

# MATHEMATICS D

Paper 4024/11

Paper 11

## Key Messages

To be successful in this examination, candidates need to have fully covered the examination syllabus, accurately remember the required formulae, show clearly all the necessary working for each question and use a suitable level of accuracy.

## General Comments

The paper contained questions which were accessible to all candidates. In general, candidates need to improve their skills in topics such as inequalities, estimation of calculations, bearings and probability.

At times candidates appeared not to have read some of the questions carefully. They should pay particular attention with questions such as **Question 7** (where approximations should have been shown), **Question 19** (answer in terms of  $\pi$ ) and **Question 21a** (answers in simplest form).

Candidates generally showed sufficient working, which was mostly clear. Some wrote in pencil and then overwrote in ink which often produced a double image which was difficult to read. Others used pencil and then rubbed out their working, which prevented any marks for intermediate steps being given.

Candidates should take care in copying their answer, found in the working space, correctly onto the answer line and should also give one answer only there. Incorrect answers are best crossed out and replaced rather than written over. There is no need to delete working if it is not to be replaced, as the original attempt may gain part marks.

## Comments on Specific Questions

### Question 1

(a) A good understanding of the method for subtracting fractions was shown although some answers were written as  $1\frac{-1}{16}$ .

(b) The rules of arithmetic were well understood, with a high proportion of correct answers.

Answers: (a)  $\frac{15}{16}$  (b) 9

### Question 2

- (a) This question on multiplying decimals was answered well. Errors were made in positioning of the decimal point and in reaching the figures 24, usually due to attempting a multiplication.
- (b) The majority of candidates gave the correct order but the common error was to write  $\frac{2}{9}$  as 0.22, thus having 2 values the same and giving the incorrect order of 0.2,  $\frac{2}{9}$ , 22%. A number of answers showed no working.

Answers: (a) 0.024 (b) 0.2 22%  $\frac{2}{9}$

### Question 3

- (a) Most candidates correctly worked with cancelling down 30:135, converting the time into minutes. Some did not always cancel sufficiently, leaving the answer as 6:27. Others cancelled too far, giving an answer of 1:4.5, unaware of the significance of  $m$  and  $n$  being integers.
- (b) The simple interest formula was well known and the majority of answers contained the figures 48. There were a number of errors in positioning the decimal point, with common wrong answers of \$48 or \$480 (rather than 480 cents). Incomplete answers such as  $\frac{24}{5}$  or  $4\frac{4}{5}$  were seen.

Answers: (a) 2:9 (b) 4.8

### Question 4

Some candidates were able to start solving the inequality correctly as far as  $3x < 7$ , thus gaining a part mark. Answers were then often given as 2 and/or  $2\frac{1}{3}$ . It was rare to see two acceptable answers between 2 and 3 (the common correct ones given were 2.1 and 2.2). Some candidates tried to solve two inequalities as the question asked for two answers and a few tried to use the quadratic formula. Testing different values of  $x$  in the inequality rarely produced correct answers.

Answer: Two numbers between 2 and  $2\frac{1}{3}$

### Question 5

Candidates generally lacked a clear understanding of the topic of bounds. A number of candidates did not attempt the question.

- (a) Few candidates were able to combine the concepts of bounds and algebra. The question was often misread as 'to the nearest 1 cm' rather than 'to the nearest 10 cm' and so the upper bound of one side usually involved  $d + 0.5$  or  $d0.5$ , giving answers such as  $4d.05$  and  $4d + 0.5$ . Those using  $d + 5$  sometimes forgot brackets, writing  $4d + 5$  rather than  $4(d + 5)$ .
- (b) Similar errors were made here with  $(d - 0.5)^2$  a common wrong answer.

Answers: (a)  $4d + 20$  (b)  $(d - 5)^2$

### Question 6

- (a) This part on evaluating expressions in standard form was generally correct. The common error was to write  $5 \times 10^0$  as 1, giving 131 as the answer.
- (b) This part was not as successful, with the common error being to leave the answer as  $12 \times 10^5$ .

Answers: (a) 135 (b)  $1.2 \times 10^6$

### Question 7

Candidates were often able to write at least one number to one significant figure in this question. Some used approximations and some gave  $\frac{(39 \text{ or } 40) \times 3}{6}$ . Those candidates using 40 were usually more successful in reaching 20 than those using 39, who did not always simplify their answers, leaving them as  $117/6$ , or incorrectly cancelling to 19.3. There were a number of candidates who attempted the calculation without approximating any of the values given or gave no response.

Answer: 20

### Question 8

- (a) There were many correct answers seen to this question on fractions, with most errors made in cancelling.
- (b) This part caused more problems with the common wrong answer being  $81/4$ .

Answers: (a)  $\frac{4}{9}$  (b)  $\frac{4}{81}$

### Question 9

- (a) There was a generally good understanding of percentage profit. The common error was to find  $\frac{400}{500} \times 100$ .
- (b) Those candidates who recognised inverse percentages were usually able to score one mark (for 50) or both marks. Most candidates tried to find 20% of 60 or used  $20\% \text{ of } x = 60$  to give  $x = 300$ . Some attempted  $60/0.8$  rather than  $60/1.2$ .

Answers: (a) 20 (b) 10

### Question 10

Both parts showed that candidates need to improve their knowledge of bearings. Some tried to write answers as compass directions. A number gave answers as measurements such as 3 cm.

- (a) Some candidates recognised that the equilateral triangle had angles of  $60^\circ$  (and not  $45^\circ$ ) but many were then often unable to measure clockwise from the north. A common wrong answer was  $150^\circ$ .
- (b) This part was less successful with little evidence that the diagram had been used to help them.
- (c) Calculating the time interval was more successful. The usual error was to use 100 minutes in an hour in the subtraction, giving an answer of 83.

Answers: (a)  $210^\circ$  (b)  $330^\circ$  (c) 43

### Question 11

- (a) Candidates who read this question on proportional objects carefully usually reached an answer with figures 375 but a significant number wrote 37.5 or 375. Some simply converted 1.5 m into cm.
- (b) It was extremely rare to see an attempt to use ratios of volumes of similar shapes. Most found  $0.005 \times 40 = 0.2$ . There were a number of candidates that did not answer this question.

Answers: (a) 3.75 or  $3\frac{3}{4}$  (b) 320

### Question 12

Candidates found the topic of histograms and frequency density difficult.

- (a) The correct table was sometimes correct. The most common error was to list the cumulative frequencies. There were a number of candidates that did not answer this question.
- (b) It was rare to see the correct answer (involving halving the frequency for the bar from 5 to 7), with  $12/43$  or  $24/43$  being the common wrong answers.

Answers: (a) All of 4, 5, 6, 6, 4 (b)  $\frac{18}{43}$

### Question 13

- (a) There were many correct answers to this question on functions, with a common mistake being to leave out the negative sign.
- (b) A good number of candidates were able to make the first step of removing the denominator but many did not then realise they had to collect all the appropriate terms on one side of the equation, and sometimes ended up back at the original equation. Some left their answer in terms of  $y$  rather than  $x$ . There were a number of numerical answers or answers of  $1 / (f(x))$ .

Answers: (a)  $-\frac{5}{8}$  (b)  $\frac{7}{2x+3}$

### Question 14

- (a) Expressing answers in set notation remains a difficult concept for many candidates. Writing  $A \cap B \cap C$  was a common incorrect answer as were answers without brackets.
- (b)(i) There was more success in this part with 6 seen fairly often but the answer  $\{a,b,c,d,e,f\}$  was also common.
- (ii) A variety of answers were seen but this part was slightly more successful than (i).

Answers: (a)  $(A \cup B) \cap C$  (b) (i) 6 (ii) d, e, f

### Question 15

There were considerable difficulties in recognising the rotational symmetry of order 3 on the diagram and there were a number of candidates that did not attempt parts (b) and (c).

- (a) The usual wrong answers were 1 or 3 lines of symmetry.
- (b) The correct answer was seen fairly frequently as were the incorrect answers of 53 and  $180 - 40 = 140$ .
- (c) The given fact of rotational symmetry of order 3 was rarely used.  $180 - 53 = 127$  was the common answer.

