

MARK SCHEME for the May/June 2015 series

9795 FURTHER MATHEMATICS

9795/01

Paper 1 (Further Pure Mathematics),
maximum raw mark 120

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

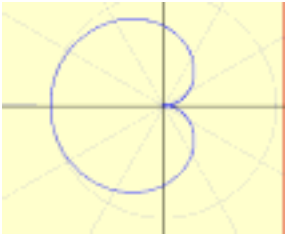
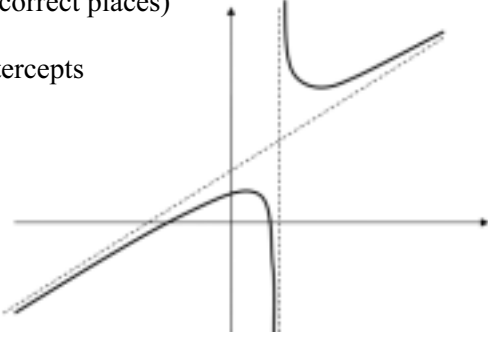
Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

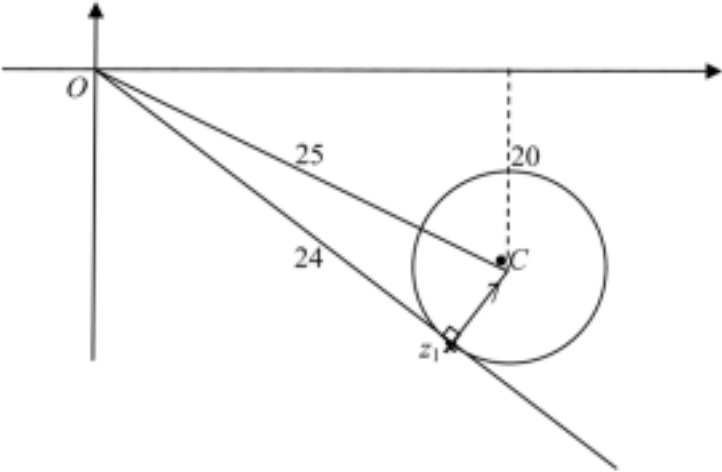
Cambridge is publishing the mark schemes for the May/June 2015 series for most Cambridge IGCSE[®], Cambridge International A and AS Level components and some Cambridge O Level components.

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1	<p>use of formula $V = \frac{1}{6} \mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$ or equivalent NB $\mathbf{b} \times \mathbf{c} = -4\mathbf{i} + 10\mathbf{j} + 2\mathbf{k}$</p> <p>attempt at relevant scalar triple product = $\begin{vmatrix} 2 & 3 & -2 \\ 2 & 0 & 4 \\ 6 & 1 & 7 \end{vmatrix} = 18$</p> <p>(or a scalar and a vector product)</p> <p>$V = 3 \text{ cso}$</p>	<p>M1</p> <p>M1</p> <p>A1</p> <p>[3]</p>
2	<p>$a_4 = \frac{y^{(4)}(1)}{4!} = \frac{(-2)^3}{1.3.5.7}$ from the Taylor series expansion</p> <p>$\Rightarrow y^{(4)}(1) = \frac{(-2)^3}{1.3.5.7} \times 4! = -\frac{64}{35}$</p> <p>ALTERNATIVE</p> <p>Diff^{te}. four times to get $\frac{d^4 y}{dx^4} = \sum_{n=4}^{\infty} \frac{(-2)^{n-1} n(n-1)(n-2)(n-3)(x-1)^{n-4}}{1.3.5\dots(2n-1)}$</p> <p>When $x = 1$, this is $\frac{(-2)^3 \times 4(3)(2)(1)}{1.3.5.7} + 0 = -\frac{64}{35}$</p>	<p>M1 A1</p> <p>A1</p> <p>M1 A1</p> <p>A1</p> <p>[3]</p>
3	<p>$n = 1, \mathbf{M} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 2(1)+1 \\ 2(1)^2 + 2(1) \\ 2(1)^2 + 2(1)+1 \end{pmatrix} \Rightarrow \text{result true for } n = 1 \text{ (both sides shown)}$</p> <p>induction hypothesis that $\mathbf{M}^k \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2k+1 \\ 2k^2 + 2k \\ 2k^2 + 2k+1 \end{pmatrix}$</p> <p>attempt at $\mathbf{M}^{k+1} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \mathbf{M} \begin{pmatrix} 2k+1 \\ 2k^2 + 2k \\ 2k^2 + 2k+1 \end{pmatrix} = \begin{pmatrix} 2k+1 - 4k^2 - 4k + 4k^2 + 4k + 2 \\ 4k+2 - 2k^2 - 2k + 4k^2 + 4k + 2 \\ 4k+2 - 4k^2 - 4k + 6k^2 + 6k + 3 \end{pmatrix}$</p> <p>$= \begin{pmatrix} 2k+3 \\ 2k^2 + 6k + 4 \\ 2k^2 + 6k + 5 \end{pmatrix}$</p> <p>$= \begin{pmatrix} 2(k+1)+1 \\ 2(k+1)^2 + 2(k+1) \\ 2(k+1)^2 + 2(k+1)+1 \end{pmatrix}$</p> <p>This is the result with k replaced by $(k+1)$. Hence IF the result is true for $n = k$ THEN it also true for $n = k+1$. Since the result is true for $n = 1$, it follows that it is true for all positive integers n.</p>	<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>E1</p> <p>[6]</p>

