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FURTHER MATHEMATICS **9795/02**

Paper 2 Further Applications of Mathematics **May/June 2016**

MARK SCHEME

Maximum Mark: 120

Published

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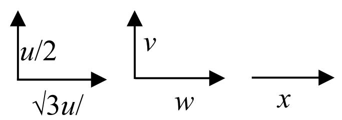
1	(i)	$75 \pm 1.96\sqrt{\frac{40^2}{500} \times \frac{500}{499}}$ $=$ **(71.5, 78.5)**	M1 B1 A1 A1 **[4]**	$75 \pm zs$, s involving 500 $z = 1.96$, allow from no 500 Variance correct Both limits correct to 3sf Condone omission of $\frac{500}{499}$
	(ii)	No, as the Central Limit Theorem applies *OR* as n is large	B1 **[1]**	"No" and mention CLT or large sample size; focus on different distributions; no irrelevancies
2	(i)	N(120, … $\sigma^2 = 0.8^2 \times 1200$ [= 768] $1 - \Phi((140 - 120)/\sqrt{768})$ $=$ **0.235**(3)	M1 M1 A1 A1 **[4]**	Normal, mean 120 or 1.20 Allow 0.8×1200 etc Both parameters correct Answer, in range [0.235, 0.236] OR: P(≥ 175) from N(150, 200) M1 $(175 - 150)/\sqrt{1200}$ A1 0.235(3) A2
	(ii)	$B_1 + B_2 + B_3 + B_4 - S_1 - S_2 - S_3$ $\sim$ N(60, …) Variance 4×1200 + 3×1500 = 9300 $\Phi\left(\frac{0-60}{\sqrt{9300}}\right) = \Phi(-0.622) =$ **0.267**	M1 M1 A1 A1 **[4]**	Consider $\pm(B_1 + B_2 + B_3 + B_4 - S_1 - S_2 - S_3)$ or $4B - 3S$ Normal, mean 60 Correct variance Answer, a.r.t. 0.267 [0.2699] [NB: $\Phi(-60/\sqrt{33700}) = 0.3700$ is 2/4]
3	(i)	$\sum t^r \mathrm{P}(r) = \sum_{r=0}^{n} t^r\, {}^nC_r p^r (1-p)^{n-r}$ $= \sum_{r=0}^{n} (pt)^r\, {}^nC_r (1-p)^{n-r}$ $= (1 - p + pt)^n$ **AG**	M1 A1 A1 **[3]**	Use $\Sigma t^r \mathrm{P}(R = r)$ and binomial probabilities Indicate correct final term Collect p^r and t^r and correctly obtain given expression *OR* $(1 - p + pt)^1$, M1A1; answer, A1
	(ii)	$\left(\frac{3}{4} + \frac{1}{4}t\right)^8 \left(\frac{1}{4} + \frac{3}{4}t\right)^8$ $= (3 + t)^8(1 + 3t)^8/4^{16}$ $= \left(\frac{3}{16} + \frac{1}{16}t(10 + 3t)\right)^8$ **AG** $= \left(\frac{3}{16}\right)^8 (1 + \left[\frac{10}{3} + t\right]t)^8$ t term: $\left(\frac{3}{16}\right)^8 \left[8 \times \frac{10}{3}\right] =$ $\mathbf{4.07 \times 10^{-5}}$	M1 A1 M1M1 A1 **[5]**	Substitute and multiply Correctly obtain given answer Select t term; method for expansion formula Answer *OR*: attempt to find G′(0) M1 $8(\frac{3}{16})^8(\frac{10}{3} + 2t)(1 + \left[\frac{10}{3} + t\right]t)^7$ A1 $= 4.07 \times 10^{-5}$ A1

