UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
Cambridge International Level 3 Pre-U Certificate
Principal Subject

FURTHER MATHEMATICS **9795/01**

Paper 1 Further Pure Mathematics **May/June 2012**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

This document consists of **4** printed pages.

UNIVERSITY *of* CAMBRIDGE International Examinations

[Turn over

1 Using any standard results given in the List of Formulae (MF20), show that

$$\sum_{r=1}^{n}(r^2 - r + 1) = \tfrac{1}{3}n(n^2 + 2)$$

for all positive integers n. [4]

2 Find the area enclosed by the curve with polar equation $r = \sin\theta + \cos\theta$, $0 \leqslant \theta \leqslant \frac{1}{2}\pi$. [4]

3 **(i)** Given that $y = \sqrt{\sinh x}$ for $x \geqslant 0$, express $\dfrac{dy}{dx}$ in terms of y only. [3]

(ii) Find $\displaystyle\int \frac{2t}{\sqrt{1+t^4}}\,dt$. [3]

4 The curve C has equation $y = \dfrac{x+1}{x^2+3}$.

(i) By considering a suitable quadratic equation in x, find the set of possible values of y for points on C. [5]

(ii) Deduce the coordinates of the turning points on C. [4]

5 **(i)** Write down the 2×2 matrices which represent the following plane transformations:

(a) an anticlockwise rotation about the origin through an angle α;

(b) a reflection in the line $y = x\tan\left(\frac{1}{2}\beta\right)$. [2]

(ii) A reflection in the x-y plane in the line $y = x\tan\left(\frac{1}{2}\theta\right)$ is followed by a reflection in the line $y = x\tan\left(\frac{1}{2}\phi\right)$. Show that the composition of these two reflections (in this order) is a rotation and describe this rotation fully. [6]

6 A group G has order 12.

(i) State, with a reason, the possible orders of the elements of G. [2]

The identity element of G is e, and x and y are distinct, non-identity elements of G satisfying the three conditions

(1) x has order 6,

(2) $x^3 = y^2$,

(3) $xyx = y$.

(ii) Prove that $yx^2y = x$. [3]

(iii) Prove that G is not a cyclic group. [2]

7 **(i)** Use de Moivre's theorem to show that $\tan 4\theta = \dfrac{4t(1-t^2)}{1-6t^2+t^4}$, where $t = \tan\theta$. [5]

(ii) Given that θ is the acute angle such that $\tan\theta = \frac{1}{5}$, express $\tan 4\theta$ as a rational number in its simplest form, and verify that

$$\tfrac{1}{4}\pi + \tan^{-1}\left(\tfrac{1}{239}\right) = 4\tan^{-1}\left(\tfrac{1}{5}\right).$$ [4]

8 The function f satisfies the differential equation

$$x^2 \mathrm{f}''(x) + (2x-1)\mathrm{f}'(x) - 2\mathrm{f}(x) = 3\mathrm{e}^{x-1} + 1, \qquad (*)$$

and the conditions $\mathrm{f}(1) = 2$, $\mathrm{f}'(1) = 3$.

(i) Determine $\mathrm{f}''(1)$. [2]

(ii) Differentiate $(*)$ with respect to x and hence evaluate $\mathrm{f}'''(1)$. [4]

(iii) Hence determine the Taylor series approximation for $\mathrm{f}(x)$ about $x = 1$, up to and including the term in $(x-1)^3$. [3]

(iv) Deduce, to 3 decimal places, an approximation for $\mathrm{f}(1.1)$. [2]

9 **(i)** Show that the substitution $u = \dfrac{1}{y^3}$ transforms the differential equation $\dfrac{\mathrm{d}y}{\mathrm{d}x} + y = 3xy^4$ into

$$\frac{\mathrm{d}u}{\mathrm{d}x} - 3u = -9x.$$ [3]

(ii) Solve the differential equation $\dfrac{\mathrm{d}y}{\mathrm{d}x} + y = 3xy^4$, given that $y = \frac{1}{2}$ when $x = 0$. Give your answer in the form $y^3 = \mathrm{f}(x)$. [9]

10 The line L has equation $\mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$ and the plane Π has equation $\mathbf{r}.\begin{pmatrix} 2 \\ -6 \\ 3 \end{pmatrix} = k$.

(i) Given that L lies in Π, determine the value of k. [2]

(ii) Find the coordinates of the point, Q, in Π which is closest to $P\,(10, 2, -43)$. Deduce the shortest distance from P to Π. [5]

(iii) Find, in the form $ax + by + cz = d$, where a, b, c and d are integers, an equation for the plane which contains both L and P. [6]

11 The complex number $w = (\sqrt{3} - 1) + \mathrm{i}(\sqrt{3} + 1)$.

(i) Determine, showing full working, the exact values of $|w|$ and $\arg w$. [4]
[You may use the result that $\tan\left(\frac{5}{12}\pi\right) = 2 + \sqrt{3}$.]

(ii) (a) Find, in the form $r\mathrm{e}^{\mathrm{i}\theta}$, the three roots, z_1, z_2 and z_3, of the equation $z^3 = w$. [4]

(b) Determine $z_1 z_2 z_3$ in the form $a + \mathrm{i}b$. [2]

(c) Mark the points representing z_1, z_2 and z_3 on a sketch of the Argand diagram. Show that they form an equilateral triangle, Δ_1, and determine the side-length of Δ_1. [5]

(d) The points representing kz_1, kz_2 and kz_3 form Δ_2, an equilateral triangle which is congruent to Δ_1, and one of whose vertices lies on the positive real axis. Write down a suitable value for the complex constant k. [1]

12 **(i)** Let $I_n = \int_0^3 x^n \sqrt{16 + x^2}\,\mathrm{d}x$, for $n \geqslant 0$. Show that, for $n \geqslant 2$,

$$(n + 2)I_n = 125 \times 3^{n-1} - 16(n - 1)I_{n-2}.$$ [6]

(ii) A curve has polar equation $r = \frac{1}{4}\theta^4$ for $0 \leqslant \theta \leqslant 3$.

(a) Sketch this curve. [2]

(b) Find the exact length of the curve. [7]

13 Define the *repunit number*, R_n, to be the positive integer which consists of a string of n 1's. Thus,

$$R_1 = 1, \quad R_2 = 11, \quad R_3 = 111, \quad \ldots, \quad R_7 = 1\,111\,111, \quad \ldots, \text{ etc.}$$

Use induction to prove that, for all integers $n \geqslant 5$, the number

$$13\,579 \times R_n$$

contains a string of $(n - 4)$ consecutive 7's. [6]