<p>4 (i)</p> <p>(ii)</p>	<p>Closed curve containing the pole, O Cusp at the pole Essentially all correct, including at least $(r, \theta) = (1, \pi)$</p>  <p>$A = \left(\frac{1}{2}\right) \int_0^{2\pi} \sin^2 \frac{1}{2} \theta \, d\theta$ Condone missing $\frac{1}{2}$ until final A1</p> <p>use of double-angle identity $A = \frac{1}{2} \int_0^{2\pi} \left(\frac{1}{2} - \frac{1}{2} \cos \theta\right) d\theta$</p> <p>correct integration. $= \frac{1}{4} [\theta - \sin \theta]$ $= \frac{1}{2} \pi$</p>	<p>B1 B1 B1</p> <p>[3]</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>[4]</p>
<p>5 (i)</p> <p>(ii)</p> <p>(iii)</p>	<p>VA $x = 3$</p> <p>$y = \frac{2x^2 + 5x - 25}{x - 3} = \frac{2x(x - 3) + 11(x - 3)}{x - 3}$ [+8]</p> <p>$y = 2x + 11$ oblique asymptote</p> <p>for $\frac{dy}{dx} = \frac{(x - 3)(4x + 5) - (2x^2 + 5x - 25)}{(x - 3)^2}$ (good attempt at the <i>Quotient Rule</i>)</p> <p>correct unsimplified OR $\frac{d}{dx} \left(2x + 11 + \frac{8}{x - 3} \right) = 2 - \frac{8}{(x - 3)^2}$</p> <p>solving quadratic equation $4x^2 - 7x - 15 = 2x^2 + 5x - 25$ i.e. $2x^2 - 12x + 10 = 0$ OR $(x - 3)^2 = 4$</p> <p>(1, 9) and (5, 25) (Give one A1 for both x's correct without either y)</p> <p>General shape (with asymptotes and turning points in approximately correct places)</p> <p>$2x^2 + 5x - 25 = 0$ solved to find x-intercepts $x = -5$ or $2\frac{1}{2}$</p> 	<p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 A1</p> <p>[5]</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>[3]</p>