4 **(i)**	Number of goals scored by home team is independent of number of goals scored by away team	B1 **[1]**	Not just *goals* independent. Extras, including conditions already implied by given Poisson distributions: B0
(ii) (a)	$e^{-4.2n}(1+4.2n+\frac{(4.2n)^2}{2!}+\frac{(4.2n)^3}{3!})$	M1 A1 A1 **[3]**	Po(4.2n) implied Correct ± 1 term Fully correct expression, aef. SR Po(4.2): Fully correct formula B1
(b)	$e^{-2.4}e^{-1.8}(1 + 2.4\times1.8)$ $= \mathbf{0.0798}$	M1 A1 A1 **[3]**	Individual Poisson distributions multiplied Correct expression [= 0.0150 + 0.0647] Answer, a.r.t. 0.080 [0.07977]
5 **(i)**	n large p close to ½	B1 B1 **[2]**	Or $np > 5$ $nq > 5$ [*not* $npq > 5$]
(ii)	$\frac{24.5-\mu}{\sigma} = \Phi^{-1}(0.8282) = 0.947$ $\frac{27.5-\mu}{\sigma} = \Phi^{-1}(0.9697) = 1.759$ $\mu = \mathbf{21},\ \sigma = \mathbf{3.69}$	M1 A1 B1 M1 A1 **[5]**	One standardised, = Φ^{-1}, allow σ^2, cc, 1– errors LHS of both equations correct including signs and cc Both z-values correct to 3 sf, ± 1 in third dp Solve to find both μ and σ μ, a.r.t. 21.0; σ, in range [3.69, 3.70]
(iii)	$q = npq/np = 21/3.694^2$ [= 0.65] $p = \mathbf{0.35},\ n = \mathbf{60}$	M1 A1ft A1 **[3]**	Correct method of solution for n, p or q, allow $\sqrt{npq}$ $npq = \sigma^2$ [not σ], ft on their npq[13.65] p, a.r.t. 0.350 and $n = 60$ [integer] *only* [*not* 60.0]
6 **(i)**	$\int_0^\infty 4xe^{-2x}e^{tx}dx = \int_0^\infty 4xe^{-(2-t)x}dx$ $=\left[\frac{4xe^{-(2-t)x}}{(t-2)}\right]_0^\infty + \int_0^\infty \frac{4e^{-(2-t)x}}{2-t}dx$ $\left[-\frac{4e^{-(2-t)x}}{(2-t)^2}\right]_0^\infty = \frac{4}{(2-t)^2}$	M1 A1 M1 A1 A1 **[5]**	Attempt $\int e^{tx}f(x)dx$, limits somewhere Combine into single e term Use parts, right way round Correct indefinite integral Correct final answer, cwo, allow $(t-2)^2$ but must use integral that visibly converges, or otherwise indicate the issue
(ii)	$t < 2$	B1 **[1]**	
(iii)	$\left[\frac{4}{(2-t)^2}\right]^3 = \frac{64}{(2-t)^6}$ $= (1-\frac{1}{2}t)^{-6} = 1+3t+\frac{21}{4}t^2+...$ E(Y) = **3** E(Y^2)/2 = 21/4 so E(Y^2) = 10.5 Var(Y) = 10.5 – 3^2 = **1.5**	M1 A1 M1 A1 M1 A1 **[6]**	$[M_X(t)]^3$ [Not cubed: M0A0 M1A0 M1A0] Series expansion *or* differentiate once $M'(t) = \frac{384}{(2-t)^7}, M''(t) = \frac{2688}{(2-t)^8}$ E(Y) = 3 correctly obtained or implied 2 × coeff of t^2 *or* $M''(0) - [M'(0)]^2$ Var(Y) = 1.5 or exact equivalent, cwo

7	**(i)**	$\int_0^k x\dfrac{3x^2}{k^3}\,dx = \tfrac{3}{4}k$ $E(\tfrac{4}{3}X) = k$, so $\tfrac{4}{3}X$ unbiased **AG**	M1 A1 A1 **[3]**	Attempt $\int x f(x)$, correct limits ¾ k, ae exact f Must state "unbiased"
	(ii)	$P(X \le x) = \int_0^k \dfrac{3x^2}{k^3}\,dx = \left(\dfrac{x^3}{k^3}\right)$ $P(M \le m) = \left(\dfrac{x^3}{k^3}\right)^3 = \dfrac{x^9}{k^9}$ $f_M(x) = 9\dfrac{x^8}{k^9}$ **AG**	B1 M1 M1 A1 **[4]**	Needs convincing derivation $[F_X(x)]^3$ Differentiate Full derivation of AG. Ignore other ranges
	(iii)	$\int_0^k x9\dfrac{x^8}{k^9}\,dx = \tfrac{9}{10}k$ Hence $E_2 = \tfrac{10}{9}M$	M1 A1 A1ft **[3]**	Attempt $\int x f_M(x)$, ignore limits Correct $E(M)$ If $E(M) = kc$, allow M/c
8		PE lost = $0.4g \times 3 \sin 20°$ [4.104] Initial KE = $\tfrac{1}{2} \times 0.4 \times 0.5^2$ [0.05] Final KE = $\tfrac{1}{2} \times 0.4 \times 2.5^2$ [1.25] Difference = Work done by friction $2.9045 = 3F$ Therefore $F =$ **0.968** N	M1 M1 M1 M1 A1 **[5]**	*mgh* attempted, with trig Both KEs attempted Work/Energy principle used, no extra/missing terms Use $\Delta E = F \times s$ Answer, a.r.t. 0.968 [SC: Energy not used: answer B2]
9	**(i)**	$R(\uparrow_Q)$: $T - 1.5g = 0$ $[T = 15]$ $R(\uparrow_P)$: $T\cos\theta - 1.2g = 0$ $[15\cos\theta = 12]$ $\Rightarrow \cos\theta = 0.8$ Distance below = $0.12/\tan\theta$ = **0.16**m	M1 M1 A1 A1 **[4]**	Resolve vertically for 1.5 kg mass, can be implied e.g. by $T = 15$ Resolve vertically for 1.2 kg mass Value of $\cos\theta$ $[\theta = 36.9°]$ Correct value of h
	(ii)	$T = 1.5g$ $R(\rightarrow_P)$: $T\sin\theta = 1.2r\omega^2$ [= 9] $\Rightarrow 1.5g\sin\theta = 1.2 \times 0.12 \times \omega^2$ $[a = 7.5]$ $\Rightarrow \omega = \sqrt{62.5} =$ **7.91** rad s^{-1}	B1 M1 A1 A1 **[4]**	Value of T *used* [= 15] Resolve horiz for P and use $r\omega^2$ or v^2/r Correct equation, and $v = r\omega$ if v used Answer, in range [7.9, 7.91] or $\dfrac{5}{2}\sqrt{10}$

