**Cambridge International Examinations**
Cambridge Pre-U Certificate

**FURTHER MATHEMATICS (PRINCIPAL)** **9795/02**

Paper 2 Further Applications of Mathematics **May/June 2015**

**3 hours**

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

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**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

The syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

This document consists of **5** printed pages and **3** blank pages.

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## Section A: Probability (60 marks)

1 The independent random variables $X$ and $Y$ are such that

$$X \sim \mathrm{N}(\mu, 11), \quad Y \sim \mathrm{N}(10, \sigma^2) \quad \text{and} \quad 2X - 5Y \sim \mathrm{N}(0, 144).$$

Find

**(i)** the values of $\mu$ and $\sigma^2$, [4]

**(ii)** $\mathrm{P}(X - Y > 10)$. [4]

2 The pH value, $X$, which is a measure of acidity, was measured for soil taken from a random sample of 20 villages in which rhododendrons grow well. The results are summarised below, where $\bar{x}$ denotes the sample mean. You may assume that the sample is selected from a normal population.

$$\Sigma x = 114 \qquad \Sigma(x - \bar{x})^2 = 2.382$$

**(i)** Calculate a 98% confidence interval for the mean pH value in villages where rhododendrons grow well, giving 3 decimal places in your answer. [6]

**(ii)** Comment, justifying your answer, on a suggestion that the average pH value in villages where rhododendrons grow well is 5.5. [2]

3 The probability generating function of the random variable $X$ is $\frac{1}{81}\left(t + \frac{2}{t}\right)^4$.

**(i)** Use the probability generating function to find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [5]

**(ii)** The random variable $Y$ is defined by $Y = \frac{1}{2}(X + 4)$. By finding the probability distribution of $X$, or otherwise, show that $Y \sim \mathrm{B}(n, p)$, stating the values of $n$ and $p$. [4]

4 **(i)** **(a)** Derive the moment generating function for a Poisson distribution with mean $\lambda$. [3]

**(b)** The independent random variables $X$ and $Y$ are such that $X \sim \mathrm{Po}(\mu)$ and $Y \sim \mathrm{Po}(\nu)$. Use moment generating functions to show that $(X + Y) \sim \mathrm{Po}(\mu + \nu)$. [2]

**(ii)** The number of goals scored per match by Camford Academicals FC may be modelled by a Poisson distribution with mean 2. The number of goals scored against Camford during a match may be modelled by an independent Poisson distribution with mean $k$. The probability that no goals are scored, **by either side**, in a match involving Camford is 0.045. Find

**(a)** the value of $k$, [2]

**(b)** the probability that exactly 3 goals are scored against Camford in a match, [2]

**(c)** the probability that the total number of goals scored, in a match involving Camford, is between 2 and 5 inclusive. [2]

5 Each year a college has a large fixed number, $n$, of places to fill. The probability, $p$, that a randomly chosen student comes from abroad is constant. Using a suitable normal approximation and applying a continuity correction, it is calculated that the probability of more than 60 students coming from abroad is 0.0187 and the probability of fewer than 40 students coming from abroad is 0.0783. Find the values of $n$ and $p$. [11]

**6** The object distance, $U$ cm, and the image distance, $V$ cm, for a convex lens of focal length 40 cm are related by the lens law

$$\frac{1}{U} + \frac{1}{V} = \frac{1}{40}.$$

The random variable $U$ is uniformly distributed over the interval $80 \leqslant u \leqslant 120$.

**(i)** Show that the probability density function of $V$ is given by

$$f(v) = \begin{cases} \dfrac{40}{(v-40)^2} & 60 \leqslant v \leqslant 80, \\ 0 & \text{otherwise.} \end{cases}$$ [5]

