CAMBRIDGE INTERNATIONAL EXAMINATIONS

Pre-U Certificate

MARK SCHEME for the May/June 2014 series

9795 FURTHER MATHEMATICS

9795/01 Paper 1 (Further Pure Mathematics), maximum raw mark 120

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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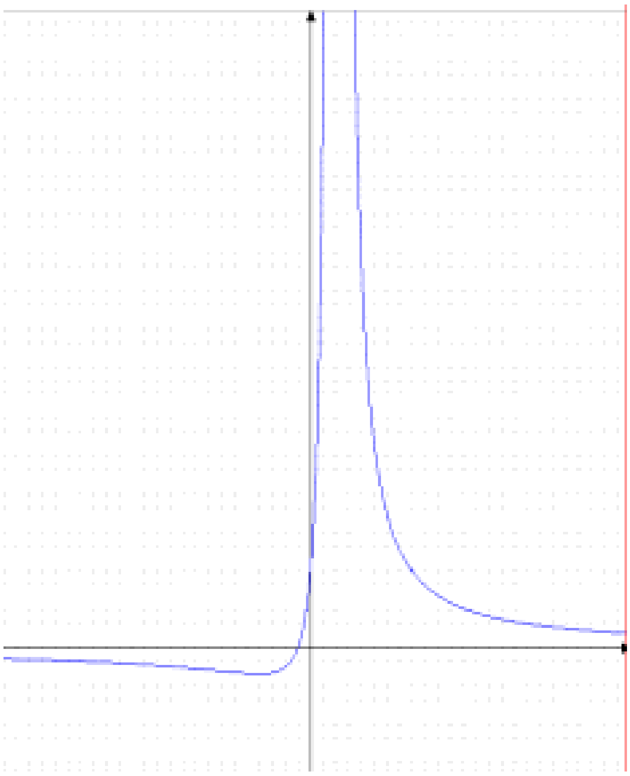
1	**(i)**	$S = N^2 + (N+1)^2 + (N+2)^2 + ... + (2N-2)^2 + (2N-1)^2 + (2N)^2$	B1 [1]
	(ii)	$= \sum_{r=1}^{2N} r^2 - \sum_{r=1}^{N-1} r^2$	M1
		$= \frac{2N}{6}(2N+1)(4N+1) - \frac{(N-1)}{6}(N)(2N-1)$	A1 A1
		$= \frac{N}{6}\left(16N^2 + 12N + 2 - [2N^2 - 3N + 1]\right) = \frac{N}{6}\left(14N^2 + 15N + 1\right)$	
		$= \frac{N}{6}(N+1)(14N+1)$	A1 [4]
		ALTERNATIVE	
		$S = \sum_{r=0}^{N}\left(N^2 + 2Nr + r^2\right)$ Splitting into three parts	M1
		$= (N+1)N^2 + 2N.\frac{N}{2}(N+1) + \frac{N}{6}(N+1)(2N+1)$ B1 M1 Use of both these standard results	B1 M1
		$= \frac{(N+1)}{6}\left(6N^2 + 6N^2 + 2N^2 + N\right)$	
		$= \frac{N}{6}(N+1)(14N+1)$	A1
2	**(i)**	$\mathbf{AB} = \begin{bmatrix} 1 & 2 & 1 \\ t & 1 & -t \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} t-2 & 0 & 5 \\ 12 & -2 & -6 \\ 3t & 4 & 7 \end{bmatrix} = \begin{bmatrix} 4t+22 & 0 & 0 \\ 12-2t-2t^2 & -2-4t & -2t-6 \\ 6t+18 & 0 & 10 \end{bmatrix}$	M1
		M1 good effort; A1 all correct	A1
		Give M0 (A0) B1 B1 for **BA** found (k, t ✓) or k, t found from 1 or 2 elements of **AB** only Give M1 (A0) B1 B1 for k, t found from most (but not all) elements of **AB**	
		$= 10\,\mathbf{I}$ when $t = -3$ Allow these correct from most of **AB** correct	A1 A1 [4]
	(ii)	$\begin{bmatrix} -5 & 0 & 5 \\ 12 & -2 & -6 \\ -9 & 4 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 12 \\ 22 \end{bmatrix}$	B1 [1]
	(iii)	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 1 & 2 & 1 \\ -3 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 8 \\ 12 \\ 22 \end{bmatrix}$ M1 for $\mathbf{x} = C^{-1}\mathbf{u}$ B1 for C^{-1} correct	M1 B1
		$x = 5.4$, $y = 5.4$, $z = 7$	A1 [3]
		B1 SC for correct x, y, z without inverse matrix method seen	