10 (i)	$N = 80$ M(X): $F\times 5 \sin\theta = 80\times 2.5 \cos\theta$ $F \le 0.4N$ $\tan\theta \ge 1.25$ θ_{min} = **51.3°** or 51.4°, 0.896	B1 M1 M1 M1 A1 **[5]**	Normal force at ground (can be implied) Moments about any point, needs both cos and sin A: $R\times 5\sin\theta = 80\times 2.5\cos\theta$ $[R = F = 0.4N = 32]$ M: $R\times 2.5\sin\theta + F\times 2.5\cos\theta = N\times 2.5\cos\theta$ Use $F \le \mu N$ or $F = \mu N$ Solve equations to obtain $\tan\theta$ Correct answer, in range [51.3, 51.4] or a.r.t. 0.896
(ii)	$F\times 5\sin\theta = (80\times 2.5 + 750d)\cos\theta$ Use 60° and μ to obtain d_{max} = **3.57**	M1* depM1 A1 **[3]**	Moments equation with variable (d) $[F = 332]$ $[332\times 5\times 0.5 = 100\sqrt{3} + 337\sqrt{3}d]$ Answer, a.r.t. 3.57 or 3.56
11 (i)	Driving force = $32000/v$ $800\dfrac{dv}{dt} = \dfrac{32000}{v} - 20v$ $\dfrac{dv}{dt} = \dfrac{1600 - v^2}{40v}$ **AG**	M1 A1 **[2]**	Use P/v and differential equation including dv/dt Correctly obtain AG, need to use 800 convincingly
(ii)	$\displaystyle\int \frac{40v}{1600 - v^2}dv = \int dt$ $c - 20\ln(1600 - v^2) = t$ $c = 20\ln 1600$ [147.56] $t = 20\ln\left(\dfrac{1600}{1600 - v^2}\right)$ $v = 40\sqrt{1 - e^{-t/20}}$ Tends to 40	M1 A1 A1 M1 A1 B1 **[6]**	Separate variables and attempt to integrate Correct indefinite integral, aef Correct value of c Make v subject, using e, allow v^2 Correct expression for v, aef Conclusion, cwo but can get from implicit formula
12 (i)	 Mom^m (→): $mu\cos 30 = mw + mx$ Rest^n: $x - w = 0.9\sqrt{3}u/2$ Solve: $w = 0.0433u$ Mom^m (↑): $mu\sin 30 = vm$ so $v = 0.5u$ ⇒ direction is $\tan^{-1}(0.5/0.0433)$ = **85.05°** to x-axis	 M1 M1 A1 B1 A1 **[5]**	$[\frac{\sqrt{3}}{2}u = w + x]$ C of M equation, needn't have m, ignore signs, needs cos and sin Restitution equation, ignore signs of LHS Correctly obtain w = a.r.t. $0.0433u$ $[=\sqrt{3}u/40]$ Obtain, state, or use $v = u/2$ Direction, [85.0°, 85.1°] to x-axis (5° or 4.9° to y)
(ii)	$u\cos 30 = w + x$, $ue\cos 30 = x - w$ ⇒ $2w = u\cos 30(1 - e)$ but $e \le 1$ so w cannot be negative	M1 M1 A1 **[3]**	One general equation Second equation, and use $e \le 1$ Correctly deduce given conclusion