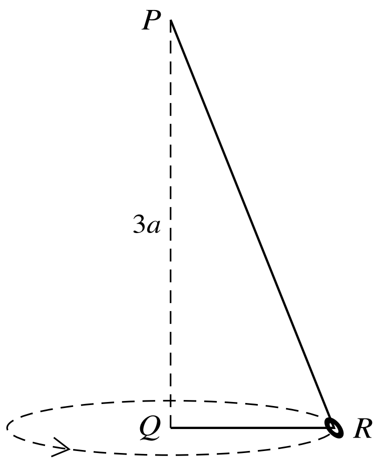
**(ii)** Find

**(a)** the median value of $V$, [2]

**(b)** the expected value of $V$. [6]

**Section B: Mechanics (60 marks)**

**7**

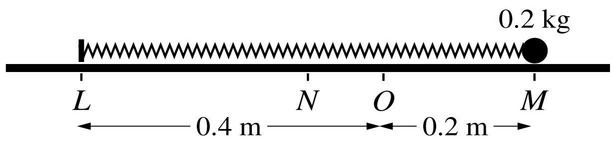



A light inextensible string of length $4a$ has one end fixed at a point $P$ and the other end fixed at a point $Q$, which is vertically below $P$ and at a distance $3a$ from $P$. A small smooth ring $R$ of mass $m$ is threaded on the string. $R$ moves in a horizontal circle with centre $Q$ and with the string taut (see diagram).

**(i)** Show that $QR = \frac{7}{8}a$. [2]

**(ii)** Find the speed of $R$ in terms of $a$ and $g$. [6]

**8**




A light spring of modulus of elasticity 8 N and natural length 0.4 m has one end fixed to a smooth horizontal table at a fixed point $L$. A particle of mass 0.2 kg is attached to the other end of the spring and pulled out horizontally to a point $M$ on the table, so that the spring is extended by 0.2 m. The particle is then released from rest. The mid-point of $LM$ is $N$ and the point $O$ is on $LM$ such that $LO = 0.4$ m (see diagram).

**(i)** Show that the particle moves in simple harmonic motion with centre $O$ and state the exact period of its motion. [4]

**(ii)** Find the exact time taken for the particle to move directly from $M$ to $N$. [4]

**9** A car of mass 800 kg is descending a straight hill which is inclined at 2° to the horizontal. The car passes through the points $A$ and $B$ with speeds $10\,\text{m s}^{-1}$ and $25\,\text{m s}^{-1}$ respectively. The distance $AB$ is 400 m.

**(i)** Assuming that resistances to motion are negligible, calculate the work done by the car's engine over the distance from $A$ to $B$. [3]

**(ii)** Assuming also that the driving force produced by the car's engine remains constant, calculate the power of the car's engine at the mid-point of $AB$. [6]

**10** A small body of mass $m$ is thrown vertically upwards with initial velocity $u$. Resistance to motion is $kv^2$ per unit mass, where the velocity is $v$ and $k$ is a positive constant. Find, in terms of $u$, $g$ and $k$,

**(i)** the time taken to reach the greatest height, [5]

**(ii)** the greatest height to which the body will rise. [5]

**11** In a training exercise, a submarine is travelling due north at $15\,\text{km h}^{-1}$. The submarine commander sees his target 5 km away on a bearing of 310°. The target is travelling due east at $20\,\text{km h}^{-1}$.

**(i)** If each of the submarine and target maintains its present course and speed, find the shortest distance between them. [3]

**(ii)** In fact, as soon as he sees the target, the submarine commander changes course, without changing speed, so as to intercept the target as quickly as possible. Find

**(a)** the course, in degrees, set by the submarine commander,

**(b)** the time taken, in minutes, to intercept the target from the moment that the course changes. [8]

**12** Points $A$ and $B$ lie on a line of greatest slope of a plane inclined at an angle $\alpha$ to the horizontal, with $B$ above $A$. A particle is projected from $A$ with speed $u$ at an angle $\theta$ to the plane and subsequently strikes the plane at right angles at $B$.

**(i)** Show that $2\tan\alpha\tan\theta = 1$. [6]

**(ii)** In either order, show that

**(a)** the vertical height of $B$ above $A$ is $\dfrac{2u^2\tan^2\alpha}{g(1+4\tan^2\alpha)}$,

**(b)** the time of flight from $A$ to $B$ is $\dfrac{2u\sec\alpha}{g\sqrt{1+4\tan^2\alpha}}$. [8]

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