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<p>6 (i)</p> <p>(ii)</p> <p>(iii)</p>	<p>$z^n = \cos n\theta + i \sin n\theta$ and $z^{-n} = \cos n\theta - i \sin n\theta \Rightarrow z^n + \frac{1}{z^n} \equiv 2 \cos n\theta$</p> <p>$\left(z + \frac{1}{z}\right)^5 \equiv \left(z^5 + \frac{1}{z^5}\right) + 5\left(z^3 + \frac{1}{z^3}\right) + 10\left(z + \frac{1}{z}\right)$</p> <p>Repeated use of result $\Rightarrow 32 \cos^5 \theta \equiv 2 \cos 5\theta + 10 \cos 3\theta + 20 \cos \theta$</p> <p>$\Rightarrow 16 \cos^5 \theta \equiv \cos 5\theta + 5 \cos 3\theta + 10 \cos \theta$ Answer Given</p> <p>$16 \cos^5 \theta = \pm \cos \theta$ Condone sign error here to allow for first A1</p> <p>$\cos \theta = 0 \Rightarrow \theta = \frac{1}{2}\pi$ or $\frac{3}{2}\pi$ (both)</p> <p>$\cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{1}{3}\pi$ or $\frac{5}{3}\pi$ (both)</p> <p>$\cos \theta = -\frac{1}{2} \Rightarrow \theta = \frac{2}{3}\pi$ or $\frac{4}{3}\pi$ (both)</p> <p>but allow one A1 for $\theta = \frac{1}{3}\pi$ and $\theta = \frac{2}{3}\pi$ if both 2nd answers missing</p>	<p>B1 [1]</p> <p>M1 A1</p> <p>M1</p> <p>A1 [4]</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>A1 [4]</p>
<p>7 (i)</p> <p>(ii)</p>	<p>Circle with centre (20, -15)</p> <p>Circle of radius = 7 stated or deducible from diagram</p> <p>Allow B1 for a circle entirely in the 4th Quad.</p> <p>Interior of circle shaded (Ignore boundary in/out)</p>  <p>for z_1 in correct place</p> <p>for (sufficient) distances</p> <p>for $\arg(z_1) = -\left(\tan^{-1} \frac{3}{4} + \tan^{-1} \frac{7}{24}\right)$ or equivalent, using other inverse trig. functions</p> <p>[NB – this is $-\tan^{-1} \frac{4}{3}$ using a result in Q13]</p> <p>for -0.927 (or $2\pi - 0.927 = 5.356$)</p>	<p>B1</p> <p>B1</p> <p>B1 [3]</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1 [4]</p>

