 840$, $q = 2820$, $r = 14$ **(ii)** 18.06 **(iii)** 16.60 **(iv)** percentages which to the nearest whole number are 6, 27, 30, 26, 11 **(v)** the city has a higher percentage of older people than the standard population **(vi)** crude death rate, since the totals for population and number of deaths would remain the same

Question 10

There were mixed answers to part **(a)**, with some candidates taking account of the different class widths, and some ignoring them.

Even though part **(b)** began with a routine mean and standard deviation calculation, marks were routinely lost. Again the accuracy instruction was not always followed. Sometimes an incorrect formula was used. Interchange of the variable and the frequency was also seen. Candidates need to realise when an answer they find is totally unreasonable and then check their work. For example, a mean of 120 in this situation is absolutely impossible. In part **(b)(ii)** the best answers were those which used the answers from part **(b)(i)**, together with the values 0.5 and 1.6, to obtain the corrected mean and standard deviation quickly and with very little working. The more laborious approach (which was still awarded full marks if successful) was to use the 0.5 and 1.6 to correct the distance values in part **(b)(i)**, then repeat all the mean and standard deviation calculations done in part **(b)(i)** with the corrected distances. Because of all the work involved, the latter approach was more likely to result in error.

Answers: **(a)(i)** 25 **(ii)** 30 **(iii)** 14.7 **(iv)** because class widths are different
(b)(i) 20.4, 1.16 **(ii)** 33.4, 1.85 to 1.86

Question 11

There were mixed answers to the Venn diagram question. In part **(a)(i)** it was common to miss the 2 in the calculation, and in part **(a)(ii)** it was common to omit the "but not France". In part **(a)(iii)** a fair number of candidates scored marks whilst making the occasional error.

Good marks were also scored in part **(b)**, with marks most commonly lost in part **(b)(ii)**, when the squares of the radii were not used. Again, in parts **(b)(ii)** and **(iii)**, the accuracy instruction was sometimes ignored.

Answers: **(a)(i)** 3 **(ii)** the number of students who had visited Germany and Italy but not France
(iii) new values: 30, 6, 26, 6, 5, 18 **(b)(i)** 108° **(ii)** 6.6 **(iii)** 166°

STATISTICS

Paper 4040/22

Paper 22

Key message

Candidates scoring the highest marks provided clear evidence of the methods they had used in logical, clearly presented solutions to the numerical problems. In questions requiring written comments, these candidates provided detailed explanations which were in the context of the problem presented.

Candidates should always read the questions carefully and re-read them after they have provided a solution in order to ensure that the question has been answered fully and, where appropriate, in numerical questions the required degree of accuracy has been given.

General comments

In general, candidates did better on questions requiring numerical calculations than on those requiring written explanations. In particular, the multi-stage problems of **Question 3(i)**, **6**, **9** and parts of **Question 11**, which required clearly laid out logical solutions, were often well presented with candidates able to achieve marks for their method, even if numerical errors or algebraic slips led to incorrect final answers. As was noted in this report last year, there continues to be an improvement in the number of candidates providing solutions to a suitable degree of accuracy.

The graphical work in **Questions 2** and **8** tended to be very accurately presented, although axis labels were often missing.

Question 11, on probability, proved to be the least popular of the optional ***Section B*** questions, although no one question proved to be particularly unpopular this year. Weaker candidates also found **Question 9**, on expectation, difficult, although some stronger candidates scored very high marks on this question. **Questions 7** and **10** proved to be the most popular of the optional questions.

Comments on specific questions

Section A

Question 1

Many candidates were successful in distinguishing between qualitative and quantitative variables and between discrete and continuous variables. The difficulty that some encountered was to distinguish between data that is, and is not, a variable.