Answers: (a) 0 (b) 40 (c) 147

### Question 16

- (a) (i) Candidates clearly knew how to calculate the mode in this data handling question.
- (ii) This was mainly answered correctly. Instead of the median, some found the middle value of unordered list, and some calculated the mean.
- (b) It was generally recognised that an equation had to be solved and candidates often started off correctly. Errors then included not multiplying -5 by 3 thus reaching an answer of 23 and giving -15 + 28 as -13. There were a number of scripts which showed no working.

Answers: (a) (i) 5 (ii) 3 (b) 13

### Question 17

- (a) Only the better candidates were able to score full marks in this part question on inequalities. Some recognised the lines  $y = 4$  and  $y = 4x$  but failed to link them with the correct inequality, sometimes using  $\leq$  or  $\geq$ . Many others gave answers containing an  $x$  term, a  $y$  term and a number term.
- (b) There was little recognition that it was helpful to find where the lines intersected in order to establish the number of required points. Common wrong answers included 4, 2, and 1 and most scripts showed no evidence of any working. There were a significant number of no responses.

Answers: (a) (i)  $y > 4$ ,  $y < 4x$  (b) 3

### Question 18

Only the better candidates were able to score full marks in this question on the angles in an irregular polygon. Problems were encountered in calculating the angle sum of the interior angles of a hexagon, with errors such as  $(6 - 2) \times 180$  becoming  $3 \times 180$ . There were also attempts to use incorrect formulae such as  $n \times 180$  or  $(n - 1) \times 180$ . It was often incorrectly thought that  $70 + 66 + 90 + 90 + 120 + x = 720$ . There were many false assumptions made about the diagram. It was acceptable to divide the shape up into 2 quadrilaterals by extending the line from the lower angle of  $90^\circ$  past the vertex with the angle  $x$  to meet the top side but candidates should not assume that by continuing lines they will meet the opposite corner.

Answer: 76

### Question 19

With the formula for the volume of a cone given, many candidates were able to score the part mark for the second cone. The main error for the first cone was to give  $r^2$  as  $2x^2$  rather than  $(2x)^2$  or  $4x^2$ , thus reaching  $\frac{1}{3}\pi \times 14x^2$ . Those correctly finding the 2 volumes often had difficulty subtracting them, with both  $\frac{1}{3}$  and  $\pi$  being cancelled in the 2 volumes, and the answer was sometimes left unsimplified as  $24\pi x^2/3$ . Some candidates did not read the question carefully enough and used  $22/7$  or  $3.14$  for  $\pi$ . A few candidates found a numerical final volume. There were a number of no responses.

Answer:  $8\pi x^3$

### Question 20

A number of candidates did not understand the concept of probability and gave answers greater than 1.

- (a) Just over half the answers were correct. Addition rather than subtraction was seen, giving an answer of  $\frac{31}{35}$ , as well as the answer  $2/7$ . Occasionally the denominator of  $5 \times 7$  was incorrectly calculated.

- (b) Some candidates realised that this combination was not possible. A common wrong answer was  $2/7 \times 0 = 2/7$ .
- (c) There were not a large number of answers which scored at least one mark. Some candidates tried to find the probability of at least one black.

Answers: (a)  $\frac{6}{35}$  (b) 0 (c)  $\frac{17}{35}$

### Question 21

- (a) (i) There were a good number of correct answers but  $2q - p$  and  $4q + 2p$  were also common. Some candidates were unaware of the form of a vector and gave numerical answers and answers containing squared terms.
  - (ii) Some candidates were able to correctly add  $2p + q$  to their answer in (i) and simplify it.
- (b) A similar number of candidates were able to add  $kp$  to their answer in (ii). There were a number of no responses.
- (c) A large number of candidates found this part particularly difficult with a significant number giving no response. There was little recognition of the significance of the lines being parallel. A number of answers were given in terms of  $p$  and  $q$ .

Answers: (a) (i)  $4q - 2p$  (ii)  $5q$  or ft their (i) +  $2p + q$  simplified (b)  $kp$  + their (ii) (c) 10

### Question 22

- (a) Some candidates used angles in a semi-circle to find the correct size of the angle. A common wrong angle was  $72^\circ$  (from attempting angle at the centre = twice the angle at the circumference).
- (b) A good number of candidates successfully used alternate angles.
- (c) This part was most commonly correct, from using angle at the centre = twice the angle at the circumference.
- (d) This multi-step part proved too difficult for many, with wrong angles including  $30.5^\circ$  and  $36^\circ$ .

Answers: (a)  $54^\circ$  (b)  $36^\circ$  (c)  $61^\circ$  (d)  $25^\circ$

### Question 23

- (a) There were a number of correct answers to this question on travel graphs with some answers unsimplified and also wrong answers such as  $1/5 = 0.5$  and  $30/6$ .
- (b) Finding the speed proved to be a difficult part for nearly all, with many wrong answers of  $6/40$ .
- (c) Distance travelled proved to be another difficult concept with many answers of  $85/10 = 8.5$ . Some attempted to use the areas under the graph but there was some confusion about the height of the trapezium, giving expressions such as  $\frac{1}{2} \times 10(u + u + 6) = 85$ . There were a significant number of no responses.

Answers: (a)  $(-\frac{1}{5})$  or  $(-0.2)$  (b) 4 (c) 11

### Question 24

- (a) The proof that the given equations were correct was not well done in this question on several occasions. Many candidates solved the equations in (b) and then substituted these values into (a). Some candidates did not attempt this part.
- (b) Those attempting to solve the equations mostly gave the correct answers to one or both letters.
- (c) Few candidates realised they needed to substitute 3 into the formula for the  $n$ th term. Many gave answers only and there were a large number of no responses.

Answers: (a)  $A + B = 5$  correctly obtained from  $15 = 10 + A + B$  and from  $4A + B = 2$  correctly obtained from  $11 = 10 + 2A + B/2$  (b) both  $A = -1$  and  $B = 6$  (c) 9

### Question 25

- (a) The correct transformation was more commonly seen than the line of symmetry. Wrong lines included the  $x$  axis, the  $y$  axis and  $y = -1$ .
- (b) There were few correct diagrams seen to this enlargement, even though some projection lines were seen. A few candidates scored a part mark for using the scale factor of  $\frac{1}{2}$ .
- (c) Some candidates recognised that the ratio of areas = (ratio of sides)<sup>2</sup> and gave the correct answer of 4. Others tried to find the two areas before simplifying. Common wrong answers included 2 and  $\frac{1}{4}$ .

Answers: (a) reflection in  $x = -1$  (b) Triangle with vertices (0,6), (-1,5), (-2,5) (c) 4

### Question 26

- (a) Many candidates were able to score at least one mark for two correct entries in this question on matrices.
- (b) This part was not as well done as (a), mainly due to some candidates squaring the individual entries of  $\mathbf{A}$ , even though the question was written as  $\mathbf{A} \times \mathbf{A}$ .
- (c) Some candidates took note of the question and simply wrote down the correct answer but many attempted to find the inverse of  $\mathbf{A}$  and then carry out a matrix multiplication which led to errors.

Answers: (a)  $\begin{pmatrix} 1 & 3 \\ 0 & -2 \end{pmatrix}$  (b)  $\begin{pmatrix} 1 & -18 \\ 6 & 13 \end{pmatrix}$  (c)  $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

# MATHEMATICS D

Paper 4024/12

Paper 12

## General comments

In order to do well in this paper, candidates need to

- have covered the whole syllabus;
- remember necessary formulae and facts;
- recognise, and carry out correctly, the appropriate mathematical procedures for a given situation;
- perform calculations accurately;
- show clearly all necessary working in the appropriate place.

At times, candidates appeared not to read the questions carefully. They should be encouraged to pay careful attention to the wording, the numbers given, the units used and the units required in the answer. Some candidates did not appear to bring geometrical instruments into the examination.

Questions that proved particularly difficult were 11**(b)**, 16, 20**(b)(ii)**, 21**(b)(c)**, 22**(d)**, 23**(b)(c)** and 26**(a)(b)(ii)**.

It was noticeable that a significant number of candidates need to improve their ability to estimate, approximate and use appropriate degrees of accuracy; and also to understand the integer class of number. Some candidates were very competent at performing standard techniques, and yet seemed unable to recognise the appropriate mathematical procedure required for a given situation.