8	(i)	<table border="1"> <tr> <td></td> <td><i>e</i></td> <td><i>a</i></td> <td><i>b</i></td> <td><i>c</i></td> <td><i>ab</i></td> <td><i>bc</i></td> <td><i>ca</i></td> <td><i>abc</i></td> </tr> <tr> <td><i>e</i></td> <td><i>e</i></td> <td><i>a</i></td> <td><i>b</i></td> <td><i>c</i></td> <td><i>ab</i></td> <td><i>bc</i></td> <td><i>ca</i></td> <td><i>abc</i></td> </tr> <tr> <td><i>a</i></td> <td><i>a</i></td> <td><i>e</i></td> <td><i>ab</i></td> <td><i>ca</i></td> <td><i>b</i></td> <td><i>abc</i></td> <td><i>c</i></td> <td><i>bc</i></td> </tr> <tr> <td><i>b</i></td> <td><i>b</i></td> <td><i>ab</i></td> <td><i>e</i></td> <td><i>bc</i></td> <td><i>a</i></td> <td><i>c</i></td> <td><i>abc</i></td> <td><i>ca</i></td> </tr> <tr> <td><i>c</i></td> <td><i>c</i></td> <td><i>ca</i></td> <td><i>bc</i></td> <td><i>e</i></td> <td><i>abc</i></td> <td><i>b</i></td> <td><i>a</i></td> <td><i>ab</i></td> </tr> <tr> <td><i>ab</i></td> <td><i>ab</i></td> <td><i>b</i></td> <td><i>a</i></td> <td><i>abc</i></td> <td><i>e</i></td> <td><i>ca</i></td> <td><i>bc</i></td> <td><i>c</i></td> </tr> <tr> <td><i>bc</i></td> <td><i>bc</i></td> <td><i>abc</i></td> <td><i>c</i></td> <td><i>b</i></td> <td><i>ca</i></td> <td><i>e</i></td> <td><i>ab</i></td> <td><i>a</i></td> </tr> <tr> <td><i>ca</i></td> <td><i>ca</i></td> <td><i>c</i></td> <td><i>abc</i></td> <td><i>a</i></td> <td><i>bc</i></td> <td><i>ab</i></td> <td><i>e</i></td> <td><i>b</i></td> </tr> <tr> <td><i>abc</i></td> <td><i>abc</i></td> <td><i>bc</i></td> <td><i>ca</i></td> <td><i>ab</i></td> <td><i>c</i></td> <td><i>a</i></td> <td><i>b</i></td> <td><i>e</i></td> </tr> </table>		<i>e</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>ab</i>	<i>bc</i>	<i>ca</i>	<i>abc</i>	<i>e</i>	<i>e</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>ab</i>	<i>bc</i>	<i>ca</i>	<i>abc</i>	<i>a</i>	<i>a</i>	<i>e</i>	<i>ab</i>	<i>ca</i>	<i>b</i>	<i>abc</i>	<i>c</i>	<i>bc</i>	<i>b</i>	<i>b</i>	<i>ab</i>	<i>e</i>	<i>bc</i>	<i>a</i>	<i>c</i>	<i>abc</i>	<i>ca</i>	<i>c</i>	<i>c</i>	<i>ca</i>	<i>bc</i>	<i>e</i>	<i>abc</i>	<i>b</i>	<i>a</i>	<i>ab</i>	<i>ab</i>	<i>ab</i>	<i>b</i>	<i>a</i>	<i>abc</i>	<i>e</i>	<i>ca</i>	<i>bc</i>	<i>c</i>	<i>bc</i>	<i>bc</i>	<i>abc</i>	<i>c</i>	<i>b</i>	<i>ca</i>	<i>e</i>	<i>ab</i>	<i>a</i>	<i>ca</i>	<i>ca</i>	<i>c</i>	<i>abc</i>	<i>a</i>	<i>bc</i>	<i>ab</i>	<i>e</i>	<i>b</i>	<i>abc</i>	<i>abc</i>	<i>bc</i>	<i>ca</i>	<i>ab</i>	<i>c</i>	<i>a</i>	<i>b</i>	<i>e</i>	B4, 3, 2, 1, 0
		<i>e</i>	<i>a</i>	<i>b</i>	<i>c</i>	<i>ab</i>	<i>bc</i>	<i>ca</i>	<i>abc</i>																																																																											
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	(ii)	<p>Subgroups of order 2: $\{e, a\}, \{e, b\}, \{e, c\}, \{e, ab\}, \{e, bc\}, \{e, ca\}, \{e, abc\}$ 6 distinct given all 7, no repeats</p> <p>Subgroups of order 4: $\{e, a, b, ab\}, \{e, b, c, bc\}, \{e, c, a, ca\}$ $\{e, a, bc, abc\}, \{e, b, ca, abc\}, \{e, c, ab, abc\}$ $\{e, ab, bc, ca\}$</p> <p>Either (all 3 of 1st type) + B1 (all 3 of 2nd type) + B1 (last one) Or (5 distinct) + B1 (6th) + B1 (all 7 and no repeats)</p>	B1 B1 B2																																																																																	
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9	(i)	<p>Comp. Fn. is $u = A \cos 2x + B \sin 2x$ Allow $Ae^{2ix} + Be^{-2ix}$ here For Part. Int. try $u = ax + b$ ($u' = a$ and $u'' = 0$) and substd. into given d.e. $a = 2, b = \frac{1}{4}$</p> <p>Gen. Soln. is $u = A \cos 2x + B \sin 2x + 2x + \frac{1}{4}$ ft</p> <p>Must have two arbitrary constants Condone apparently complex coefficients here Don't allow $Ae^{2ix} + Be^{-2ix}$ here</p>	M1 A1 M1 A1 B1																																																																																	