A majority of the candidates scored some, but not all, of the available marks in part **(i)** of this question. The most common errors were to incorrectly identify 'the number of pages in the book' as a variable and to incorrectly identify 'the number of letters in each of a random selection of 100 words' as not a variable. In part **(ii)(b)** many candidates were able to suggest suitable continuous variables, such as 'the length of the leaves' or the 'height of the trees'. There were often errors in part **(ii)(a)**, as the most common suggestion was 'the number of trees', which is not a variable. The most commonly seen correct suggestions in this part were discrete, quantitative variables such as 'the number of fruits' or 'the number of leaves in each tree', although qualitative variables such as 'the types of tree' would also have scored the mark.

Answers: **(i)** B, D, A, B, C.

Question 2

This question was particularly well done with the majority of candidates calculating appropriate perc in order to produce a percentage sectional bar chart. These tended to be accurately drawn wi appropriate scale and key. The most common error was for the label of 'percentage' to be missing from vertical scale. In part **(ii)** candidates needed to correctly state that a greater proportion or percentage o country B's electricity comes from renewable sources than country A's. Only the most able candidates expressed the comparison in this way, with the most common incorrect answer being that 'more electricity comes from renewable sources in country B than country A'.

Answers: **(i)** Percentages rounding to 62%, 30% and 8% for A and 54%, 26% and 20% for B.

Question 3

This was an example of a multi-stage problem in which many candidates were able to score marks for an appropriate method even though there were often numerical slips. The most common cause of the numerical errors was candidates struggling to cope with the combination of hours and minutes for the times to complete a marathon. There were, however, marks available for attempting to find midpoints, appropriate use of an assumed mean and correct use of the formula for finding the mean from a grouped frequency distribution. The majority of candidates provided clearly set out working and thus were able to score marks where the methods used were correct. The numerical errors tended to be a result of candidates trying to work in decimals of an hour rather than finding midpoints in hours and minutes. This latter approach leads to straightforward values in minutes only, once the assumed mean has been applied. In part **(ii)** many candidates did not notice that the data does not contain extreme values, or that it is approximately symmetrical, making the mean an appropriate measure of central tendency in this case.

Answers: **(i)** 3 hours 8 minutes.

Question 4

It was rare to see all three midpoints correctly calculated in part **(a)**. The most common error was in part **(a)(i)** where 15 was a commonly seen incorrect answer. Candidates appeared to assume that an integer value was required for the midpoint in this case. In part **(b)(i)** most candidates were able to attempt a pair of frequency polygons, although the plots were sometimes at the upper class boundaries rather than at the midpoints of the intervals, suggesting a possible confusion with cumulative frequency polygons. It was common in part **(b)(ii)** for candidates to incorrectly suggest that the waiting times at hospital A were longer when in fact the waiting times at hospital B were longer.

Answers: **(a)(i)** 14.5; **(ii)** 14.5; **(iii)** 15.

Question 5

This question was done well by many candidates, particularly parts **(i)** and **(iii)**. Most were able to use an appropriate formula but there were occasional algebraic or arithmetic errors in the expansion of brackets and rearrangement of equations, particularly where negative signs were involved. Some candidates left part **(ii)** entirely blank, whilst others inserted their answer to part **(i)** where one of the occurrences of the unknown should have appeared.

Answers: **(i)** 54; **(ii)** 24; **(iii)** 50.

Question 6

Part **(i)** of this question was generally done well, with most candidates correctly locating the position of the median and many providing a fully correct solution. A few candidates missed the instruction to give the answer to one decimal place. Part **(ii)** was a little less well done, although many of the most able candidates were successful with it. Some candidates tried to divide the whole population proportionally rather than just those in the 60 – 65 interval. Some candidates who had the correct basic idea added 4.8, rather than 3.2, to the 3 + 1 from the intervals above. Most candidates appreciated the need, in this case, to give the final answer as a whole number of eggs.

Answers: **(i)** 58.7; **(ii)** 7.

Section B

Question 7

This was a popular ***Section B*** question on sampling.

