CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Pre-U Certificate

MARK SCHEME for the May/June 2015 series

9795 FURTHER MATHEMATICS

9795/02 Paper 2 (Further Application of Mathematics), maximum raw mark 120

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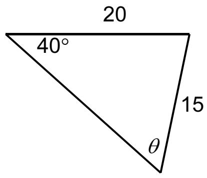
CAMBRIDGE
International Examinations

1	**(i)**	$2\mu - 50 = 0 \Rightarrow \mu = \mathbf{25}$	M1A1	
		$44 + 25\sigma^2 = 144 \Rightarrow \sigma^2 = \mathbf{4}$	M1A1 [4]	$44 + 5\sigma^2$ or $22 + 5\sigma^2$ $\to$ 20 or 24.4: M1A0
	(ii)	$X - Y \sim \mathrm{N}(15, 15)$	M1	$\mathrm{N}(\mu - 10,\ 11 + \sigma^2)$
		$z = \frac{10-15}{\sqrt{15}} = -1.291$	M1 A1	Standardise, including $\sqrt{\sigma^2}$ Allow +1.29(1)
		$\mathrm{P}(X - Y > 10) = \mathbf{0.902}$	A1 [4]	[N(15, 31) $\to$ 0.898 $\to$ 0.815: M1M1A0A0]
2	**(i)**	$\bar{x} = \frac{114}{20} = 5.7$, $s^2 = \frac{2.382}{19} = 0.1254$	B1B1	
		$t_{19} = 2.5395$	B1	
		98% c.l.: $5.7 \pm \left(2.5395 \times \frac{\sqrt{0.1254}}{\sqrt{20}}\right)$	M1	Normal: B2B0M1A0
		98% C.I. is **(5.499, 5.901)**	A1A1 [6]	
	(ii)	5.5 is (just) within the confidence interval. Some evidence to suggest that the average pH is 5.5 in villages where rhododendrons grow well.	B1FT B1FT [2]	Relate to CI Conclusion FT on their confidence interval
3	**(i)**	$\mathrm{G}'(t) = \frac{1}{81} \times 4\left(t + \frac{2}{t}\right)^3\left(1 - \frac{2}{t^2}\right)$	M1	For information: $G(t) = \frac{1}{81}\left(t^4 + 8t^2 + 24 + \frac{32}{t^2} + \frac{16}{t^4}\right)$
		$\mathrm{E}(X) = \mathrm{G}'(1) = \frac{4}{81} \times 27 \times (-1) = -\frac{4}{3}$	A1	$G'(t) = \frac{1}{81}\left(4t^3 + 16t - \frac{64}{t^3} - \frac{64}{t^5}\right)$
		$\mathrm{G}''(t) = \frac{4}{27}\left(t + \frac{2}{t}\right)^2\left(1 - \frac{2}{t^2}\right)^2 + \frac{4}{81}\left(t + \frac{2}{t}\right)^3 \times \frac{4}{t^3}$	M1	$G''(t) = \frac{1}{81}\left(12t^2 + 16 + \frac{192}{t^4} + \frac{320}{t^6}\right)$
		$\mathrm{G}''(1) = \frac{4}{27} \times 9 \times 1 + \frac{4}{81} \times 27 \times 4 = \frac{20}{3}$		
		$\mathrm{Var}(X) = \frac{20}{3} + \left(-\frac{4}{3}\right) - \frac{16}{9} = \mathbf{\frac{32}{9}}$	M1A1 [5]	
	(ii)	x: –4, –2, 0, 2, 4	B1	x and probabilities
		$y = \frac{1}{2}(x+4)$: 0, 1, 2, 3, 4	B1	y
		$\mathrm{P}(X = x) = \mathrm{P}(Y = y)$: $\frac{16}{81}$, $\frac{32}{81}$, $\frac{24}{81}$, $\frac{8}{81}$, $\frac{1}{81}$		Or: $G_{X+4}(t) = t^4 \mathrm{G}_X(t)$ $G_{\frac{1}{2}(X+4)}(t) = G_{X+4}\left(\sqrt{t}\right)$
		Recognising as terms of the expansion of $\left(\frac{2}{3} + \frac{1}{3}\right)^4$		
		State $\boldsymbol{n} = \mathbf{4}$ and $\boldsymbol{p} = \mathbf{\frac{1}{3}}$	B1B1 [4]	Independent of method BUT max 3 if binomial not shown