	(ii)	<p>$xy = u \Rightarrow x \frac{dy}{dx} + y = \frac{du}{dx}$ and $x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = \frac{d^2u}{dx^2}$</p> <p>$\Rightarrow (*)$ becomes $x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 4xy = 8x + 1$</p> <p>$\Rightarrow y = \frac{A \cos 2x}{x} + \frac{B \sin 2x}{x} + 2 + \frac{1}{4x}$ ft</p> <p>Accept $xy = \dots$ Must have two real arbitrary constants</p>	M1 A1 A1 B1																																																																																	
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<p>10 (i)</p>	<p>d.v. = $\begin{pmatrix} 1 \\ 7 \\ -6 \end{pmatrix} \times \begin{pmatrix} 3 \\ -5 \\ 8 \end{pmatrix} = \begin{pmatrix} 26 \\ -26 \\ -26 \end{pmatrix} \equiv \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$</p> <p>finding one point on line: e.g. (0, 8, 11), (8, 0, 3), (11, -3, 0)</p> <p>ft Line equation in vector form $\mathbf{r} = \begin{pmatrix} 8 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ (must have $\mathbf{r} = \dots, t = \dots$)</p> <p>ALTERNATIVE 1</p> <p>finding two points on line: e.g. (0, 8, 11), (1, 7, 10), (8, 0, 3), (11, -3, 0), ...</p> <p>ft d.v. $\mathbf{r} = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ (e.g.)</p> <p>ft Line equation in vector form $\mathbf{r} = \begin{pmatrix} 8 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ (must have $\mathbf{r} = \dots$)</p> <p>ALTERNATIVE 2</p> <p>Eliminating x (say): $z = y + 3$ Setting $y = \lambda$ (or equivalent) and finding z and x in terms of the parameter $z = \lambda + 3$ and $x = 8 - \lambda$</p> <p>ft Line equation in vector form $\mathbf{r} = \begin{pmatrix} 8 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ (must have $\mathbf{r} = \dots$)</p>	<p>M1 A1</p> <p>M1 A1</p> <p>B1</p> <p>[5]</p> <p>M1 A1 A1</p> <p>B1</p> <p>B1</p> <p>[5]</p> <p>M1 A1 M1 A1</p> <p>B1</p> <p>[5]</p>
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<p>(ii)</p>	$\begin{vmatrix} 1 & 7 & -6 \\ 3 & -5 & 8 \\ k & 2 & 3 \end{vmatrix} = 26k - 130$ <p>= 0 when $k = 5$ ft from a linear eqn.</p> $x + 7y - 6z = -10$ <p>e.g. $3x - 5y + 8z = 48 \rightarrow -33y + 33z = 66 \rightarrow -y + z = 2$ $5x + 2y + 3z = 16 \rightarrow -26y + 26z = 78 \rightarrow -y + z = 3$</p> <p>eliminating one variable (twice); correctly Inconsistency noted/explained (allow valid ft)</p> <p>ALTERNATIVE</p> <p>Substituting $x = 8 - \lambda$, $y = \lambda$ and $z = 3 + \lambda$ into $kx + 2y + 3z = 16$ $\Rightarrow \lambda(5 - k) + (11k - 22) = 0$ λ terms; other terms $k = 5$ gives non-unique soln. Then $33 = 0$ ft and system inconsistent (allow valid ft)</p>	<p>M1 A1</p> <p>A1</p> <p>M1 A1 A1 [5]</p> <p>M1 A1 A1 A1 A1 B1 [5]</p>
<p>11 (a)</p>	<p>Roots $\{qr, rp, pq\} = \left\{ \frac{pqr}{p}, \frac{pqr}{q}, \frac{pqr}{r} \right\}$</p> $= \left\{ \frac{4}{p}, \frac{4}{q}, \frac{4}{r} \right\}$ from $pqr = 4$, so subst. $y = \frac{4}{x}$ <p>Setting $x = \frac{4}{y} \Rightarrow \frac{64}{y^3} + 2 \cdot \frac{16}{y^2} + 3 \cdot \frac{4}{y} - 4 = 0$</p> $\Rightarrow 64 + 32y + 12y^2 - 4y^3 = 0$ $\Rightarrow y^3 - 3y^2 - 8y - 16 = 0$ <p>ALTERNATIVE (for last four-mark section)</p> $\sum \alpha' = qr + rp + pq = 3$ $\sum \alpha' \beta' = pqr(p + q + r) = 4 \times (-2) = -8$ $\sum \alpha' \beta' \gamma' = (pqr)^2 = 16$ <p>New eqn. is $x^3 - 3x^2 - 8x - 16 = 0$ (must have “= 0”)</p> <p>(b) (i) β and γ are complex conjugates (since the coefficients are real)</p> <p>(ii) $\alpha\beta\gamma = 4 \Rightarrow \beta\gamma = \frac{4}{\alpha}$ and $\beta = \gamma$ since conjugates $\Rightarrow \beta\gamma = \beta \cdot \gamma = \beta ^2 = \frac{4}{\alpha}$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>M1 A1</p> <p>A1 [6]</p> <p>B1</p> <p>B1</p> <p>B1</p> <p>B1 [6]</p> <p>B1 [1]</p> <p>M1</p>