In part **(i)**, those candidates who recognised that the question was asking why a sample was preferable to a census usually gave sensible answers. Unfortunately, some candidates misunderstood and answered a different question such as 'Why might the government want to collect data about schools?' or 'Why would the government collect data from candidates (as opposed to parents or teachers)?' In part **(ii)** some candidates simply said that the sample would be biased because it would not be fair, without explaining why. Many though suggested, correctly, in **(ii)(a)** that candidates arriving together may have something in common such as their gender, proximity to the school, keenness to attend etc. which might lead to their having similar opinions. Fewer correctly gave the most likely reason for the method described in part **(b)** being biased, namely that the list may be organised in some way, for example by year group or form. Most candidates were able to identify the simple random sample in part **(iii)**, with just a few candidates missing a value out or repeating the 23 a second time. Very few candidates answered parts **(iv)(a)** and **(b)** correctly, missing the significance of leading zeros in the 3-digit numbers, but many were successful in part **(c)**, providing a suitable list of numbers with gaps of 100. In part **(v)** most candidates recognised that six represents a greater proportion of School A than School B and answers to part **(vi)** were also usually correct. In part **(vii)** some candidates suggested, incorrectly, that the interviewer would select 'at random' or 'on a first come first served basis', rather than that the interviewer would be free to choose the candidates, having been given the number to survey from each school.

Answers: **(iii)** 57, 04, 23, 16, 38, 17; **(iv)(a)** 000, 099; **(b)** 083; **(c)** 183, 283, 383, 483, 583;
(vi)(a) Stratified; **(b)** 15.

Question 8

This was a fairly popular question on moving averages.

In part **(i)** the graphical work tended to be accurate with the majority of candidates choosing an appropriate scale for the vertical axis. A few candidates chose inappropriate scales, such as scales that did not cover the full range of values to be plotted, and it was quite common for the label of '$' to be missing. In part **(ii)** some candidates incorrectly felt that the explanation was connected to the fact that seven is an odd number, rather than with the fact that the pattern can be seen to repeat itself every seven days. In parts **(iii)** and **(iv)** the values were usually correctly inserted in the table and given to the required degree of accuracy, with plots and trend lines tending to be very accurately drawn. Most candidates provided a correct interpretation of the trend line in part **(v)** giving this in the context of the question. Many candidates were successful with part **(vi)**, adding the given seasonal component to the reading from their trend line to find the required estimate. In part **(vii)** some candidates incorrectly stated that Esme was correct, thinking that a smaller value for the seasonal component on a Wednesday as compared to a Monday was correct because of the decreasing trend. There were also, however, a substantial number of correct responses recognising from the graph that the seasonal component for a Wednesday should be negative. The need to centre the moving average values in part **(viii)** was recognised by the majority of candidates.

Answers: **(iv)(a)** 146.3, 145.3, 143.4, 141.3, 140.4, 139.4, 138.9, 138.6.

Question 9

This question on probability and expectation was popular with the more able candidates, who frequently scored well on it, although it was often the question omitted by the weaker candidates.

Most candidates were successful with parts **(a)(i)** and **(ii)** of this question. In parts **(iii)** and **(iv)** some candidates did not take account of the fact that each of the outcomes can occur in three ways. Most candidates set their working out clearly and correctly in part **(v)** and thus, even when errors had been made in an earlier part of the question, they were often able to score full marks here. In part **(b)** many candidates were able to correctly deal with the instruction that the counters were now to be drawn without replacement. In part **(i)** some candidates did not realise that three counters, one of each colour, could be selected in six different ways, and in part **(ii)** it was rare to see solutions where it was appreciated that the probability that two or more counters are the same colour is one minus the probability that they are different colours.

Answers: **(a)(i)** 1/216; **(ii)** 1/6; **(iii)** 5/72; **(iv)** 2/3; **(v)** $3; **(b)(i)** 27/136; **(ii)** 109/136.