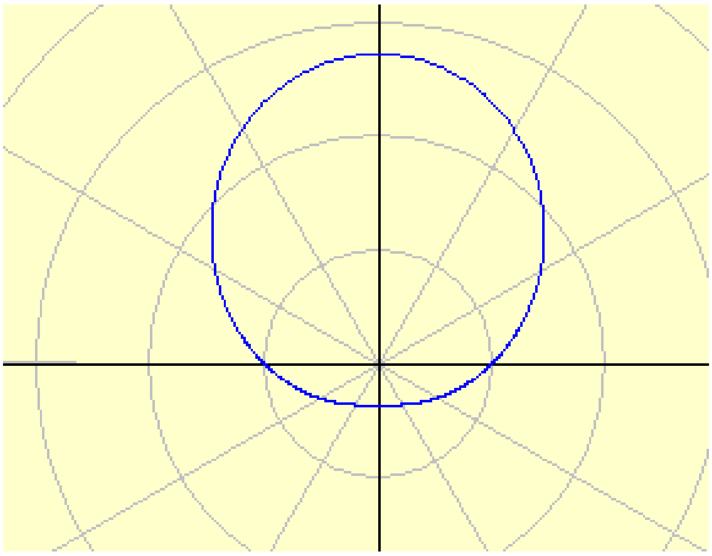
3	**(i)**	$\lvert z+2 \rvert = 3$ Circle Centre –2 + 0i, radius 3	M1 A1
		$\arg(z-\mathrm{i}) = -\frac{1}{4}\pi$ Half-line From 0 + i at 45° downwards	M1 A1 [4]
		[Allow full line through i for M1 A0]	
	(ii)	1 (+ 0i)	B1 [1]
4		$I_n = \left[x^n.\frac{1}{3}(2x+1)^{\frac{3}{2}}\right]_0^4 - \int_0^4 nx^{n-1}.\frac{1}{3}(2x+1)^{\frac{3}{2}}\,dx$ Use of parts	M1
		For $\int\sqrt{2x+1} = \frac{1}{3}(2x+1)^{\frac{3}{2}}$ at any stage	B1
		$3\,I_n = \left(4^n.27-0\right) - n\int_0^4 x^{n-1}(2x+1)\sqrt{2x+1}\,dx$ Splitting the power of 2*x* + 1	M1
		$-n\left(2I_n + I_{n-1}\right)$	A1
		$\Rightarrow\ (2n+3)I_n = 27\times 4^n - n\,I_{n-1}$ Legitimately (**Answer Given**)	A1 [5]

5	(i)	$y = \dfrac{12(x+1)}{(x-2)^2} \Rightarrow \dfrac{dy}{dx} = 12\left\{\dfrac{(x-2)^2 - (x+1).2(x-2)}{(x-2)^4}\right\}$	M1 A1
		$= 0$ when $x - 2 = 2x + 2$ i.e. $x = -4$, $y = -1$	A1 A1 [4]
	(ii)	VA $x = 2$ HA $y = 0$ Stated or clearly shown on diagram	B1 B1
		Intercepts $(0, 3)$ and $(-1, 0)$ Stated or clearly shown on diagram	B1 B1
		 Basic shape correct	B1
		All details correct (Allow **ft** on min. point provided it doesn't ruin overall shape or position)	B1 [6]
6		$\dfrac{dy}{dx} + \dfrac{2}{x}y = \dfrac{4\ln x}{x}$ I.F. is $\exp\left\{\int \dfrac{2}{x}\,dx\right\} = x^2$	M1 A1
		$x^2\dfrac{dy}{dx} + 2xy = 4x\ln x \Rightarrow x^2 y =$ LHS correct	B1
		RHS $= 4\ln x.\dfrac{1}{2}x^2 - \int \dfrac{1}{2}x^2.\dfrac{4}{x}\,dx$ M1 Use of parts, right way round	M1
		$= 2x^2\ln x - x^2 + C$ Ignore the "+ C" here	A1
		$y = 2\ln x - 1 + \dfrac{C}{x^2}$	A1
		Use of $x = 1, y = 1$ to find C $C = 2$ i.e. $y = 2\ln x - 1 + \dfrac{2}{x^2}$ (any correct form)	M1 A1 [8]