Candidates need to improve their skills in manipulating fractions and in basic arithmetic. Work, that used correct methods, was spoilt by wrong calculations like  $10 - 7.62 = 3.38$ ;  $12/3 = 6$ ;  $1.86 \times 3 = 5.48$ ;  $3(4/3 \times \pi \times 8a^3) = 4 \times \pi \times 24a^3$ . Careless mistakes like these were made by candidates of all abilities. It was noticeable that a significant number of candidates could not solve an equation of the type  $ax = b$  correctly.

Presentation of the work was usually good. Candidates should bear in mind that it is to their advantage to make sure they provide sufficient working and that this working is set out neatly and legibly. Working makes it possible for marks, where they are available, to be awarded for correct methods and intermediate results. Care must always be taken to ensure that answers obtained in the working are accurately transferred to the answer space.

A few candidates did not heed the instructions on the front page - to write in dark blue or black pen. Except for diagrams and graphs, candidates should not write in pencil. Nor should they overwrite pencil answers in ink, as this makes a double image which can be difficult to read.

Candidates should be made aware that only their final answer to each question should be written in the answer space. Alternative offerings and working should not be written there. When an answer is to be changed, it is far better to delete and replace the original one rather than attempt to write over it.



**Comments on specific questions**

**Question 1**

- (a) This part showed that some candidates needed to read the question carefully, and to note that three drinks cost \$1.86 *each*, not \$1.86 for all three. Others obtained the wrong answer by arithmetic mistakes such as  $1.86 \times 3 = 5.48$  and  $10 - 7.62 = 3.38$ . Those who read the question carefully, and calculated  $10 - 1.86 \times 3 = 2.04$  accurately, obtained the correct answer.
- (b) This part was well answered. The usual wrong answers were 45, from  $\frac{180}{4}$ , or 144, from  $180 \times \frac{4}{5}$ .

Answer: (a) 2.38 (b) 80

**Question 2**

- (a) Attempts at this question showed that some candidates need to improve their ability to manipulate fractions and to work with a common denominator. Many obtained  $\frac{65-36}{20}$ , but sometimes miscalculated this as  $\frac{39}{20}$  or  $\frac{19}{20}$ .
- (b) This part showed that many candidates struggle with place value in decimal numbers. Most obtained the figures 602, but not all placed the decimal point correctly. Some candidates attempted an unnecessary long multiplication.

Answer: (a)  $1\frac{9}{20}$  (b) 0.0602

**Question 3**

- (a) The majority of candidates knew to substitute  $-\frac{1}{2}$  for  $x$  in this question on functions. Errors were made with signs by writing  $-1 - 6 = 7$ , or by writing  $2\left(-\frac{1}{2}\right) = -\frac{5}{2}$ , or  $-\frac{2}{4}$ .
- (b) Most candidates seemed to know how to deal with this type of question and could find the inverse of the function. The usual wrong answer was  $\frac{x-6}{2}$ . A few obtained  $\frac{x+6}{2}$  and then spoilt their answer by cancelling 6 and 2 to get  $x + 3$ . Some gave an answer that was not expressed in terms of  $x$ .

Answer: (a)  $-7$  (b)  $\frac{x+6}{2}$

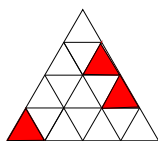
**Question 4**

- (a) Many candidates obtained the correct answer to this question on time. The main error was in using 100 minutes in an hour. Common wrong answers were 3 hours 88 minutes; 4 hours 28 minutes; 4 hours 48 minutes. A few subtracted the hours correctly but reversed the minutes, to get 4 hours 12 minutes.
- (b) Frequently no working was shown to this question on ordering fraction, decimals and percentages. Many candidates obtained the correct answer. A common difficulty was to place  $\frac{4}{9}$  correctly.

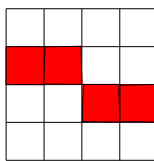
Answer: (a) 3 hours 48 minutes (b)  $\frac{2}{5}$  44%  $\frac{4}{9}$

### Question 5

There were some correct answers given to this question on symmetries. On the whole, attempts that many candidates need to improve their understanding of the symmetries.



(b)



Answer: (a)

### Question 6

Most candidates could answer this question on proportionality competently. The most common error was to obtain  $k = 10$ , incorrectly, from  $20 = \frac{k}{2}$ . Some who obtained  $k = 40$  then went on to write  $y = \frac{20}{5}$ . A few candidates either did not read, or misunderstood, the word “inversely”, and answered the question as if  $y$  is directly proportional to  $x$ .

Answer: 8

### Question 7

This was answered well by those who know what is meant by standard form.

- (a) Many candidates gave the correct answer. Common wrong answers were  $35 \times 10^6$ ;  $3.5 \times 10^{-7}$ ;  $35^6$ ; 35 million.
- (b) The most common wrong answer was  $1.4 \times 10^6$ . Some tried to use ordinary numbers rather than standard form, or left their answer as a fraction. Errors were made in subtracting the indices or by multiplying, instead of dividing, 4.2 by 3. Some candidates were unable to evaluate  $4.2 \div 3$  correctly.

Answer: (a)  $3.5 \times 10^7$  (b)  $1.4 \times 10^{-6}$

### Question 8

Many candidates made a sensible attempt at this question on solving an algebraic equation which included fractions. Errors in removing fractions, expanding the bracket, or gathering terms to one side were very common, as were errors like  $7x + 3 = 6$  therefore  $7x = 9$ ;  $7x = 3$  therefore  $x = \frac{7}{3}$ .

Answer:  $\frac{3}{7}$

### Question 9

The topic of estimation remains a difficult concept for many, shown by the number of “no attempts” and those who tried to work with numbers that were not corrected to one or two significant figures. Some thought that square and cube roots involve division by 2 and 3 respectively. At times, 0.3012 was wrongly approximated to 0 or 0.4; the square root by 4 and the cube root by 100 or by 1. Many gave good approximations, and showed them clearly as instructed in the question. Attempts at evaluating  $\frac{6 \times 10}{0.3}$  were not always correct, particularly in the positioning of the decimal point.

Answer: 200

### Question 10

The usual correct answer to this question on inequalities was 4.5. The usual wrong answer was given following on from the correct inequality  $x < 5$ , or from  $x > 5$  obtained incorrectly from  $-3x > -15$ .

*Answer:* Any number between 4 and 5.

### Question 11

- (a) This part on bounds when measuring angles was often answered correctly. Common wrong answers were 45; 46; 45.95. Some answers used the numbers around the point  $O$ , indicating that the writer did not understand which angle was being referred to in the question.
- (b) Few candidates realised that to find the lower bound of angle  $BOC$  it was necessary to evaluate  $360 - 162.5 - 46.5$ . The most common mistake was to write  $360 - 162 - 46 = 152$  and to give the answer 151.5. A few calculated  $360 - 161.5 - 45.5$ .

*Answer:* (a)  $45.5^\circ$  (b)  $151^\circ$

### Question 12

Candidates who knew the laws of indices answered this question well. Many showed that they need to improve their knowledge of these laws.

- (a) The usual wrong answers were  $\frac{25}{9}$ ;  $\frac{5}{9}$ .
- (b) The usual wrong answers were  $\frac{3}{t^6}$ ;  $\frac{9}{t^3}$ ;  $\frac{4.5}{t^3}$ .
- (c) Wrong answers to this part varied, with incorrect indices for  $x$  and/or  $y$  and not cancelling the 2 with the 6.

*Answer:* (a)  $\frac{9}{25}$  (b)  $\frac{3}{t^3}$  (c)  $\frac{x^2}{3y}$

### Question 13

Most candidates showed that they could make a good attempt at solving the simultaneous equations, but errors in subtraction, signs, gathering terms and transferring the obtained answers correctly to the  $x$  and  $y$  lines in the Answer Space occurred very frequently. Errors like  $10x = 5$  therefore  $x = 2$  and  $-5y = 20$  therefore  $y = 4$  were quite common.

*Answer:* Both  $x = \frac{1}{2}$  and  $y = -4$

### Question 14

Many candidates made a good attempt at this question on cumulative frequencies.

In parts (a) and (b), the most common type of error was to read the horizontal scale incorrectly.

- (a) Common wrong answers were 1.7; 2.
- (b) Common wrong answers were 1.05; 1.2.
- (c) Common wrong answers were 16, from  $120 - 104$ ; 102.