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(iii)	and $ \beta = \frac{2}{\sqrt{\alpha}}$ (since $\alpha > 0$ given) Answer Given	A1	[2]
	$p(2.695) = -0.003... < 0$ and $p(2.705) = 0.049... > 0$ so $2.695 < \alpha < 2.705$ (by the “ <i>Change-of-Sign Rule</i> ”) $\Rightarrow \alpha = 2.70$ to 3s.f. (Allow numerical methods, e.g. <i>Newton-Raphson</i> – to find $\alpha = 2.70$ to 3s.f.)	B1	
	$\alpha + \beta + \gamma = 4 \Rightarrow \beta + \gamma = 4 - \alpha$ Then $2.695 < \alpha < 2.705 \Rightarrow 1.295 < 4 - \alpha < 1.305$	M1	
	Now, if $\beta = u + iv$, then $\gamma = u - iv$ and $\beta + \gamma = 2u$, i.e. $2 \operatorname{Re}(\beta)$ Thus $1.295 < 2 \operatorname{Re}(\beta) < 1.305 \Rightarrow 0.6475 < \operatorname{Re}(\beta) < 0.6525$	M1	
and $\operatorname{Re}(\beta) = 0.65$ to 2 s.f. (A0 for failure to justify 2s.f. accuracy properly)	A1	[4]	

