Cambridge International Examinations
Cambridge Pre-U Certificate

FURTHER MATHEMATICS (PRINCIPAL) **9795/01**

Paper 1 Further Pure Mathematics **May/June 2016**

3 hours

0431896259

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

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1 Using standard summation results, show that $\sum_{r=1}^{n}(8r^3 + r) \equiv \frac{1}{2}n(n+1)(2n+1)^2$. [4]

2 Find a vector which is perpendicular to both of the lines

$$\mathbf{r} = \begin{pmatrix} 11 \\ 5 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ 2 \\ 5 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} 1 \\ 7 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} -6 \\ 1 \\ 4 \end{pmatrix}$$

and hence find the shortest distance between them. [6]

3 A curve has equation $y = \dfrac{2x^2 - x - 1}{2x - 3}$.

(i) Show that the curve meets the line $y = k$ when $2x^2 - (2k+1)x + (3k-1) = 0$, and hence show that no part of the curve exists in the interval $\frac{1}{2} < y < \frac{9}{2}$. [4]

(ii) **Deduce** the coordinates of the turning points of this curve. [4]

4 A 3×3 system of equations is given by the matrix equation $\begin{pmatrix} -1 & 3 & 1 \\ 5 & -1 & 2 \\ -1 & 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 16 \\ -2 \end{pmatrix}$.

(i) Show that this system of equations does not have a unique solution. [2]

(ii) Solve this system of equations and describe the geometrical significance of the solution. [6]

5 Find the general solution of the differential equation $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 5y = 24e^{2x}$. [8]

6 The equation $\sinh x + \sin x = 3x$ has one positive root α.

(i) Show that $2.5 < \alpha < 3$. [2]

(ii) By using the first two non-zero terms in the Maclaurin series for $\sinh x + \sin x$, show that $\alpha \approx \sqrt[4]{60}$. [3]

(iii) By taking the third non-zero term in this series, find a second approximation to α, giving your answer correct to 4 decimal places. [3]

7 **(i)** Find all values of z for which $z^3 = 2 + 2i$. Give your answers in the form $re^{i\theta}$, where $r > 0$ and θ is an exact multiple of π in the interval $0 < \theta < 2\pi$. [6]

(ii) The vertices of a triangle in the Argand diagram correspond to the three roots of the equation $z^3 = 2 + 2i$. Sketch the triangle and determine its area. [3]

8 **(i)** S is the set $\{1, 2, 4, 8, 16, 32\}$ and $\times_{63}$ is the operation of multiplication modulo 63.

(a) Construct the multiplication table for $(S, \times_{63})$. [2]

(b) Show that $(S, \times_{63})$ forms a group, G. (You may assume that $\times_{63}$ is associative.) [3]

(ii) The group H, also of order 6, has identity element e and contains two further elements x and y with the properties

$$x^2 = y^3 = e \qquad \text{and} \qquad xyx = y^2.$$

(a) Construct the group table of H. [4]

(b) List all the proper subgroups of H. [2]

(c) State, with justification, whether G and H are isomorphic. [1]

9 The cubic equation $x^3 - ax^2 + bx - c = 0$ has roots α, β and γ.

(i) State, in terms of a, b and c, the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. [2]

(ii) Find, in terms of a, b and c, the values of $\alpha^2 + \beta^2 + \gamma^2$ and $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$. [4]

(iii) Show that $(\alpha - 2\beta\gamma)(\beta - 2\gamma\alpha)(\gamma - 2\alpha\beta) = c(2a + 1)^2 - 2(b + 2c)^2$. [4]

(iv) Deduce that one root of the equation $x^3 - ax^2 + bx - c = 0$ is twice the product of the other two roots if and only if $c(2a + 1)^2 = 2(b + 2c)^2$. [1]

10 **(i)** Sketch the curve with polar equation $r = \left|\frac{1}{2} + \sin\theta\right|$, for $0 \leqslant \theta < 2\pi$. [6]

(ii) Find in an exact form the total area enclosed by the curve. [4]