*Answer:* (a) 1.35 (b) 1.1 (c) 104

### Question 15

Answers to this question on inequalities varied greatly. There were some very good answers. It showed that there is a need for some candidates to get a deeper understanding of graphical inequalities.

- (a) Answered correctly by many. Some omitted to include  $B$ . Others gave points that lie in the region defined by  $x + y < 4$ .
- (b) The usual wrong answers were  $F$  and  $A$ . Some did not heed the question to write down *one* point.
- (c) Some candidates obtained the correct equation for the line through  $O$  and  $C$ , but of these most gave the wrong inequality. Many could not obtain the correct equation.

Answer: (a)  $B C D$  (b)  $E$  (c)  $y < \frac{1}{2}x$

### Question 16

Some candidates made a very superficial attempt at this question and seemed to have difficulty in understanding the situation described, and could not relate the volumes of water, cylinder and spheres. Those who made a reasonable attempt very often made mistakes such as using the wrong formula (usually  $2\pi r^2 h$  or  $2\pi r h$  or  $\frac{1}{3}\pi r^2 h$  for the volume of a cylinder and  $\frac{4}{3}\pi r^2$  for the volume of a sphere) and simplifying  $3\left(\frac{4}{3} \times \pi \times 8a^3\right)$  to  $4\pi \times 24a^3$ . The most common errors were to simplify  $(2a)^3$  as  $2a^3$  or  $8a$ ;  $(3a)^2$  as  $3a^2$ , or  $9a$ . It was not unusual to see an early  $8a^3$  and  $9a^2$  reappear as  $8a$  and  $9a$  at a later stage. A few misread the information in the question and used  $3a$  cm as the radius of a sphere. Some attempted, unnecessarily, to use numerical approximations to  $\pi$  and invariably made arithmetic errors.

Answer: 76

### Question 17

There seemed to be a distinct improvement in this type of question in that fewer candidates offered “solutions” and instead factorised the expressions.

- (a) Many realised that this part involved the “difference of squares” and gave the correct factors. Common wrong answers were  $(5t - 2)^2$ ;  $(25t + 2)(25t - 2)$ ;  $(5t + 4)(5t - 4)$ .
- (b) Some candidates only gave partial factors, such as  $2r(3rH - rh)$ . A few did not realise that  $H$  and  $h$  are different symbols.
- (c) Many candidates paired off the terms to produce the correct factors. Those who did not often gave a correct partial factorisation. Candidates who used a grid or a table seldom reached any stage worthy of merit.

Answer: (a)  $(5t - 2)(5t + 2)$  (b)  $2r^2(3H - h)$  (c)  $(4x - 3)(2y + 1)$

### Question 18

- (a) Those who were familiar with histograms and frequency density answered this part correctly. Some candidates failed to attach any significance to the width of the bars, and gave the answer 8.
- (b) The usual answer involved a rectangle, base 7 cm to 9 cm, height 4 units, and, if it was not overlooked, the correct rectangle on a base of 2 cm to 3 cm.
- (c) This part was often correct. The common error was to overlook the first interval and to give a fraction with a numerator of 9. Most showed some knowledge of probability and gave the denominator as the sum of the frequencies.

Answer: (a) 16 (b) Rectangle, base 2 to 3, height 6 units: Rectangle, base 7 to 9, height 2 units  
(c)  $\frac{15}{31 + \text{their}(p)}$

### Question 19

Parts (a) and (b) were usually attempted correctly in this question on coordinates and lines, though arithmetic and sign errors occurred rather frequently.

- (a) There was a very good success rate in this part. Some started by subtracting, instead of adding, the coordinates. Common wrong answers were  $(-2, 1)$ ;  $(-2, 2)$ .
- (b) Occasionally the  $x$  and  $y$  components were confused. Common wrong answers were  $\frac{2}{3}$ ;  $-\frac{3}{2}$ ; 0.6.
- (c) Some candidates used an incorrect form of the formula for the distance between two points. Others simplified  $\sqrt{6^2 + (-4)^2}$ , or its equivalent, to  $\sqrt{6^2 - 4^2}$ , or to  $6 - 4$ . Many were able to get as far as  $PQ = \sqrt{52}$ , but were unable to proceed, via  $52 = 4 \times 13$  and  $\sqrt{52} = \sqrt{4} \times \sqrt{13}$ , to find  $n = 13$ . The common wrong answer was 26.

Answer: (a) (2, 1) (b)  $-\frac{2}{3}$  (c) 13

### Question 20

- (a) Many identified that the transformation was a reflection, but fewer could complete the description by giving the equation of the line in which triangle  $A$  is reflected. The usual wrong answer was to name the transformation as a rotation. Candidates should realise that “mirrored” is not a recognised name for a transformation.
- (b)(i) This part was less well answered, and quite often omitted. Many drew a triangle which was the reflection of the correct triangle in the  $x$ -axis. Some rotated in the wrong direction. Others rotated about the point  $(0, 1)$ ; or rotated triangle  $B$  instead of  $A$ ; or rotated triangle  $A$  by  $180^\circ$ .
- (ii) There were some completely correct answers, though often a matrix that did not have an order of 2 by 2 was offered. Few candidates knew how to approach this part. Many just gave the coordinates of the vertices of triangles  $A$  or  $C$  in their final answer.

Answer: (a)(i) Reflection in  $y = x$  (b)(i) Triangle with vertices  $(-1, 0)$ ,  $(-3, 0)$ ,  $(-3, 1)$  (b)(ii)  $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

### Question 21

The fact that this question was sometimes omitted, or answers given were whole numbers with no working, suggests that some candidates need to gain a better understanding of probability. A common misunderstanding was that the first card was replaced before the second card was drawn. Occasionally an answer was not expressed in its simplest form, as directed in the question. A very small number of candidates attempted this question by means of a possibility space. Most of these diagrams, however, contained 36 elements instead of 30.

- (a) Some candidates realised that it was a certainty that the number is greater than 20 and gave the answer 1. A few gave an equivalent value in the form of an unsimplified fraction, such as  $\frac{6}{6}$ .
- (b) Correct answers to this part were not common. Typical wrong answers were  $\frac{1}{30}$ ;  $\frac{1}{36}$ . Some did write down  $\frac{2}{6} \times \frac{1}{5}$ , but cancelled it incorrectly.
- (c) Reasonable attempts at this part sometimes went wrong by assuming that the first card was replaced, leading to the expression  $\frac{3}{6} \times \frac{2}{6} + \frac{2}{6} \times \frac{1}{6}$  instead of the correct  $\frac{3}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{1}{5}$ . Others used  $(\frac{3}{6} + \frac{2}{5}) \times (\frac{2}{6} + \frac{1}{5})$  or obtained the fractions  $\frac{1}{5}$  and  $\frac{1}{15}$  but failed to add them.

Answer: (a) 1 (b)  $\frac{1}{15}$  (c)  $\frac{4}{15}$

### Question 22

Answers to this part showed that many candidates need to get a better understanding of the angle properties of polygons drawn on a circle, and not to apply them to polygons that are not drawn on a circle. In general, attempts at this question showed that it is vital to look carefully at the diagram, and not to assume facts, such as triangle  $ADB$  is isosceles, that are not true. Many candidates obtained answers that they should have realised were clearly the wrong size just by looking at the diagram. For example, giving an obtuse angle for something that is clearly acute.

Some otherwise correct attempts were ruined by careless arithmetic errors.

- (a) The most common answer was  $66^\circ$ .
- (b) The correct answer was given most often. A common wrong answer was  $114^\circ$ , from using angle  $BDC$  is half reflex angle  $AOB$ . Other wrong answers were  $90^\circ$ ;  $42^\circ$ .
- (c) A common wrong answer was  $48^\circ$ , from equating angle  $BDC$  to angle  $BOC$ . Others were  $31^\circ$ ;  $59^\circ$ ;  $90^\circ$ . Candidates who had found angle  $BDA$  correctly could not always use it to correctly find angle  $BDC$ , as they did not identify the  $90^\circ$  angle in the semicircle.
- (d) This part, which involved a multistep solution, was rarely correct. Most candidates gave the answer  $59^\circ$ , from equating angle  $OBD$  to angle  $DCA$ . Other common wrong answers were  $31^\circ$ ;  $48^\circ$ .