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<p>12 (i)</p>	<p>(a) $I_1 = \int_0^2 x\sqrt{1+2x^2} \, dx = \left[\frac{1}{6}(1+2x^2)^{\frac{3}{2}} \right]_0^2 = \frac{13}{3}$</p> <p>(b) $I_n = \int_0^2 x^{n-1} \cdot x\sqrt{1+2x^2} \, dx$ Correct splitting and use of parts</p> $= \left[x^{n-1} \cdot \frac{1}{6}(1+2x^2)^{\frac{3}{2}} \right]_0^2 - \int_0^2 (n-1)x^{n-2} \cdot \frac{1}{6}(1+2x^2)^{\frac{3}{2}} \, dx$ $= 2^{n-1} \cdot \frac{27}{6} - 0 - \frac{1}{6}(n-1) \int_0^2 x^{n-2} \cdot (1+2x^2)\sqrt{1+2x^2} \, dx$ $= 2^{n-1} \cdot \frac{27}{6} - \frac{1}{6}(n-1)\{2I_n + I_{n-2}\}$ $\Rightarrow 6I_n = 27 \times 2^{n-1} - 2(n-1)I_n - (n-1)I_{n-2}$ $\Rightarrow (2n+4)I_n = 27 \times 2^{n-1} - (n-1)I_{n-2} \quad \text{Answer Given}$ <p>(c) $I_0 = \int_0^2 \sqrt{1+2x^2} \, dx$</p> <p>Let $x\sqrt{2} = \sinh \theta$ ($\sqrt{2} \, dx = \cosh \theta \, d\theta$, $\sqrt{1+2x^2} = \cosh \theta$)</p> <p>full substitution $I_0 = \frac{1}{\sqrt{2}} \int \cosh^2 \theta \, d\theta$ (Ignore limits for now)</p> <p>trig. identity $= \frac{1}{2\sqrt{2}} \int (1 + \cosh 2\theta) \, d\theta$</p> $= \frac{1}{2\sqrt{2}} \left[\theta + \frac{1}{2} \sinh 2\theta \right] \text{ft ln. of } a + b \cosh 2\theta \text{ only}$ <p>$(0, 2) \rightarrow (0, \sinh^{-1} 2\sqrt{2})$ Limits properly dealt with</p> $= \frac{1}{2\sqrt{2}} \left[\sinh^{-1} 2\sqrt{2} + 6\sqrt{2} \right]$ <p>$\sinh^{-1} 2\sqrt{2} = \ln(3 + 2\sqrt{2})$ to at least here $= \ln(1 + \sqrt{2})^2 = 2 \ln(1 + \sqrt{2})$</p> <p>$I_0 = 3 + \frac{1}{\sqrt{2}} \ln(1 + \sqrt{2})$ legitimately Answer Given</p>	<p>M1 A1 A1 [3]</p> <p>M1 A1 A1 M1 A1 A1 [6]</p> <p>M1 M1 A1 M1 A1 M1 M1 A1 A1 [8]</p>
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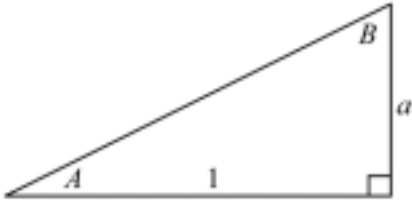
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	<p>ALTERNATIVE (partial)</p> <p>Let $x\sqrt{2} = \tan \theta$ ($\sqrt{2}dx = \sec^2 \theta d\theta$)</p> <p>full substitution $I_0 = \frac{1}{\sqrt{2}} \int \sec^3 \theta d\theta$ (Ignore limits for now)</p> <p>use of \int by parts on $\int \sec \theta \sec^2 \theta d\theta$ and use of $\tan^2 \theta = \sec^2 \theta - 1$</p> $2\sqrt{2}I_0 = \sec \theta \tan \theta + \ln \sec \theta + \tan \theta $ <p>Further progress (limits, etc.) essentially impossible</p> <p>ALTERNATIVE</p> $I_0 = \int_0^2 \sqrt{1+2x^2} \times 1 dx = x\sqrt{1+2x^2} - \int \frac{1}{2}(1+2x^2)^{-\frac{1}{2}} \cdot 4x \cdot x dx \quad \int \text{ by parts}$ $= x\sqrt{1+2x^2} - \int \frac{(2x^2+1)-1}{\sqrt{1+2x^2}} dx$ $= x\sqrt{1+2x^2} - I_0 + \int \frac{1}{\sqrt{1+2x^2}} dx$ $\Rightarrow 2I_0 = x\sqrt{1+2x^2} + \int \frac{1}{\sqrt{1+2x^2}} dx$ $\Rightarrow I_0 = \frac{1}{2} \left(x\sqrt{1+2x^2} + \frac{1}{\sqrt{2}} \sinh^{-1}(x\sqrt{2}) \right) \quad (\text{from MF20})$ <p>Use of limits (0, 2) $\Rightarrow I_0 = 3 + \frac{1}{2\sqrt{2}} \sinh^{-1}(2\sqrt{2})$</p> $\sinh^{-1} 2\sqrt{2} = \ln(3+2\sqrt{2}) = \ln(1+\sqrt{2})^2 = 2 \ln(1+\sqrt{2})$ <p>$I_0 = 3 + \frac{1}{\sqrt{2}} \ln(1+\sqrt{2})$ legitimately Answer Given</p>	M1 M1 A1 M1 A1 M1 M1 M1 A1 M1 M1 M1 A1 M1 A1
(ii)	$y = \frac{1}{\sqrt{2}}x^2 \Rightarrow \frac{dy}{dx} = x\sqrt{2} \quad \text{and} \quad S = 2\pi \int_0^2 \frac{x^2}{\sqrt{2}} \cdot \sqrt{1+2x^2} dx$ $= \pi\sqrt{2} (I_2)$ <p>use of R.F. for $n = 2$: $8 I_2 = 54 - I_0$</p> <p>use of <i>their</i> (i) result $= 51 - \frac{1}{\sqrt{2}} \ln(1+\sqrt{2})$</p> $\Rightarrow S = \frac{\pi\sqrt{2}}{8} \left(51 - \frac{1}{\sqrt{2}} \ln(1+\sqrt{2}) \right) \quad \text{or} \quad \frac{\pi}{8} (51\sqrt{2} - \ln(1+\sqrt{2}))$	M1 A1 M1 M1 A1