11 **(i)** The sequence of *Fibonacci Numbers* $\{F_n\}$ is given by

$$F_1 = 1, \quad F_2 = 1 \quad \text{and} \quad F_{n+1} = F_n + F_{n-1} \quad \text{for } n \geqslant 2.$$

Write down the values of F_3 to F_6. [1]

(ii) The sequence of functions $\{\mathrm{p}_n(x)\}$ is given by

$$\mathrm{p}_1(x) = x + 1 \quad \text{and} \quad \mathrm{p}_{n+1}(x) = 1 + \frac{1}{\mathrm{p}_n(x)} \quad \text{for } n \geqslant 1.$$

(a) Find $\mathrm{p}_2(x)$ and $\mathrm{p}_3(x)$, giving each answer as a single algebraic fraction, and show that $\mathrm{p}_4(x) = \dfrac{3x + 5}{2x + 3}$. [3]

(b) Conjecture an expression for $\mathrm{p}_n(x)$ as a single algebraic fraction involving Fibonacci numbers, and prove it by induction for all integers $n \geqslant 2$. [5]

12 The curve C has equation $y = \ln(\tanh \frac{1}{2}x)$, for $x > 0$.

(i) Show that $\dfrac{dy}{dx} = \operatorname{cosech} x$. [3]

(ii) For positive integers n, the length of the arc of C between $x = n$ and $x = 2n$ is L_n.

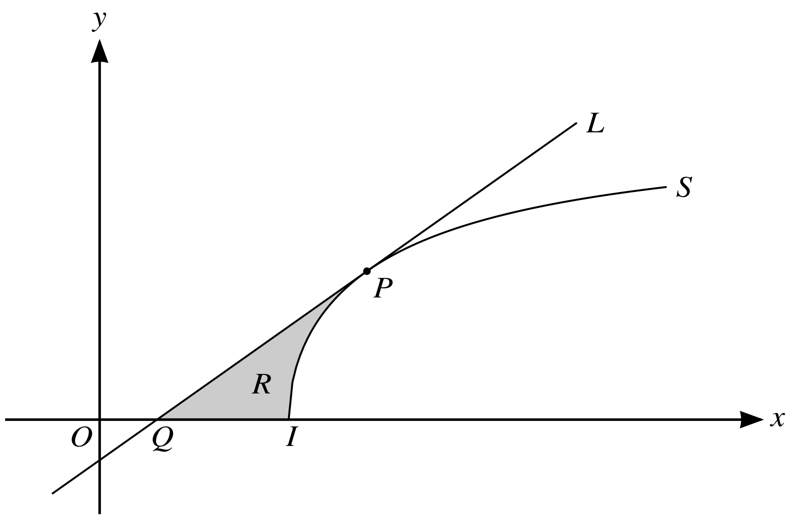
(a) Show by calculus that, when n is large, $L_n \approx n$. [5]

(b) Explain how this result corresponds to the shape of C. [2]

13 **(i)** **(a)** Given that $x \geqslant 1$, show that $\sec^{-1} x = \cos^{-1}\left(\dfrac{1}{x}\right)$, and deduce that $\dfrac{d}{dx}(\sec^{-1} x) = \dfrac{1}{x\sqrt{x^2 - 1}}$. [3]

(b) Use integration by parts to determine $\displaystyle\int \sec^{-1} x \, dx$. [4]

(ii)



The diagram shows the curve S with equation $y = \sec^{-1} x$ for $x \geqslant 1$. The line L, with gradient $\dfrac{1}{\sqrt{2}}$, is the tangent to S at the point P and cuts the x-axis at the point Q. The point I has coordinates $(1, 0)$.

(a) Determine the exact coordinates of P and Q. [6]

(b) The region R, shaded on the diagram, is bounded by the line segments PQ and QI and the arc IP of S. Show that R has area

$$\ln\left(1 + \sqrt{2}\right) - \frac{\pi(8 - \pi)\sqrt{2}}{32}.$$ [4]