4 (i) (a)	$\mathrm{M}(t)=\sum\frac{\lambda^r}{r!}\mathrm{e}^{-\lambda}.\mathrm{e}^{tr}=\mathrm{e}^{-\lambda}\sum\frac{(\lambda\mathrm{e}^t)^r}{r!}$	M1	Allow via expansion
	$=\mathrm{e}^{-\lambda}.\mathrm{e}^{\lambda\mathrm{e}^t}=\mathrm{e}^{\lambda(\mathrm{e}^t-1)}$	M1A1 [3]	Needs to recognise series for M1 PGF quoted: M0M0A0
(b)	$\mathrm{M}_Z(t)=\mathrm{M}_X(t).\mathrm{M}_Y(t)=\mathrm{e}^{\mu(\mathrm{e}^t-1)}.\mathrm{e}^{\nu(\mathrm{e}^t-1)}$	M1	Multiply two MGFs
	$=\mathrm{e}^{(\mu+\nu)(\mathrm{e}^t-1)}\Rightarrow Z\sim\mathrm{Po}(\mu+\nu)$	A1 [2]	Needs one intermediate step
(ii) (a)	From tables $2+k=3.1\Rightarrow k=1.101=$ **1.10** (3sf)	M1A1 [2]	Or $\mathrm{e}^{-(2+k)}=0.045$ $\Rightarrow 2+k=\ln 22.22$ $\Rightarrow k=1.101=1.10$ (3sf)
(b)	$\mathrm{P}(3)=\mathrm{e}^{-1.1}\times\frac{1.1^3}{3!}=$ **0.0740**	M1 A1 [2]	Or $\mathrm{P}(\le 3)-\mathrm{P}(\le 2)=0.9743-0.9004$ Answer in range [0.0738, 0.0740] $\lambda=k$ needed for M1
(c)	Using mean of 3.1: $\mathrm{P}(\le 5)-\mathrm{P}(\le 1)$ $=0.9057-0.1847=0.7210$ $=$ **0.721** (3 sf)	M1 A1 [2]	Or from series ±1 term
5	$\frac{60.5-\mu}{\sigma}=2.083$, $\frac{39.5-\mu}{\sigma}=1.417$	B1M1 A1A1	z correct to 3 sf,
	$\boldsymbol{\mu=48.0,\ \sigma=6}$	M1A1	Allow 3 sf, can be implied
	$\mu=np$, $\sigma=\sqrt{npq}$	B1B1	
	$1-p=36\div 48=\frac{3}{4}$	M1	
	$\Rightarrow$ $\boldsymbol{p=0.25,\ n=192}$	A1 A1 [11]	$p\in[0.248, 0.250]$ 192 or 193, must be integer
6 (i)	$\mathrm{f}_u(u)=\frac{1}{40}$ $U=\mathrm{g}^{-1}(V)=\frac{40V}{V-40}$	B1 M1	PDF of u U in terms of V
	$\mathrm{f}_v(v)=\mathrm{f}_u(\mathrm{g}^{-1}(v))\times\lvert\mathrm{g}^{-1\prime}(v)\rvert$	M1A1	Formula; mod sign needed for A1
	$=\frac{1}{40}\times\left\lvert\frac{-1600}{(v-40)^2}\right\rvert=\frac{40}{(v-40)^2}$ (AG)	A1 [5]	Mod sign needed for A1
	Or: $\mathrm{F}_u(u)=\frac{u-80}{40}$; $U=\frac{40V}{V-40}$	B1 M1	CDF of U U in terms of V
	$\mathrm{P}(V<v)=\mathrm{P}\left(U>\frac{40V}{V-40}\right)=1-\mathrm{P}\left(U<\frac{40V}{V-40}\right)$	M1	Turn $\mathrm{F}_U(u)$ into $\mathrm{F}_V(v)$, allow no 1–
	$=2-\frac{40}{v-40}$ so $\mathrm{f}_v(v)=\frac{40}{(v-40)^2}$ (AG)	A1A1 [5]	Correct $\mathrm{F}_V(v)$; correctly obtain AG