Answer: (a)  $48^\circ$  (b)  $66^\circ$  (c)  $24^\circ$  (d)  $35^\circ$

### Question 23

- (a) This part of this question on sequences was often answered correctly. The usual wrong answers were to have  $13^2$  or  $17^2$  instead of  $15^2$ ; to continue the right-hand side to 8.
- (b) Most candidates gave a numerical expression and not one in terms of  $n$ . Other wrong answers were  $n^2 - 1^2$ ;  $(n + 2)^2 - 1^2$ ;  $2n^2 - 1^2$ .
- (c) Few candidates could start from the  $n$ th line of the pattern and establish the required result by mathematically correct statements. A number did simplify  $(2n + 1)^2 - 1^2$  to  $4n^2 + 4n$ , for which there was some credit. Some showed, in fact, that  $(2n + 1)^2 - 1^2$  is equal to  $\frac{n(n+1)}{2}$ . Many omitted this part, or else merely verified that the result was true for a particular value of  $n$ .

Answer: (a)  $15^2 - 1^2 = 8 \times (1 + 2 + 3 + 4 + 5 + 6 + 7)$  (b)  $(2n + 1)^2 - 1^2$  or (c) Correct proof

### Question 24

This question was quite often omitted, or showed that many candidates need to get a better understanding of loci and the difference between perpendicular bisectors of sides and bisectors of angles. Those who understood loci usually did quite well, though accuracy was sometimes lost because arcs with small radii were used in the constructions. Occasionally constructed lines did not cover all points *inside* the triangle as instructed in the question.

- (a) This part was often correct, with  $83^\circ$ , or  $92^\circ$  being common wrong answers. There were a few instances where a length was given, which showed a need to understand the way angles are labelled.
- (b)(i) Most attempts were good. A few drew appropriate arcs and then omitted to draw a line.
- (ii) Some attempts were correct. Incorrect constructions were aimed at finding a bisector of the line  $AC$ ; a bisector of the line  $BC$ ; or the bisector of the angle  $BAC$ .
- (c) Those who constructed good loci in part (b) usually obtained an acceptable answer.

Answer: (a)  $96^\circ$  to  $98^\circ$  (b)(i) Perp. bisector of  $AB$  (b)(ii) Bisector of angle  $ABC$  (c) 10 to 10.3

### Question 25

Attempts at this question showed that some candidates need to get a better understanding of the properties of a speed-time graph, and to appreciate that the "D-S-T triangle" is not a valid method when there is an acceleration.

- (a) Usually answered correctly by those who used  $20 \times \frac{12}{15}$ , particularly when this expression was cancelled down to  $4 \times 4$  rather than trying to evaluate  $\frac{240}{15}$ . Common wrong answers were 15; 17; 1.6
- (b) Many candidates recognised that the distance travelled is obtained by finding the area under the graph and obtained the correct answer. The usual wrong answer was 300.
- (c) Those candidates who used the fact that the distance travelled is given by the area under the graph, together with a correct expression for this area, usually obtained the correct answer – though some did make errors in expanding brackets or gathering like terms. Some obtained 30 s as the base of the rectangle but omitted to add the 15 s. Many candidates obtained a wrong answer by dividing 750 by 20 or by evaluating  $15 \times \frac{750}{\text{their (b) answer}}$ .

- (d) Those who used the geometry of the right-hand triangle and the result  $2 = \frac{20}{\text{time}}$  were successful in answering this part. Attempts that used  $\frac{\text{change in velocity}}{\text{change in time}} = \text{acceleration}$  with  $k$  and  $x$  were sometimes successful, but often obtained incorrect answers by using +2 instead of -2 for the acceleration.

Answer: (a) 16 (b) 150 (c) 45 (d) 10

### Question 26

- (a) Completely correct explanations, establishing that the three angles of each triangle are equal, were very rare. A few noticed that angles  $ABD$  and  $BDC$  are equal, and some of these gave the essential reason of “alternate” angles. Most, though, tried to involve a side and quoted a test for congruency, or else gave very vague statements, such as “triangles between parallel lines”.
- (b)(i) There were many correct answers from those who started with  $\frac{4.2}{4} = \frac{BC}{6}$ , or an equivalent expression, and did not make a careless slip, or an error in evaluating  $\frac{25.2}{4}$ . Others used an incorrect expression or tried to use Pythagoras’ theorem. A few gave 6.2 from  $6 + 4.2 - 4$ .
- (ii) Those candidates who knew that, with similar triangles, the ratio of their areas is equal to the ratio of the squares of corresponding sides, usually obtained the correct answer. Others tried to use  $\text{Area} = \text{Base} \times \text{Height}$ , or  $\text{Area} = \frac{1}{2} ab \sin C$ .

Answer: (a) Correct proof (b)(i) 6.3 (b)(ii)  $\frac{4}{9}$



# MATHEMATICS D (CALCULATOR VERSION)

Paper 4024/21

Paper 21

## Key Messages

To succeed in this paper, candidates need to have completed full syllabus coverage, remember necessary formulae, show all necessary working and use a suitable level of accuracy.

## General comments

Most candidates were reasonably well prepared for this paper, which offered them the opportunity to demonstrate their mathematical ability. When questions were not attempted, this appeared to be due to a problem in understanding what was required rather than a lack of time to complete the paper.

Some candidates continue to have problems with questions that require explanations involving mathematical reasoning.

Candidates are again advised to check that their calculator is set in degree mode before evaluating trigonometrical expressions.

Candidates scored well in **Questions 1(a)(i), 1(a)(ii), 1(a)(iii), 2(a), 3(a), 3(b), 4(a), 4(c)(i), 4(c)(ii), 4(d)(i), 5(a), 5(b)(i), 5(b)(ii), 5(c)(ii), 5(d), 8(b)(iii), 9(a), 9(b) and 9(c).**

## Comments on specific questions

### **Section A**

#### **Question 1**

- (a) (i) Invariably \$468 was correctly seen from  $300 \times 1.56$  in this question on currency conversions. Occasionally some candidates calculated  $300 \div 1.56$  to give \$192.30.
- (ii) Almost always \$700 was found correctly from  $770 \div 1.1$  with only a small minority opting for  $770 \times 1.56 = \$1201.2$  or  $770 \times 1.1 = \$847$ .
- (iii) Generally less success was seen in this part, which required a two stage approach. The correct answer of \$550 was given on many occasions, but some candidates carried out only one stage of the calculation such as  $780 \times 1.1 = \$858$  or made an error in the two stage calculation by dividing \$780 by 1.56 and then by 1.1 to give \$455
- (b) The formation of one linear equation was sufficient to answer this question. Some candidates used simultaneous equations successfully. Clearly some had difficulty in formulating their ideas in algebraic form and were unable to make any substantial progress. This part was sometimes omitted.

Answer: (a)(i) 468 (ii) 700 (iii) 550 (b) 19 926

### Question 2

- (a) The constructions seen were mostly correct. When inaccurately drawn, the angle was more likely to be incorrect than the length  $AC$ . A number of candidates put their angle of  $40^\circ$  at  $B$ . There were some solutions that looked as though the angle had been drawn without the use of a protractor.
- (b) This part on loci was not answered well. Several candidates made a partial attempt to draw the locus of points. Often, either the lines parallel to the sides were absent, or only one of the three arcs was drawn.
- (c) The implications of this part were often understood and  $P$  was successfully located. The most common misunderstanding here was to use the bisector of angle  $A$  instead of the perpendicular bisector of the side  $BC$ . Some candidates did not indicate the position of  $P$ .

### Question 3

- (a) This question on vectors was generally well answered. Sometimes the correct method was seen for the coordinates, but the subsequent simplification went astray. In some instances, the relevant  $x$  and  $y$  coordinates were subtracted, leading to the answer  $(4, 2)$ . Some candidates thought the length of  $AB$  was required.
- (b) This part was well answered. Sometimes the reciprocal of the gradient was given.
- (c) Candidates using  $y = mx + c$ , with  $m$  from the previous part, followed by a relevant substitution to find  $c$ , were generally more successful than those finding the equation of  $AB$  by equating gradients.
- (d) Few candidates were able to find the correct vector joining the two points. Many were uneasy about the connection between coordinates and column vectors. Some simply offered the coordinates of  $A$  as a column vector.
- (e) This question proved demanding for many. There were some neat solutions using column vectors. In some cases, the vector method produced only one solution instead of two. A few candidates worked successfully through gradients and lengths. However, the formula for the length between two points did get confused with other things, coordinates being added instead of subtracted, and squares being subtracted instead of added.