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<p>13 (i)</p>	 $\tan A = \frac{a}{1}, \tan B = \frac{1}{a}$ $\Rightarrow \tan^{-1} a + \tan^{-1} \frac{1}{a} = A + B = \frac{\pi}{2}$	<p>B1</p> <p>[1]</p>
<p>(ii)</p>	$\tan(\tan^{-1} a \pm \tan^{-1} b) = \frac{\tan(\tan^{-1} a) \pm \tan(\tan^{-1} b)}{1 \mp \tan(\tan^{-1} a) \tan(\tan^{-1} b)} = \frac{a \pm b}{1 \mp ab}$	<p>M1 A1</p> <p>[2]</p>
<p>(iii)</p>	$\tan^{-1}\left(\frac{1}{n-1}\right) - \tan^{-1}\left(\frac{1}{n+1}\right) = \tan^{-1}\left(\frac{2}{n^2}\right) \text{ noted at any stage}$ $\sum_{n=1}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right) = \tan^{-1}(2) + \tan^{-1}\left(\frac{1}{2}\right) + \sum_{n=3}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right) \text{ Splitting off 1st 2 terms}$ $= \frac{\pi}{2} + \sum_{n=3}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right) \text{ using (i)'s result}$ <p>use of difference method (finite or infinite series)</p> $\Rightarrow \sum_{n=1}^{\infty} \tan^{-1}\left(\frac{2}{n^2}\right) = \frac{\pi}{2} +$ $\left(\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{4} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{6} + \tan^{-1} \frac{1}{7} \dots\right)$ $-\left(\tan^{-1} \frac{1}{4} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{6} + \tan^{-1} \frac{1}{7} + \dots\right)$ <p>Cancelling of terms made clear</p> $= \frac{\pi}{2} + \left(\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3}\right)$ <p>and $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = \tan^{-1}\left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}}\right) = \tan^{-1} 1 = \frac{\pi}{4}$ using (ii)'s result</p> <p>leading to ... Answer Given</p>	<p>M1 A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>[7]</p>

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	<p>ALTERNATIVE</p> <p>$\tan^{-1}\left(\frac{1}{n-1}\right) - \tan^{-1}\left(\frac{1}{n+1}\right) = \tan^{-1}\left(\frac{2}{n^2}\right)$ noted at any stage</p> <p>Use of difference method (finite or infinite series)</p> $\sum_{n=1}^N \tan^{-1}\left(\frac{2}{n^2}\right) = \left(\tan^{-1}\frac{1}{0} + \tan^{-1}1 + \tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} + \dots\right)$ $- \left(\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} + \dots\right)$ <p>Cancelling of remaining terms made clear</p> $\tan^{-1}\infty + \tan^{-1}1 = \frac{\pi}{2} + \frac{\pi}{4}$ $= \frac{3\pi}{4} \quad \text{Given Answer}$	<p>M1 A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>
		[7]