7	For $n = 1$, $f(1) = 39$ (= 3 × 13) which is divisible by 13 (so result true for $n = 1$)	B1
	Assume that $f(k) = 13m$ (for some (positive) integer m)	M1
	Considering $f(k + 1) = 11^{2k+1} + 7 \times 4^{k+1}$ (must be simplified powers)	M1
	Expressing $f(k + 1)$ in terms of $f(k)$	
	$f(k + 1) = 121\, f(k) - 819 \times 4^k$ **or** $f(k + 1) = 4\, f(k) + 117 \times 11^{2k-1}$ etc.	M1
	$= 13\{121m - 63 \times 4^k\}$ **or** $= 13\{4m + 9 \times 11^{2k-1}\}$ etc.	A1
	Explanation that $13 \mid f(k) \Rightarrow 13 \mid f(k + 1)$ and $13 \mid f(1) \Rightarrow$ general result	E1 [6]
8 (i)	$\begin{bmatrix} 2+5\lambda \\ -5-2\lambda \\ 7+3\lambda \end{bmatrix}$ substd. into $x + 4y + z + 11 = 0$	M1
	$\Rightarrow 2+5\lambda-20-8\lambda+7+3\lambda+11=0$ (for all λ) Shown correct **ALTs.** 2 points of l shown to be in Π **or** Π's nml. perpr. to l and 1 point of l in Π	A1 [2]
(ii)	$\overrightarrow{PR} = \begin{bmatrix} 1+5\lambda \\ -7-2\lambda \\ 7+3\lambda-k \end{bmatrix}$ Setting their $\overrightarrow{PR} = m\begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$	B1 M1
	Solving for λ in **i** & **j** components: $4(1 + 5\lambda) = -7 - 2\lambda \Rightarrow \lambda = -\frac{1}{2}$	M1 A1
	Substituting back into **i** & **k** components to find k: $1+5\left(-\frac{1}{2}\right) = 7+3\left(-\frac{1}{2}\right) - k \Rightarrow k = 7$	M1 A1 [6]
	ALTERNATIVE	
	$\begin{bmatrix} 1 \\ -7 \\ 7-k \end{bmatrix} \bullet \begin{bmatrix} 5 \\ -2 \\ 3 \end{bmatrix} \times \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} 1 \\ -7 \\ 7-k \end{bmatrix} \bullet \begin{bmatrix} -14 \\ -2 \\ 22 \end{bmatrix} = 0 \Rightarrow -14 + 14 + 22(7 - k) = 0$	M2 M1 A1 A1
	$\Rightarrow k = 7$	A1