(ii) (a)	$F(v) = 2 - \frac{40}{(v-40)}$ $60 \le v \le 80$	M1	(Or by integration of pdf from 60 to median = 0.5; M1 needs limits or c) Allow from 1 – F(v)
	$2 - \frac{40}{v-40} = \frac{1}{2} \Rightarrow v = \mathbf{\frac{200}{3}}$ (OE)	A1 [2]	
(b)	$E(V) = \int_{60}^{80} \frac{40v}{(v-40)^2}\,dv = \int_{60}^{80} \frac{40}{v-40} + \frac{1600}{(v-40)^2}\,dv$	M1 M1A1	Or by $x = v - 40$: $40\left[\ln x - \frac{40}{x}\right]_{20}^{40}$
	$= \left[40\ln\lvert v-40\rvert - \frac{1600}{v-40}\right]_{60}^{80}$	M1A1	
	$=$ **40 ln 2 + 40** (= 67.7)	A1 [6]	
7 (i)	$x^2 + 9a^2 = (4a-x)^2 \Rightarrow \ldots \Rightarrow x = \frac{7}{8}a$ (AG)	M1A1 [2]	
(ii)	$T\cos\theta = mg$	B1	
	$T + T\sin\theta = \frac{mv^2}{r}$	M1*A1	M1 needs two forces
	$\cos\theta = \frac{24}{25}$, $\sin\theta = \frac{7}{25}$ or $\tan\theta = \frac{7}{24}$	B1	One correct, may be implied
	Solve to obtain $v = \sqrt{\frac{7ag}{6}}$	dep*M1 A1 [6]	Solve
8 (i)	Hooke's law: $T = \frac{8x}{0.4} = 20x$	B1	Can be specific x
	Newton II: $\frac{1}{5}\ddot{x} = -20x$	M1	Needs general x, – sign
	$\Rightarrow \ddot{x} = -100x$ which is SHM	A1	A.e. exact f.
	Period = $\frac{2\pi}{10}$ $= \frac{1}{5}\pi$ seconds	A1 [4]	
(ii)	$-0.1 = 0.2\cos 10t$	M1A1	Or: Obtain $\frac{1}{60}\pi$
	$\Rightarrow 10t = \frac{2\pi}{3}$ $\Rightarrow t = \frac{1}{15}\pi$ seconds	M1A1 [4]	Add quarter period to get $\frac{1}{15}\pi$