Answer: (a)  $(2,3)$  (b)  $\frac{4}{8}$  (c) 2 (d)  $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$  (e)  $(-3,-2)$  and  $(13,6)$

### Question 4

- (a) Most candidates found the modal class correctly, with the appropriate inequalities transcribed. A common incorrect answer was 25, the modal frequency.
- (b) The given scales were usually accurately drawn. A few candidates chose to use other scales. There were some good solutions, with the plots at mid-interval values as expected. Some plotted the points either at the upper class or lower class boundary. Clearly, a significant number of candidates were not familiar with the idea of a frequency polygon, opting to draw a histogram instead. Sometimes this part was omitted.
- (c) (i) Many gave the correct cumulative frequencies in the table with only the occasional error.  
(ii) A good response was seen, with accurate plots and well drawn curves.
- (d) (i) The correct value for the median mass was usually given  
(ii) The tenth percentile was generally understood, but a significant number of candidates gave 10 as the answer.

Answer: (a)  $3.5 < x \leq 4$  (c)(i) 35 59 84 98 (d)(i) 3.4 (ii) 2.3

### Question 5

- (a) Almost all candidates gave the correct solution to this equation. A small minority opted for  $x = 5$ .
- (b)(i) This factorising question was well answered. A common incorrect answer was  $10xy$ .
- (ii) Many candidates recognised the expression as the difference of two squares and gave the correct factors. Common incorrect answers were  $(3x - 4)^2$ ,  $(9x - 4)(9x + 4)$  and  $(3x + 2)(3x - 2)$ .
- (c)(i) The factorisation of the trinomial was usually correct.
- (ii) A good number of correct solutions to the quadratic were seen. Some candidates resorted to the use of the formula to solve the equation.
- (d) The required algebra was confidently handled in this question with a good number scoring full marks. Incorrect attempts usually started off with either  $L = kd^2$  or  $L = \frac{k}{d}$

Answer: (a) 1 (b)(i)  $5(x + y)$  (ii)  $(3x + 4)(3x - 4)$  (c)(i)  $(2x - 3)(x + 4)$  (ii)  $\frac{3}{2} - 4$  (d) 4

### Question 6

- (a)(i) Candidates were clearly familiar with the appropriate trigonometrical ratios that can be used in a right-angled triangle. Some candidates cannot resist using more elaborate methods that are more appropriate in general triangles. These methods were often unsuccessful. Candidates were expected to evaluate  $CD$  to at least 3 places of decimals, in this case, 19.926, to show that the value would round to the given 19.93.
- (ii) Candidates were reasonably successful here. Again, more complicated methods were popular, with combinations of both the sine rule and the cosine rule being used. In particular, the use of, for example, the cosine rule to find  $CD$  in a right-angled triangle is to be discouraged.
- (b)(i) Many candidates struggled with this part question on angles of depression. There were some correct answers, but  $10^\circ$  (angle  $PSQ$ ),  $180^\circ - (10^\circ + 55^\circ) = 115^\circ$  and  $360^\circ - 65^\circ = 295^\circ$  were common wrong answers.
- (ii) A good number of candidates spotted the expected strategy based on right-angled triangles, and were usually successful. Candidates using other methods gained credit as appropriate.

Answer: (a)(i) 19.93 from correct rounding (ii) 28.3 (b)(i)  $25^\circ$  (ii) 37.2 or 37.3

### Question 7

- (a)(i) Only the stronger candidates were able to state the 3 facts needed to establish congruency. Most gained credit for stating  $AE = AC$  and some went on to state that angle  $EAD =$  angle  $DAC$ , but fewer recognised the fact that  $AD$  was common to both triangles. Weaker candidates thought that  $CD = AD$ .
- (ii) Most candidates found this part difficult with few achieving the correct answer. A common incorrect answer was  $x = 180 - y - z$ .
- (b) Unless candidates were working with the half angles at  $Q$  and  $R$ , success was rather limited. A common error here was to give the answer as  $132^\circ$  instead of the reflex angle.

Answer: (a)(ii)  $z - y$  (b)  $228^\circ$

### Question 8

- (a) The key here was to draw the radius to the point of contact with the tangent and then to join it to either  $A$  or  $B$ . Pythagoras' theorem can then be applied to the resulting right-angled triangle. Many candidates resorted to trivial statements such as  $r = 14 - 10 = 4$  or  $r = 10 \div 2 = 5$  or  $r = 14 \div 2 = 7$ . Some began by working out the area of the large circle.
- (b) (i) Many explanations to this question on circles theorems lacked depth. Most candidates were able to give one pair of equal angles, but coming up with 2 or 3 pairs of equal angles with a reason was beyond the majority. Candidates who considered the ratios of sides could not be given any credit.
- (ii) This part was often omitted with very few candidates able to use the similarity to establish the result expected.
- (iii) Many handled the quadratic formula confidently and achieved the correct solutions to the required degree of accuracy. Others misquoted the formula with some not using a complete division line.
- (iv) Candidates found this part difficult with only a few obtaining the correct answer. 14.1 was a value that was given frequently.

Answer: (a) 7.14 (ii) 3.9 14.1 (iv) 10.2

### Question 9

- (a) Usually the table was completed correctly indicating that most candidates understood what was happening in this question on exponential growth.
- (b) The plotting was mostly correct, with acceptable curves drawn. Some candidates did not join the point at  $t = 1$  to the point at  $t = 0$ .
- (c) Many candidates were able to read the value from the  $y$ -axis accurately. The scale on the horizontal axis caused a problem for some who read the value at  $t = 3.1$ .
- (d) (i) The connection between gradient and tangent was generally understood. Some attempts at tangents were clearly chords. Reading from the horizontal axis required care. Not all attempts at tangents were at  $t = 2.5$ .
- (ii) This part was omitted by many candidates. A few interpreted the gradient correctly. Others wrote about the number of bacteria at a given time and did not refer to the change that had taken place over time.
- (e) This question defeated all but the most able. Some did manage to establish that  $k = 50$ .
- (f) (i) Candidates realised that this graph was a straight line. The correct line was usually drawn.
- (ii) A common error here was to state the number of bacteria rather than the value of  $t$ .

Answer: (a) 4050 (c) 1700 (d)(i) 870 (ii) Rate of increase (e)  $k = 50$   $a = 3$  (f)(ii) 3.45

### Question 10

- (a) (i) There was little awareness among candidates on how to tackle this problem involving a segment of a circle. Some confusion with volume rather than surface area was apparent, and there were many attempts to include triangle  $AOB$  into the calculation.
- (ii) Very few candidates were able to make any progress with this question. The solution was sometimes seen as the answer to part (i).
- (iii) A few understood the concept of the percentage remaining. Weaker candidates found this part demanding.
- (b) (i) Some correct answers were seen to this question on volumes, but there were a large number of incorrect answers.  $4.5 \times 300 = 1350$  was frequently given as the answer.

- (ii) Many candidates omitted this question. Much of the work seen was not progressing to a correct answer. The omission of a division by 60 was frequent.

Answer: (a)(i) 11.9 (ii) 1.73 or 1.74 (iii) 9.1% (b)(i) 19 100 (ii) 22

### Question 11

- (a) (i) The transformation was often recognised as a shear, but the factor not always given. It was not sufficient in this case simply to state which points of triangle *A* had been mapped to points on triangle *B*.
- (ii) A few candidates remembered the matrix for a shear. When not remembered, there was generally no indication of a method for finding it.
- (b) (i) There was some success in drawing the required triangle. Again, no indication of a method. Some candidates had a small triangle that had one vertex at the origin.
- (ii) Few were able to identify that triangle *C* resulted from a stretch, preferring instead to state an enlargement.
- (iii) Although finding the inverse of a  $2 \times 2$  matrix is a standard part of matrix theory, few candidates answered this question correctly.
- (iv) Those that had triangle *C* correct usually made a successful attempt in calculating the ratio of the areas.
- (c) Few attempts were made at this part involving finding a matrix to represent a transformation. There was very little evidence of the application of appropriate methods.