9	(i)	All non-identity elements pair up with their inverses. If $\mid G \mid = 2k$, then there are $2k - 1$ elements to pair up $\Rightarrow$ at least one element "pairs up" with itself.	M1 A1 [2]
	(ii)	$ab = (ab)^{-1} = b^{-1}a^{-1} = ba$ **ALT.** $(ab)^2 = i \Rightarrow abab = i \Rightarrow aba = b$ (post-multiplying by $b = b^{-1}$) $\Rightarrow ab = ba$ (post-multiplying by $a = a^{-1}$)	M1 A1 [2]
	(iii)	E.g. Let $x = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$ and $y = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$, each of order 2 Then $xy = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 2 & 3 & 1 \end{pmatrix}$ (transforming) OR $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \\ 3 & 1 & 2 \end{pmatrix}$ (transposing) of order 3	B1 B1 [2]
	(iv)	**(a)** H closed since $x(yx) = x(xy) = x^2y = y$ and $y(xy) = y(yx) = y^2x = x$ using result of **(ii)** [Associativity follows from that of G] The identity is in H and each element has its own inverse (itself) in H Hence H a subgroup of G However, Lagrange's Theorem states that $o(H) \mid o(G)$ and $4 \nmid 4n + 2$ contradicting the assumption that G can contain all self-inverse elements.	M1 A1 B1 B1 E1 [5]

10 (i)	$\cos 6\theta = \text{Re}(c + \text{i}s)^6$	M1	
	Use of Binomial expansion for $(c + \text{i}s)^6$ Re terms only required	M1	
	$= c^6 - 15c^4s^2 + 15c^2s^4 - s^6$	A1	
	Use of $s^2 = 1 - c^2$ (and powers)	M1	
	$\Rightarrow \cos 6\theta = c^6 - 15c^4(1-c^2) + 15c^2(1-2c^2+c^4) - (1-3c^2+3c^4-c^6)$		
	$\Rightarrow 2\cos 6\theta = 64c^6 - 96c^4 + 36c^2 - 2$ **Answer Given**	A1	[5]
(ii)	Setting $x = 2\cos\theta$ in given eqn.	M1	
	$\Rightarrow (2c)^6 - 6(2c)^4 + 9(2c)^2 - 3 = 2\cos 6\theta - 1 = 0$	A1	
	$\cos 6\theta = \frac{1}{2} \Rightarrow 6\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}$	M1	
	$\Rightarrow \theta = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{7\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}$	A1	
	$x = 2\cos\theta$ for each θ Must be 6 distinct θ's **OR** $x = \pm 2\cos\theta$ for $\theta = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{7\pi}{18}$	B1	[5]
(iii)	Product of SIX roots = –3 = $\left(-4\cos^2\frac{\pi}{18}\right)\left(-4\cos^2\frac{5\pi}{18}\right)\left(-4\cos^2\frac{7\pi}{18}\right)$	M1	
	Signs justified; e.g. via $\cos(\pi-\theta) = -\cos\theta$	M1	
	$\Rightarrow \cos\frac{\pi}{18}\cos\frac{5\pi}{18}\cos\frac{7\pi}{18} = \frac{\sqrt{3}}{8}$ [+ve √ taken as read, since all angles acute]	A1	[3]

11	(i)	$(r, \theta) = (1, 0)$ Condone (r, θ) given as (θ, r) throughout	B1 [1]
	(ii)	$e^{\sin\theta}$ min./max. when $\sin\theta$ min./max. giving min. at $(r, \theta) = \left(e^{-1}, -\frac{\pi}{2}\right)$ and max. at $(r, \theta) = (e, \frac{\pi}{2})$	B1 B1 [2]
	(iii)		
		Symmetry in y-axis	B1
		Closed curve	B1
		Shape essentially correct – don't penalise kinks. **ft** from (ii) where suitable	B1 [3]
	(iv)	$A = \int_0^{0.3} \frac{1}{2} e^{2\sin\theta}\, d\theta$ Use of formula; correct	M1 A1
		$f(\theta) = e^{2\sin\theta} \Rightarrow f'(\theta) = e^{2\sin\theta}.\, 2\cos\theta$ and $f''(\theta) = e^{2\sin\theta}(4\cos^2\theta - 2\sin\theta)$	B1 B1
		$f(0) = 1,\ f'(0) = 2,\ f''(0) = 4$	B1
		$\Rightarrow f(\theta) = 1 + 2\theta + 2\theta^2 \ldots$	M1 A1
		$A = \frac{1}{2}\left[\theta + \theta^2 + \frac{2}{3}\theta^3\right]_0^{0.3} = 0.204$ or $\frac{51}{250}$ 1st A1 for correct ∫n. of a 3-term quadratic	A1
		Accept 0.205 from correct ∫n. of quartic $(1 + 2\theta + 2\theta^2 + \theta^3 + \frac{1}{4}\theta^4)$	A1 [9]
		NB Correct answer is 0.204 98 …	
		ALTERNATIVES (middle 5 marks)	
		Alt. I Ignoring terms in θ^3 and above, $\sin\theta \approx \theta$ ...	B1
		$e^{\sin\theta} \approx 1 + \theta + \frac{1}{2}\theta^2 \ldots$	M1 A1
		$\Rightarrow (e^{\sin\theta})^2 \approx 1 + 2\theta + \theta^2 \ldots + 2 \times \frac{1}{2}\theta^2 \ldots = 1 + 2\theta + 2\theta^2 \ldots$	M1 A1
		Alt. II $f(\theta) = e^{\sin\theta}$ … $f'(\theta) = \cos\theta\, e^{\sin\theta}$ and $f''(\theta) = (\cos^2\theta - \sin\theta)\, e^{\sin\theta}$	B1 B1
		$f(0) = f'(0) = f''(0) = 1$	B1
		$\Rightarrow f(\theta) = 1 + \theta + \frac{1}{2}\theta^2 + \ldots$ Maclaurin attempt at as a function of θ	M1
		$\Rightarrow [f(\theta)]^2 = 1 + 2\theta + 2\theta^2 + \ldots$ Ignore higher power terms	A1