Question 10

This question on price relatives was both very popular with all candidates, and generally very well answ

In part **(a)(i)** most candidates were able to explain that the price relative of 108 implies an increase in the price of 8% between 2009 and 2010. Some candidates lost a mark because the dates or time interval were missing from their description. The calculations in parts **(ii)** and **(iii)** were often correct, although some candidates misunderstood the instruction to give the answers to the nearest cent, and instead provided answers (in dollars) to one decimal place. In part **(b)(i)** calculations of the price relatives were often accurate, although some candidates did not insert price relatives of 100 in the table for 2009, the base year. In part **(ii)** many candidates were successfully able to calculate a weighted aggregate cost index, although some weaker candidates used the hourly rates of pay rather than the expenditure as the weights in their calculation. The most able candidates explained that a change in the weights may have occurred if the number of employees at each grade had changed. Some candidates, however, incorrectly suggested that a change in the wage rates might be responsible.

Answers: **(a)(ii)** 0.69; **(iii)** 0.71; **(b)(i)** 100, 116, 121, 100, 111, 111, 100, 97, 100; **(ii)** 115 (if 111 used for grade B) or 114 (if 110.5… used for grade B).

Question 11

This question on probability was the least popular of the optional questions.

Most candidates were able to find two of the three correct pairs of mutually exclusive events in part **(a)**. In parts **(b)(i)** and **(ii)** there were often correct reasons given to explain that events *E* and *F* are not mutually exclusive and are independent. In part **(iii)(a)** most candidates were able to correctly find the required probability, but in part **(b)** some candidates appeared unfamiliar with the notation for the complement of *E*. In part **(c)** calculations were often correct in part **(i)** and partially correct in part **(ii)**. The most common error in part **(ii)** was for candidates to consider the probability that she eats breakfast and then wins the race, but to not consider also the probability that she does not eat breakfast and then wins the race.

Answers: **(a)** *A* and *B*, *A* and *D*, *B* and *C*; **(b)(iii)(a)** 0.89; **(b)** 0.09; **(c)(i)** 3/10; **(ii)** 8/15.

STATISTICS

Paper 4040/23

Paper 23

Key message

Candidates scoring the highest marks provided clear evidence of the methods they had used in logical, clearly presented solutions to the numerical problems. In questions requiring written comments these candidates provided detailed explanations which were in the context of the problem presented.

Candidates should always read the questions carefully and re-read them having provided a solution to ensure that the question has been answered fully and, where appropriate, in numerical questions the required degree of accuracy has been given.

General comments

In general candidates did better on questions requiring numerical calculations than on those requiring written explanations. In particular, the multi-stage problems of **Questions 3(ii)**, **4(ii)** and parts of **Questions 7** and **8**, which required clearly set out logical solutions, were often well presented with candidates able to achieve marks for their method even if numerical errors or algebraic slips lead to incorrect final answers. As was noted in this report last year, there continues to be an improvement in the number of candidates providing solutions to a suitable degree of accuracy.

Question 9, on probability, proved to be both the least popular ***Section B*** question, and the one which candidates found the most testing. Weaker candidates also found **Question 7**, on expectation, difficult, although some stronger candidates scored very high marks on this question. The accuracy of the graphical work in **Question 11** was generally very good, with **Questions 8** and **11** proving to be the most popular of the optional questions.

Comments on specific questions

Section A

Question 1

The majority of candidates were able to use the laws of probability associated with mutually exclusive and independent events to correctly find the probabilities in parts **(i)** and **(ii)** of this question. In part **(iii)** there were very few correct comments, with the majority of candidates not noticing that P($E \cap F$) should not be bigger than P(E).

Answers: **(i)** 0.4; **(ii)** 0.8.

Question 2

Most candidates were able to find the true lower and upper class limits for the continuous quantity in part **(i)** of this question. In part **(ii)**, where the variable was discrete, many candidates correctly stated one, but not necessarily both, of the limits. In part **(iii)** many candidates were unable to correctly state the limits for the 'age next birthday' of people, giving instead the limits associated with the way age is more usually measured.

Answers: **(i)** 24.5, 29.5; **(ii)** 25, 29; **(iii)** 24, 29.