9	**(i)**	Gain in KE = $\frac{1}{2}\times 800\times\left(25^2-10^2\right)=$ 210 000 J Loss in PE = $800\times 10\times 400\sin 2^\circ$ = 111 678 J Work done = 210 000 – 111 768 = **98 322** J (= 98.3 kJ)	M1 A1 A1 [3]	$\frac{1}{2}mv^2-\frac{1}{2}mu^2-mg\times 400\sin 2^\circ$ Sign wrong: M1A0
	(ii)	$a=\frac{25^2-10^2}{800}=0.65625$ or $\frac{21}{32}$ $v^2 = 10^2 + 400 \times 0.65625$ $\Rightarrow v = 19.0394$ $F + 800 \times 10 \times \sin 2° = 800 \times 0.65625$ $\Rightarrow F = 245.8$ $P = Fv = 4680$ $\Rightarrow$ Power is **4.68 kW**.	M1A1 A1 M1A1 A1 [6]	Allow in part (i) only if used in part (ii) 3 terms needed for M1
10	**(i)**	$-\left(mg+mkv^2\right)=m\frac{\mathrm{d}v}{\mathrm{d}t}$ $-\int_0^T \mathrm{d}t=\int_u^0\frac{1}{g+kv^2}\mathrm{d}v$ $=\frac{1}{k}\int_u^0\frac{1}{\frac{g}{k}+v^2}\mathrm{d}v$ $T=\frac{1}{k}\left[\sqrt{\frac{k}{g}}\tan^{-1}\sqrt{\frac{k}{g}}v\right]_0^u$ $=\frac{1}{\sqrt{gk}}\tan^{-1}\sqrt{\frac{k}{g}}u$	B1 M1 M1 A1 A1 [5]	Allow + only if ↓ explicit Separate and insert integral signs Or indefinite integral and find *c* Correct indefinite integral
	(ii)	$-\left(mg+mkv^2\right)=mv\frac{\mathrm{d}v}{\mathrm{d}x}$ $-\int_0^H \mathrm{d}x=\frac{1}{2k}\int_u^0\frac{2kv}{g+kv^2}\mathrm{d}v$ $H=\frac{1}{2k}\left[\ln\left\|g+kv^2\right\|\right]_0^u$ $=\frac{1}{2k}\ln\left(\frac{g+ku^2}{g}\right)$	B1 M1 M1 A1 A1 [5]	Allow + only if ↓ explicit Or indefinite integral and find *c* Correct indefinite integral

11 (i)	Right angled triangle with angle $\tan^{-1}\frac{4}{3}-50°=3.13°$	B1	$(20t-5\sin 50)^2+(5\cos 50-15t)^2$ $\Rightarrow t=0.1997$
	Shortest distance = 5 sin 3.13° = **0.273 km**	M1A1 [3]	
(ii) (a)	A correct velocity triangle. 	B1	
	$\frac{\sin\theta}{20}=\frac{\sin 40°}{15}\Rightarrow\theta=58.986\ldots°$	M1A1	
	Bearing is **008.99°**	A1 [4]	(Accept 9° or 8.99°)
(b)	$\frac{v}{\sin 81.013\ldots°}=\frac{15}{\sin 40°}\Rightarrow v=23.049$	M1A1	
	Time $=\frac{5}{23.049}\times 60=$ **13.0** minutes.	M1A1 [4]	(Accept 13 minutes.)
12 (i)	Taking axes along and perpendicular to plane:		
	$\dot{x}=u\cos\theta-gt\sin\alpha$; $\dot{x}=0\Rightarrow t=\frac{u\cos\theta}{g\sin\alpha}$	M1 A1	Equation for $\dot{x}$, equated to 0 Find (eliminate) t
	$y=ut\sin\theta-\frac{1}{2}gt^2\cos\alpha$; $y=0\Rightarrow t=\frac{2u\sin\theta}{g\cos\alpha}$	M1 A1	Equation for y, equated to 0 Find (eliminate) t
	$\frac{u\cos\theta}{g\sin\alpha}=\frac{2u\sin\theta}{g\cos\alpha}\Rightarrow 2\tan\alpha\tan\theta=1$ (AG)	M1A1 [6]	Equate and simplify; get AG
(ii) (a)	$x=u\cos\theta.\frac{u\cos\theta}{g\sin\alpha}-\frac{1}{2}g\sin\alpha.\frac{u^2\cos^2\theta}{g^2\sin^2\alpha}$	M1	Equation for x, substitute t
	$\Rightarrow x\sin\alpha=\frac{u^2}{2g}\cos^2\theta$	A1	Correct expression for $x\sin\alpha$
	But $\cos^2\theta=\frac{1}{1+\tan^2\theta}=\frac{1}{1+\frac{1}{4\tan^2\alpha}}=\frac{4\tan^2\alpha}{1+4\tan^2\alpha}$	M1A1	$\cos\theta$ in terms of $\tan\alpha$
	$\Rightarrow x\sin\alpha=\frac{2u^2\tan^2\alpha}{g(1+4\tan^2\alpha)}$ (AG)	A1 [5]	Obtain given answer
(b)	Time of flight $=\frac{u\cos\theta}{g\sin\alpha}=\frac{u}{g\sin\alpha}\sqrt{\frac{4\tan^2\alpha}{1+4\tan^2\alpha}}$	M1	Find t in terms of $\tan\alpha$
	$=\frac{2u}{g\cos\alpha\sqrt{1+4\tan^2\alpha}}=\frac{2u\sec\alpha}{g\sqrt{1+4\tan^2\alpha}}$ (AG) (N.B. Other orders of doing this may be seen.)	M1A1 [3]	Simplify to given answer