13 (i)	Solution 1: ***V,H***		
	$y = 30t\sin 50° - 5t^2$; $x = 30t\cos 50°$	M1A1	Both equations attempted; both correct
	$[y = 26.2]$ $\quad x = 48.2$	A1	Correct value of x SC: If M0, $y = 26.2$ gets B1
	Height of slope $= x\tan 10° = 8.5$	M1	Find height
	Difference $=$ **17.7**	A1 **[5]**	Correct answer [17.703]
	Solution 2: **‖, ⊥**		
	$y = 30t\sin 40° - 5t^2\cos 10°$ $= 17.43$	M1A1 A1	
	Height above field $= y \div \cos 10°$ $=$ **17.7**	M1 A1	
(ii)	Solution 1: ***V,H***		
	$30t\sin 50° - 5t^2 = 30t\cos 50°\tan 10°$	M1	Put $y = x\tan 10°$ and solve
	$t = 3.916$	A1	Correct value of t, can be implied
	$PX = 30t\cos 50° \div \cos 10°$	M1	Calculate $x \div \cos 10°$ or $y \div \sin 10°$
	$X =$ **76.7**	A1 **[4]**	X, a.r.t. 76.7 [76.684]
	Solution 2: **‖, ⊥**		
	$Y = 30t\sin 40° - 5t^2\cos 10°$	M1	Y equation, allow sign/trig errors (3 terms)
	$Y = 0$ at $\quad t = 3.916$	A1	Correct value of t, can be implied
	$X = 30t\cos 40° - 5t^2\sin 10°$	A1	X equation, allow sign/trig errors (3 terms)
	$PX =$ **76.7**	A1	X, a.r.t. 76.7 [76.684]
	Solution 3: **traj**		
	$Y = X\tan 50 - \dfrac{10X^2\sec^2 50}{2\times 30^2}$ $= x\tan 10°$	M1 A1	Use trajectory equation All correct
	$PX = 180(\tan 50 - \tan 10)\cos^2 50 =$ **76.7**	M1 A1	Solve for X X, a.r.t. 76.7 [76.684]

14 (i)	$T - mg = 0 \quad \Rightarrow \dfrac{3}{0.5} \times e = 0.3g$	M1	Use N2 for equilibrium
	$\Rightarrow \quad e = \mathbf{0.5}$	A1 **[2]**	Equilibrium extension 0.5, a.r.t. 0.500
(ii)	GPE lost = EPE gained: $3(0.5 + x) = \frac{1}{2}\dfrac{3}{0.5}x^2 \quad [= 3x^2]$	M1*	Use cons of energy, need variable on both sides, e.g. $3(1+h) = 3(h+\frac{1}{2})^2$ or $3h = 3(h-\frac{1}{2})^2$
		depM1	Obtain and solve quadratic equation
	Solve to get $x = \dfrac{1+\sqrt{3}}{2} = \mathbf{1.37}$	A1	Correct equation, add/subtract 0.5 etc if necessary
		A1 **[4]**	Solve to get a.r.t. 1.37 *only*, allow surds
(iii)	$0.3\ddot{x} = 0.3g - \dfrac{3}{0.5}(x+e)$	M1	Use N2 including extension
	$\ddot{x} = -20x$	A1	All correct, check signs
		B1 **[3]**	Obtain correct value of ω^2, no wrong working
(iv)	$x = \dfrac{\sqrt{3}}{2}\cos\sqrt{20}t$	M1	Use $x = a\cos\omega t$ or $a\sin\omega t$, allow $a = 1$
		A1ft	a correct ft (= their (ii) – their (i)), ω from (iii)
	$x = 0.5$ at $\omega t = 0.9553$ or 5.3279	M1	Equate to ($\pm$)e and use correct trig method e.g. $\dfrac{2}{\sqrt{20}}\cos^{-1}\left(-\dfrac{1}{\sqrt{3}}\right)$ or $\dfrac{1}{\sqrt{20}}\left(\dfrac{\pi}{\omega} + 2\sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)\right)$
	$t = 0.2136$ or 1.1913	A1	One correct value of t
	Difference = **0.978**	A1 **[5]**	Correct final answer [0.9777]