12	**(i)**	**(a)** $\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{\frac{1}{2}\left(e^{x}-e^{-x}\right)}{\frac{1}{2}\left(e^{x}+e^{-x}\right)} \times \dfrac{e^{x}}{e^{x}} = \dfrac{e^{2x}-1}{e^{2x}+1}$	M1
		Answer Given so MUST justify final answer	A1 [2]
		(b) $\tanh x = \dfrac{e^{2x}-1}{e^{2x}+1} = \dfrac{1}{k} \Rightarrow ke^{2x}-k = e^{2x}+1$ Equated for k	M1
		$\Rightarrow (k-1)e^{2x} = k+1 \Rightarrow x = \dfrac{1}{2}\ln\left(\dfrac{k+1}{k-1}\right)$ **Answer Given**	A1
		$\sinh 2x = \dfrac{1}{2}\left(e^{2x}-e^{-2x}\right) = \dfrac{1}{2}\left(\dfrac{k+1}{k-1} - \dfrac{k-1}{k+1}\right)$	M1
		or via $t = \tanh(\frac{1}{2}$–"angle") identity	A1 [4]
		$= \dfrac{1}{2}\left(\dfrac{\left(k^2+2k+1\right)-\left(k^2-2k+1\right)}{k^2-1}\right) = \dfrac{2k}{k^2-1}$	
	(ii)	$y = \dfrac{1}{2}\ln(\tanh x) \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2}.\dfrac{1}{\tanh x}.\text{sech}^2 x$	M1 A1
		$= \dfrac{1}{\sinh 2x}$ Here or later (or equivalent)	A1
		$1 + \left(\dfrac{dy}{dx}\right)^2 = 1 + \dfrac{1}{\sinh^2 2x} = \dfrac{\cosh^2 2x}{\sinh^2 2x}$	M1 A1
		M0 if no progress towards an integrable form	
		$L = \displaystyle\int_{\alpha}^{\beta} \dfrac{\cosh 2x}{\sinh 2x}\,dx = \left[\dfrac{1}{2}\ln(\sinh 2x)\right]_{\alpha}^{\beta}$ For the integration	M1 M1 A1
		NB Alternative approaches lead to $\dfrac{1}{2}\displaystyle\int(\tanh x + \coth x)\,dx = \dfrac{1}{2}\left[\ln(\cosh x) + \ln(\sinh x)\right]$	
		For β, $k = 2 \Rightarrow \sinh 2x = \dfrac{4}{3}$; for α, $k = 3 \Rightarrow \sinh 2x = \dfrac{3}{4}$	M1
		$\Rightarrow L = \dfrac{1}{2}\ln\dfrac{4}{3} - \dfrac{1}{2}\ln\dfrac{3}{4} = \ln\dfrac{4}{3}$	A1 [10]