Question 3

This question was well answered by the majority of candidates, with many achieving full marks. In p the vast majority of candidates correctly found the mean in part **(i)**. It has been noticed in the pas commented on in a recent report, that some candidates had been unable to correctly recall and use formula for standard deviation. It was therefore particularly pleasing to see that most candidates were abl to correctly use the formula and apply it to the situation presented, providing, in many cases, a fully correct solution to part **(ii)**.

Answers: **(i)** 38; **(ii)** 8.

Question 4

In this question, having correctly found the cumulative frequencies in part **(i)**, many candidates were able to attempt a correctly structured solution to the calculation of the median using linear interpolation. The boundaries of these classes representing 'age' were, however, often incorrectly stated, with many candidates either incorrectly giving the lower boundary of the 32 – 38 class as 31.5, rather than 32, or incorrectly giving the class width as 6, rather than 7. Those candidates who had provided clearly set out working for their solution were able to score some of the available marks even if they had made one of these errors. In part **(iii)** only a few candidates provided a suitable description of the shape of the distribution, namely that it has a longer upper tail, in order to achieve the mark here.

Answers: **(i)** 20, 55, 80, 98, 110, 117, 120; **(ii)** 33.4.

Question 5

In part **(i)** of this question most candidates were able to correctly identify whether statements **(a)**, **(c)** and **(d)** were true or false. In part **(b)** it was common to see candidates wrongly state that the presence of one extreme value meant that the standard deviation was an appropriate measure of dispersion. In part **(ii)** many candidates wrongly stated that Robert's decision was correct. Those that realised the decision was incorrect were often able to correctly express the reason as being because the distances quoted are per journey and not per day.

Answers: **(i)(a)** True; **(b)** False; **(c)** False; **(d)** True.

Question 6

Many candidates were able to successfully show how to achieve the result presented to them in part **(i)** and were then able to use the same method to achieve correct results in part **(ii)(a)**. In part **(ii)(b)** the majority of candidates expressed that they were looking for the highest scaled value in order to establish which of the athletes' performances was the best, but most did not notice that in the timed event of the 100 metres the lower the value the better the performance.

Answers: **(i)** (35 – 50)/10; **(ii)(a)** –0.69, 0.46, 0.67; **(b)** Ikram.

Section B

Question 7

Some of the most able candidates scored full marks on this question; however, some of the weaker candidates scored no marks or chose this as the question to leave out.

In part **(i)** many candidates were able to identify the four ways of achieving outcome *A* and were hence able to correctly calculate the probability of *A*. Part **(ii)** was also usually correctly calculated. In part **(iii)** those candidates who demonstrated some evidence of correct understanding of the problem usually provided enough working to earn all of the marks. There were, however, a large number of candidates who had not successfully identified the eight ways in which outcome *C* could occur, and were thus unable to score any of the available marks. Most candidates realised that in order to find the final probability in part **(iv)** they had to use the fact that the four probabilities needed to sum to one. Thus, even where errors had been made in earlier parts of this question, many candidates were able to score full marks here. In part **(v)** it was pleasing to see clearly set out working by those candidates who progressed this far with the question. It was noticed,

however, that some weaker candidates who had embarked upon this question left parts **(v)** and [illegible] part **(v)** some good candidates incorrectly rounded the final answer to \$0.40 and it was only the candidates who provided a correct solution to part **(vi)**.

Answers: **(i)** 1/9; **(ii)** 1/6; **(iv)** ½; **(v)** \$0.39; **(vi)** \$4.

Question 8

This was a popular ***Section B*** question and, for the able candidates, one where the numerical parts, **(i)**–**(iv)**, were usually completely correctly.