Alternative method for 12:

12 (i)	Taking axes horizontally and vertically, where $\beta = \alpha + \theta$		
	$y = x\tan\beta - \frac{gx^2}{2u^2}\left(1+\tan^2\beta\right)$ and $y = \tan\alpha$ (on landing)	M1 A1	
	$\Rightarrow \tan\alpha = \tan\beta - \frac{gx}{2u^2}\left(1+\tan^2\beta\right)$ (I)	M1A1	
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)\quad \tan\beta - \frac{gx}{u^2}\left(1+\tan^2\beta\right) = \frac{-1}{\tan\alpha}$ (II)		
	(I) & (II) $\Rightarrow \tan\beta + \frac{1}{\tan\alpha} = 2\left(\tan\beta - \tan\alpha\right)$	M1	$\left(= \frac{gx}{u^2}\left(1+\tan^2\beta\right)\right)$
	$\Rightarrow \tan\beta = 2\tan\alpha + \frac{1}{\tan\alpha}$ (III)		
	$\Rightarrow \frac{\tan\alpha+\tan\theta}{1-\tan\alpha\tan\theta} = \frac{2\tan^2\alpha+1}{\tan\alpha} \Rightarrow \ldots$ $\Rightarrow 2\tan\alpha\tan\theta = 1$ (AG)	A1	
		[6]	
(ii) (a)	(I) & (III) $\Rightarrow \frac{gx^2}{2u^2}\left(1+\tan^2\beta\right) = \frac{2\tan^2\alpha+1}{\tan\alpha} - \tan\alpha$ $= \frac{\tan^2\alpha+1}{\tan\alpha}$	M1A1	
	$\Rightarrow x\tan\alpha = \frac{2u^2}{g}\left\{\frac{1+\tan^2\alpha}{1+\tan^2\beta}\right\} = \frac{2u^2}{g}\left\{\frac{1+t^2}{1+\left(\frac{2t^2+1}{t}\right)^2}\right\}$,	M1A1	
	where $t = \tan\alpha$. (Required height is $x\tan\alpha$.)		
	$\Rightarrow \ldots \Rightarrow x\tan\alpha = \frac{2u^2\tan^2\alpha}{g\left(1+4\tan^2\alpha\right)}$ (AG)	A1	
		[5]	
(b)	$T = \frac{x}{u\cos\beta} = \frac{2u^2t^2}{g\left(1+4t^2\right)}.\frac{1}{t}.\frac{1}{u\cos\beta}$	M1	
	$\Rightarrow T^2 = \frac{4u^2t^2\left(1+\tan^2\beta\right)}{g^2\left(1+4t^2\right)^2} = \frac{4u^2t^2\left(1+\left[2t+\frac{1}{t}\right]^2\right)}{g^2\left(1+4t^2\right)^2} = \ldots$	M1	
	$= \frac{4u^2}{g^2}\left(\frac{1+t^2}{1+4t^2}\right) \Rightarrow T = \frac{2u\sec\alpha}{g\sqrt{1+4\tan^2\alpha}}$ (AG)	A1	
		[3]	