Answer: (a)(i) Shear, scale factor  $\frac{3}{2}$  (ii)  $\begin{pmatrix} 1 & 1.5 \\ 0 & 1 \end{pmatrix}$  (b)(i) Triangle vertices (8,2), (12,2), (24,6) (ii) Stretch  
(iii)  $\frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$  (iv) 2: 1 (c)  $\begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix}$

### Question 12

- (a) (i) Most candidates showed some knowledge of the sine rule and its manipulation. Some realised that  $AC = BC - 5$ , but several believed that  $AC$  was 5 cm. Only the better candidates attempted the final stage successfully. Weaker candidates did none of this and simply evaluated the expression given in the question.
- (ii) The expression given in the question was not always evaluated correctly. A variety of incorrect answers were given, the most frequent being 4.29. This answer resulted from an attempt to cancel the expression by  $\sin 65$  and ending up with  $5 - \sin 45 = 4.29$ .
- (b) (i) The cosine rule was generally applied well. Sometimes the answer was given in decimal form rather than the fractional form asked for in the question. Some candidates went on to work out the value of angle  $PRQ$ . Others treated  $PQR$  as a right-angled triangle and said that  $\cos PRQ = \frac{10}{13}$ .
- (ii) Candidates were not always clear about the relationship between the cosines of supplementary angles. Many gave an angle as their answer.
- (c) A correct answer was rare. Many simply drew a reflection of the original triangle.
- (d) There was a mixed response to this question. Some comfortably obtained the correct value for the area of the triangle. Others had the correct equation after applying the sine area formula but were unable to manipulate it correctly.

Answer: (a)(ii) 22.7 or 22.8 (b)(i)  $-\frac{11}{40}$  (ii)  $\frac{11}{40}$  (d) 6

# MATHEMATICS D (CALCULATOR VERSION)

Paper 4024/22

Paper 22

## Key messages

To succeed in this paper, candidates need to have completed full syllabus coverage and remember necessary formulae. All working must be clearly shown, and, in numerical work, a suitable level of accuracy must be used in any given calculation, and maintained throughout a series of linked calculations.

## General Comments

Many scripts were well presented, using the working and answer spaces effectively. Clarity and neatness in presentation remain a problem for some candidates. Candidates generally kept within the rubric when answering **section B** questions.

The standard algebraic techniques examined in **Questions 3** and **11** produced confident solutions from most candidates. In **Questions 2(a)(ii)** and **11(b)(ii)**, candidates were prompted to express their thinking in algebraic terms. Many candidates found this difficult or even impossible. In the last part of **Question 3**, very few candidates worked algebraically. In **Question 6(a)(iii)** and **(b)**, algebra could be used to express thinking from first principles rather than relying on poorly remembered formulae. Candidates could be encouraged to use the algebraic techniques they have acquired in order to do this.

Plotting and drawing in the graphical question was generally successful.

**Questions 1** and **8** required the use of remembered formula to deal with areas and perimeters. These were generally used successfully. A number of candidates were not clear which formulae were areas, and which formulae were perimeters. **Question 7** contained some standard work on  $2 \times 2$  matrices. This was well done. Many candidates seemed to be taken by surprise by the notation used in part **(c)(ii)** of this question.

## Comments on specific questions

### **Section A**

#### **Question 1**

- (a) Complete solutions were seen to this question on finding the area of a compound shape. As expected, in these cases, the field was seen as two right angled triangles and one trapezium. The formulae for the required areas were well remembered. A common error was to use 38 and 58 in the trapezium formula in the wrong order. Candidates splitting the trapezium into a right-angled triangle and a rectangle were apt to make errors, such as omitting  $\frac{1}{2}$  in the triangle formula, or in totalling all the areas. Some candidates thought that by calculating the unknown sides, the area could be calculated using a single formula. Although "area" was clearly stated in the question, some candidates gave perimeters for their answers.
- (b) Many candidates succeeded in finding the appropriate right-angled triangle and applying Pythagoras' Theorem accurately.

- (c) This question was generally well done, using the tangent of the acute angle. Some candidates gave the answer as  $54^\circ$ . A number of candidates gave the tangent of the complementary angle and went on without adjusting to the angle required. Some more elaborate methods were used. These usually lost accuracy at some point in the process.

Answers: (a) 3760 (b) 42.0 (c)  $54.1^\circ$ .

### Question 2

- (a) (i) Most candidates knew that they had to share out a total in this question on data handling. A good number did this correctly. A common error in evaluating  $\sum fx$  was  $4 \times 0 = 4$ . Some candidates used the number of families (50) for this total. A common error in sharing was to share by 4. It did not seem clear to these candidates that the second row of the table gave frequencies. Some candidates were uncomfortable with accepting the non-integer value 1.24 as being the correct answer.
- (ii) There were some well thought out solutions to this question, using correct algebra. Some candidates used a valid method to find  $y$  then gave  $x = 0$ . Others knew intuitively that  $x + y$  would need to be 8, offering such as 4 and 4, often with no working.
- (b) (i) When understood,  $\frac{1}{12}$  was usually reached in this question on equivalent fractions. Some fractions were not reduced to their lowest terms, and there were some percentages given. It was important to find the fraction of the category asked for in the question. Many candidates gave answers in numbers of litres, omitting to find the fraction.
- (ii) Candidates going astray in the previous part often managed to recover here and complete the pie chart. Candidates should be reminded that angles need to be accurately measured when drawing pie charts. In some cases, it would have been helpful if the calculations for the angles had been shown in the working. A number of pie charts were left incomplete.

Answers: (a)(i) 1.24 (ii)  $x = 3$   $y = 5$  (b)(i)  $\frac{1}{12}$  (ii) Correct pie chart

### Question 3

- (a) Candidates who included brackets when making the initial substitutions in this algebra question usually did well. Those who omitted them did not always recover the correct values, for example reaching  $-40$  in the denominator or  $\sqrt{-25}$  in the numerator.
- (b) Although there were many correct answers to this question on expanding brackets, this question did contain problems for many candidates.  $6x^2$  was frequently seen instead of  $6x^3$  for example. As usual, the removal of brackets when negative signs are involved needs care. The  $x$  was frequently omitted from the final term  $2x$ . A minority of candidates appeared not to understand the implications of the phrase "expand the brackets".
- (c) (i) This question on factorising was quite well done. Some evaluations of the quadratic formula were given here instead of factors.
- (ii) As the question stated, it was essential to have shown factors to gain credit here. Solutions by formula only were not accepted.
- (d) Correct answers were seen to this algebra question, often without any algebra in the working. The phrase "consecutive integers" was not always understood. Answers such as 28, 28, 28 were seen. Some candidates thought this question was about prime factors.

Answers: (a)  $-\frac{1}{8}$  (b)  $6x^3 - 3$  (c)(i)  $(9x - 4)(x + 1)$  (ii)  $\frac{4}{9} - 1$  (d) 27, 28 and 29

#### Question 4

- (a) The correct answer was usually given to this question on angles in circles, but not always justified. Some candidates thought the question was about the relationship between angles centre and angles at the circumference of the circle.
- (b)(i) There were a good many solutions that stated which sides were equal, often with a correct reason, and many of these gained full credit for this part. Some solutions quoted equal angles that could not be justified and so did not gain full credit. Some candidates showed working on the diagram. This can be unclear. Candidates should be encouraged to write their observations in the appropriate working space.
- (ii) (a) Many candidates were able to name the special quadrilateral.
- (b) Some candidates omitted this question on the properties of a kite.

Answers: (a) 72 because  $\widehat{O\hat{D}B}$  and  $\widehat{O\hat{E}B}$  are right-angles (b)(i) Congruency using three pairs of equal sides (ii)(a) Kite (b)  $90^\circ$

#### Question 5

- (a)(i) This question on sets was quite well done. There were some candidates who were not clear whether the answer should be 3 or 1.
- (ii) This question on the complement of the union of the two sets was generally well done. A common error was to include an extra element in this set.
- (b) This question on drawing a Venn diagram was less well done. Candidates could be encouraged to draw clearer working diagrams. 52 was often seen within working that was not always clear. Candidates could be encouraged to use such as  $x$  for an unknown number.
- (c)(i) A reasonable response was seen to this question on shading sets on a Venn diagram.
- (ii) Again, this question on interpreting a Venn diagram was quite well done. Some near misses included the omission of brackets or the omission of the intersection sign.

