Cambridge Pre-U

Cambridge International Examinations
Cambridge Pre-U Certificate Principal Subject

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FURTHER MATHEMATICS **9795/01**

Paper 1 Further Pure Mathematics **May/June 2014**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

This document consists of **4** printed pages.

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1 The series S is given by $S = \sum_{r=0}^{N} (N + r)^2$.

(i) Write out the first three terms and the last three terms of the series for S. [1]

(ii) Use the standard result $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n + 1)(2n + 1)$ to show that $S = \frac{1}{6}N(N + 1)(aN + 1)$ for some positive integer a to be determined. [4]

2 **(i)** Show that there is a value of t for which $\mathbf{AB}$ is an integer multiple of the 3×3 identity matrix $\mathbf{I}$, where

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 1 \\ t & 1 & -t \\ 3 & 2 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{B} = \begin{pmatrix} t-2 & 0 & 5 \\ 12 & -2 & -6 \\ 3t & 4 & 7 \end{pmatrix}.$$ [4]

(ii) Express the system of equations

$$\begin{aligned} -5x \quad\quad + 5z &= 8 \\ 12x - 2y - 6z &= 12 \\ -9x + 4y + 7z &= 22 \end{aligned}$$

in the form $\mathbf{Cx} = \mathbf{u}$, where $\mathbf{C}$ is a 3×3 matrix, and $\mathbf{x}$ and $\mathbf{u}$ are suitable column vectors. [1]

(iii) Use the result of part **(i)** to solve the system of equations given in part **(ii)**. [3]

3 **(i)** On a single copy of an Argand diagram, sketch the loci defined by

$$|z + 2| = 3 \quad \text{and} \quad \arg(z - \mathrm{i}) = -\tfrac{1}{4}\pi.$$ [4]

(ii) State the complex number z which corresponds to the point of intersection of these two loci. [1]

4 Let $I_n = \int_0^4 x^n \sqrt{2x + 1}\,\mathrm{d}x$ for $n \geqslant 0$. Show that, for $n \geqslant 1$,

$$(2n + 3)I_n = 27 \times 4^n - nI_{n-1}.$$ [5]

5 The curve C has equation $y = \dfrac{12(x + 1)}{(x - 2)^2}$.

(i) Determine the coordinates of any stationary points of C. [4]

(ii) Sketch C. [6]

6 Solve the first-order differential equation $x\dfrac{\mathrm{d}y}{\mathrm{d}x} + 2y = 4\ln x$ given that $y = 1$ when $x = 1$. Give your answer in the form $y = \mathrm{f}(x)$. [8]

7 Let $\mathrm{f}(n) = 11^{2n-1} + 7 \times 4^n$. Prove by induction that $\mathrm{f}(n)$ is divisible by 13 for all positive integers n. [6]

8 **(i)** Show that the line l with vector equation $\mathbf{r} = \begin{pmatrix} 2 \\ -5 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ -2 \\ 3 \end{pmatrix}$ lies in the plane Π with cartesian equation $x + 4y + z + 11 = 0$. [2]

(ii) The plane Π is horizontal, and the point $P\,(1,\ 2,\ k)$ is above it. Given that the point in Π which is directly beneath P is on the line l, determine the value of k. [6]

9 **(i)** Explain why all groups of even order must contain at least one self-inverse element (that is, an element of order 2). [2]

(ii) Prove that any group in which every non-identity element is self-inverse is abelian. [2]

(iii) Simon believes that if x and y are two distinct self-inverse elements of a group, then the element xy is also self-inverse. By considering the group of the six permutations of $(1\ \ 2\ \ 3)$, produce a counter-example to prove him wrong. [2]

(iv) A group G has order $4n + 2$, for some positive integer n, and i is the identity element of G. Let x and y be two distinct self-inverse elements of G. By considering the set $H = \{i,\ x,\ y,\ xy\}$, prove by contradiction that G cannot contain all self-inverse elements. [5]

10 **(i)** Use de Moivre's theorem to show that $2\cos 6\theta \equiv 64\cos^6\theta - 96\cos^4\theta + 36\cos^2\theta - 2$. [5]

(ii) Hence find, in exact trigonometric form, the six roots of the equation

$$x^6 - 6x^4 + 9x^2 - 3 = 0.$$ [5]

(iii) By considering the product of these six roots, determine the exact value of

$$\cos\left(\tfrac{1}{18}\pi\right)\cos\left(\tfrac{5}{18}\pi\right)\cos\left(\tfrac{7}{18}\pi\right).$$ [3]

11 A curve has polar equation $r = \mathrm{e}^{\sin\theta}$ for $-\pi < \theta \leqslant \pi$.

(i) State the polar coordinates of the point where the curve crosses the initial line. [1]

(ii) State also the polar coordinates of the points where r takes its least and greatest values. [2]

(iii) Sketch the curve. [3]

(iv) By deriving a suitable Maclaurin series up to and including the term in θ^2, find an approximation, to 3 decimal places, for the area of the region enclosed by the curve, the initial line and the line $\theta = 0.3$. [9]

12 **(i)** **(a)** Show that $\tanh x = \dfrac{e^{2x} - 1}{e^{2x} + 1}$. [2]

(b) Hence, or otherwise, show that, if $\tanh x = \dfrac{1}{k}$ for $k > 1$, then $x = \frac{1}{2}\ln\left(\dfrac{k+1}{k-1}\right)$ and find an expression in terms of k for $\sinh 2x$. [4]

(ii) A curve has equation $y = \frac{1}{2}\ln(\tanh x)$ for $\alpha \leqslant x \leqslant \beta$, where $\tanh\alpha = \frac{1}{3}$ and $\tanh\beta = \frac{1}{2}$. Find, in its simplest exact form, the arc length of this curve. [10]

13 The complex number w has modulus 1. It is given that

$$w^2 - \frac{2}{w} + k\mathrm{i} = 0,$$

where k is a positive real constant.

(i) Show that $k = \left(3 - \sqrt{3}\right)\sqrt{\frac{1}{2}\sqrt{3}}$. [8]

(ii) Prove that at least one of the remaining two roots of the equation $z^2 - \dfrac{2}{z} + k\mathrm{i} = 0$ has modulus greater than 1. [2]