13 (i)	If w is a root with $\lvert z \rvert = 1$, then $w = \cos\theta + i\sin\theta$	Modelling	M1	
	Then $\cos 2\theta + i\sin 2\theta - 2(\cos\theta - i\sin\theta) + ki = 0$	Use of de Moivre's theorem	M1	
	$\cos 2\theta - 2\cos\theta = 0$	Considering real parts	M1	
	$2c^2 - 2c - 1 = 0$	Solving	M1	
	$\lvert c \rvert \le 1 \Rightarrow \cos\theta = \frac{1}{2}\left(1-\sqrt{3}\right)$		A1	
	$\sin\theta = \pm\sqrt{1-\frac{1}{4}\left(4-2\sqrt{3}\right)} = \pm\sqrt{\frac{1}{2}\sqrt{3}}$		A1	
	$k = -2\sin\theta - 2\sin\theta\cos\theta$ or $-2\sin\theta(1+\cos\theta)$	Considering imaginary parts	M1	
	$= \mp 2\times\sqrt{\frac{1}{2}\sqrt{3}}\times\frac{1}{2}\left(3-\sqrt{3}\right) = \left(3-\sqrt{3}\right)\sqrt{\frac{1}{2}\sqrt{3}}$	**Answer Given**	A1	[8]
	ALTERNATIVE I			
	Let $w = a + \mathrm{i}b$ where $a^2 + b^2 = 1$ Then $w^2 = a^2 - b^2 + \mathrm{i}.2ab$ and $\frac{1}{w} = \frac{a - \mathrm{i}b}{a^2+b^2} = a - \mathrm{i}b$ (since $\lvert w \rvert = 1$)		B1 B1	
	giving $\left(a^2 - 2a - b^2\right) + \mathrm{i}\left(2ab + 2b + k\right) = 0$			
	$\Rightarrow a^2 - 2a - b^2 = 0$ **and** $2ab + 2b + k = 0$		M1	
	Using $b^2 = 1 - a^2$ in Real part = 0 and solving a quadratic in a: $2a^2 - 2a - 1 = 0$ $\Rightarrow a = \frac{1}{2}\left(1 \pm \sqrt{3}\right)$		M1	
	However, $\lvert a \rvert < 1 \Rightarrow a = \frac{1}{2}\left(1-\sqrt{3}\right)$ or similar argument from $a^2 = 1 - \frac{\sqrt{3}}{2}$		A1	
	Thus $b^2 = \frac{\sqrt{3}}{2}$ and $b = \pm\sqrt{\frac{\sqrt{3}}{2}}$		A1	
	Substituting for a and b in Im part = 0		M1	
	$\Rightarrow k = -2b(a+1) = \mp 2 . \sqrt{\frac{1}{2}\sqrt{3}} . \frac{1}{2}\left(3-\sqrt{3}\right)$ $= \left(3-\sqrt{3}\right)\sqrt{\frac{1}{2}\sqrt{3}}$ (since told $k > 0$) **Answer Given**		A1	
	ALTERNATIVE II			
	Using the form $w^3 - 2 + kw\mathrm{i} = 0$ with $w = x + \mathrm{i}y$ where $x^2 + y^2 = 1$		M1	
	$\Rightarrow \left(x^3 - 3xy^2 - 2 - ky = 0\right) + \mathrm{i}\left(3x^2y - y^3 + kx = 0\right)$		B1 B1	
	Equating for k and eliminating y using $x^2 + y^2 = 1 \Rightarrow 2x^2 - 2x - 1 = 0$ etc.		M1	

(ii)	If $w^3 + kiw - 2 = 0$ has roots α, β, γ, then $\alpha\beta\gamma = 2$	B1	
	Then $\lvert \alpha \rvert = 1 \Rightarrow \lvert \beta\gamma \rvert = 2$ and at least one of β, γ has magnitude > 1	B1	[2]