Answers: (a)(i) 3 (ii) 4, 8, 10 (b) 66 (c)(i) Correct region shaded (ii)  $C' \cap (A \cup B)$

#### Question 6

- (a)(i) This question on reverse percentages was generally understood. But here, as elsewhere, the question had to be read carefully.
- (ii) Candidates seemed to make this percentages question difficult for themselves by not working out and writing down the actual amount of money the price of the bicycle had been reduced by.
- (iii) Many answered this question well. There were a number of candidates who worked algebraically from first principles. More candidates could be encouraged to do this. A common error was to take 20% off \$1080.
- (b) This question on simple interest was very poorly answered. While it was clear that the formula for simple interest was well known, many candidates were not able to use it correctly in this question. Expressions that should have used the amount of interest, \$81, instead were using \$600 or \$681.

Answers: (a)(i) 899 (ii) 33.5 (iii) 900 (b) 4.5



### Question 7

- (a) The principles of column vectors were generally understood. The required arithmetic was always accurate. This often involved sign errors.
- (b) Again, quite a good response was seen to this question on matrices. There was some confusion about the shape of the product matrix.  $2 \times 3$  was a popular alternative.
- (c) (i) This question on finding the inverse of a matrix was generally well done. A number of 6's appeared when evaluating the relevant determinant, arising from  $2 \times 0 = 2$ . Some rearrangements of the original matrix went astray, leading to an incorrect adjoint matrix.
- (ii) The main problem here was expressing  $3I$  as a  $2 \times 2$  matrix.

Answers: (a)  $\begin{pmatrix} 6 \\ 7 \\ 13 \end{pmatrix}$  (b)  $\begin{pmatrix} 13 \\ 10 \end{pmatrix}$  (c)(i)  $\frac{1}{4} \begin{pmatrix} 4 & 0 \\ 2 & 1 \end{pmatrix}$  (ii)  $\begin{pmatrix} -2 & 0 \\ -2 & 1 \end{pmatrix}$

### Section B

### Question 8

- (a) This question on fractions of circles was dependent upon remembering the correct formula for the circumference of a circle and how to adapt it to evaluate arc length. Full marks were dependent upon using the correct angle for the arc and adding  $6+6$  to complete the perimeter. Occasionally, candidates used area formulae.
- (b) Less confusion between area and length was seen here. The formula for the area of a circle was usually remembered accurately. A good number of candidates were able to adapt this to calculate the correct sector area.
- (c) (i) There were some correct solutions to this part. Candidates needed to do some work on the diagram to find the right-angled triangle within triangle  $AOB$ . Some who did this successfully to find  $6\cos 25$  doubled it instead of adding 6. Others found  $AB$  and thought that this would lead directly to  $w$ . There were a number who thought that  $w$  could be calculated using previous area results. When not omitted, there were a number of solutions that simply assumed  $w$  was 12.
- (ii) There were a number of completely successful solutions. Usually, the area of triangle  $AOB$  was omitted from the calculation. When included, the expected area sine formula was not always used. Solutions usually revealed clear ideas about the area of a rectangle and the formulation of the required percentage.

Answers: (a) 44.5 (b) 97.4 (c)(i) 11.4 (ii) 19.0

### Question 9

- (a) This question on plotting points and drawing a curve was generally well done.
- (b) Most of the candidates understood how to estimate the gradient. Sometimes the negative sign was lost in the calculation.
- (c) (i) Some correct answers seen. A number of correct substitutions using  $-a$  were seen, but these were not converted into  $-b$ .
- (ii) Candidates generally gained this mark, usually by re-calculating each value.
- (iii) The graph was generally accurately plotted. There were a number of candidates who plotted this part of the curve in the second quadrant.

- (iv) Some candidates appreciated the pattern here and realised that the gradient should be the same as in part (b). Others assumed that the gradient was opposite in sign. There were some working solutions that got within range of their previous answer.
- (d)(i) The drawing of this line was often incorrect. Lines such as  $y = 4$  were drawn.
- (ii) Candidates who did draw the correct line in part (i) usually read their graphs accurately, although the scales did need care. For example, 0.7 instead of 1.7 was sometimes seen. Many of the lines drawn in (i) did not intersect the curve twice. Two values were nevertheless given as the answer. It seemed that candidates were reading off the intersection of their line and the tangent drawn earlier.
- (iii) The expected method, equating  $f(x)$  and  $4 - x$  was seen a number of times, and the equation formed was successfully transformed into the required form.

Answers: (a) Correct plots and curve (b) Tangent at  $x = 0.75$  with correct follow through gradient (c)(i) - b (ii) Completed table (iii) Correct curve (iv) Same gradient as in part (b) (d)(i) Correct straight line (ii) Intersections of their curve and line (iii)  $2x^2 - 4x + 1 = 0$

### Question 10

- (a)(i) Candidates usually recognised that the cosine rule was required here. It was often well remembered and accurately evaluated. Some candidates stopped before taking the square root. Some candidates tried to use Pythagoras' theorem or a ratio more appropriate for an acute angled right-angled triangle. There were some who simply added 8 and 6.
- (ii) Bearings were generally understood. Some candidates had partially correct answers with such as 85 or 95 seen in the working. Incorrect ideas led to answers such as 45, 65 or 275. Angles of 86, 266 or 267 were seen, which could have come from measuring angles on the diagram.
- (b)(i) There were many successful proofs seen. A number of candidates just evaluated  $PR$ .
- (ii) Again a number of successful proofs. A common error here was to equate ratios from two different triangles. Again, some candidates just evaluated  $SR$ .
- (iii) The evaluation of the expression was generally well done.
- (iv) A number of correct answers using the expected method were seen, and also a number using longer methods, involving the calculation of separate areas. These often went astray by calculating an incorrect area. Some candidates found the correct ratio and then squared it.

Answers: (a)(i) 11.9 (ii)  $265^\circ$  (b)(i) See the marking scheme (ii) See the marking scheme (iii) 267 (iv) 2.34

### Question 11

- (a) The manipulation of algebraic fractions appeared to be a familiar topic, with candidates generally being successful. The most common error occurred in the removal of the brackets from the numerator, leading to +8 instead of - 8. In a question such as this, candidates could be advised to leave the denominator in factor form.
- (b)(i) Most of the candidates were able to use the given information to construct the an expression for the average speed of the train.
- (ii) Generally a poorer response was seen here, largely due to the fact that many candidates were unable to produce the correct term,  $\frac{320}{x + 2.5}$ , for the average speed of the car.
- (iii) There were many accurate solutions of the quadratic equation. The most common difficulty was in dealing successfully with the (- 20) in the discriminant.

- (iv) Only a minority were able to calculate the correct average speed of the car. Not many corrected their value to the nearest km/h.

Answers: (a)  $\frac{10p-29}{(p+2)(2p-3)}$  (b) (i)  $\frac{320}{x}$  (ii)  $2x^2 + 5x - 20 = 0$  (iii) 2.15 - 4.65 (iv) 69

### Question 12

- (a) (i) This question on vectors was quite well done.
- (ii) Some success was seen here in this question on adding vectors. Candidates need to realise that in vectors, when making a general statement, the order of the letters is important.
- (iii) Many of the candidates were able to complete this question.
- (iv) There were some correct completions here based on previous correct work.
- (v) There were some carefully constructed solutions using  $\overline{AB}$  to find, for example,  $\overline{CD}$  using a variety of vector paths. Most of the attempts at this part were restricted to showing that  $\overline{CD} = 2\overline{CG}$ .
- (b) (i) Some correct solutions, and some near misses were seen to this question on enlargement. Many attempts clearly did not understand the idea of enlargement.
- (ii) This part was not always attempted, even by those candidates who were successful in the previous part.
- (iii) A number of correct responses were seen, using either areas or scale factors.

Answers: (a)(i) 6.08 (ii)  $\begin{pmatrix} 2 \\ -1.5 \end{pmatrix}$  (iii)  $\begin{pmatrix} 2 \\ -1.5 \end{pmatrix}$  (iv) Equal and parallel (b)(i) Correct triangle  
(ii) Correct triangle (iii) 1 : 9