Most candidates were able to successfully find the price relatives in part **(i)**. It was common in part **(ii)** for candidates to misunderstand the instruction and attempt to assign weights and find a weighted rather than an unweighted average. In part **(iii)** only the most able candidates appreciated the need to multiply the costs per unit from the table by the quantity ratios in order to establish expenditure ratios. Many simply took the costs per unit, and simplified the ratio of these, ignoring the information provided regarding quantities purchased. In part **(iv)** it was common for weaker candidates to use the costs per unit from 2010 in place of price relatives. The five marks available for this question should perhaps have alerted these candidates to the fact that price relatives would need to be calculated before the formula for the weighted average could be applied. Those that were successful tended to give this answer to the required degree of accuracy. In part **(v)** it was necessary for candidates to notice that the weightings were very different from each other, and this was often missed by candidates. In part **(vi)** the fact that the quantities purchased may have altered considerably over the time interval was the important fact to note. Unfortunately for many candidates the quantities purchased had not formed a part of their calculation, and hence, did not form a part of the answer here.

Answers: **(i)** 150, 164, 110; **(ii)** 141.3; **(iii)** 9 : 20 : 4; **(iv)** 179.4.

Question 9

This was the least popular ***Section B*** question by some margin, and of those who attempted it only the very able were successful.

In part **(i)** candidates often calculated the probability that only one rather than 'at least' one of the pair will function. Part **(ii)(a)** was, for many who attempted this, the only part of this question that they got correct. In parts **(ii)(b)** and **(c)** again the fact that the phrase 'at least one' means one or more was often misunderstood. In part **(iii)** candidates needed to appreciate that 6000 microchips means that 2000 of design *A* or 1000 of designs *B* or *C* can be produced. This was misunderstood by the majority and hence it was rare to see a correct solution to this part of the question.

Answers: **(i)** 0.99; **(ii)(a)** 0.729; **(b)** 0.927; **(c)** 0.970; **(iii)** *A*, 1458.

Question 10

This was a fairly popular ***Section B*** question, although some who attempted it seemed to struggle with the topic of sampling methods and the use of random number tables to find various samples.

It was surprising that some who attempted this question were unable to find the simple random sample requested in part **(i)**, sometimes giving just one value from the random number table, rather than listing seven values to provide a sample of the required size. In part **(ii)(a)** only the most able candidates found the smallest and largest possible number of the first candidate for a systematic sample. Common errors in part **(ii)(b)** were to see seven values picked from the random number table, or a list of values at intervals of 10 rather than 11. In parts **(iii)** and **(iv)** it was again only the most able candidates who were successful, with some candidates seeming to misunderstand the questions. For example, some candidates provided just a single sample size rather than the sizes of the various sub-samples, in both parts **(iii)(a)** and **(iv)(a)**. In part **(v)** it was necessary for candidates to appreciate that, in order to accurately represent local Physics candidates, the samples would need to contain one such candidate. This fact appeared to be missed by many, and thus it was rare to see correct answers indicating for each of the four samples whether it over, under or accurately represented the local Physics candidates. In part **(vi)** very few candidates expressed the fact that a sample of size seven is very small.

Answers: **(i)** 67, 40, 15, 60, 32, 02, 59; **(ii)(a)** 01, 11; **(b)** 03, 14, 25, 36, 47, 58, 69; **(iii)(a)** 2, 3, 2;
(b) 14, 62, 21, 61, 24, 48, 54; **(iv)(a)** 4, 2, 1; **(b)** 12, 46, 14, 45, 74, 13, 01.

Question 11

This was a popular ***Section B*** question, with most candidates successfully finding the unknowns in the in part **(ii)**, plotting the moving average values in part **(iv)**, accurately drawing and interpreting the trend in part **(v)** and finding the unknown quarterly component in part **(vi)**. Thus this question often provided a good source of marks for candidates.

It was surprising that many candidates did not know why it was necessary to centre the moving average values in part **(i)**. In part **(iii)** most candidates were able to provide a suitable scale covering the required range, but the labelling of this axis was often missing. In part **(vii)** it was common to see, as in previous years, a reading simply being taken from the trend line, without the required quarterly component adjustment being made – in this case the subtraction of 66. Only the most able candidates spotted, in part **(viii)**, that the previous year the quarter II value had decreased, contrary to the general trend.

Answers: **(ii)** $w = 1479$, $x = 3520$, $y = 459.5$; **(vi)** 289.