



FURTHER MATHEMATICS

9795/01

Paper 1 Further Pure Mathematics

May/June 2017

MARK SCHEME

Maximum Mark: 120

Published

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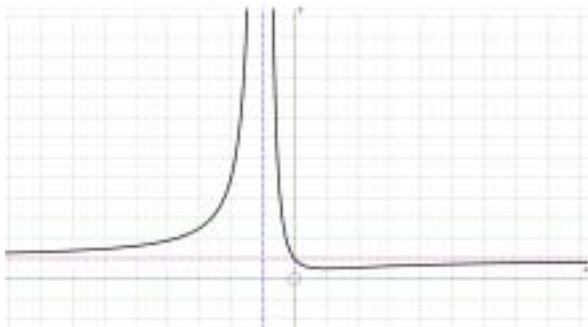
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Question	Answer	Marks	Part Marks
1	$(a + ib)^2 = (a^2 - b^2) + i.2ab$	B1	
	$(a^2 - b^2) = 21$ and $ab = -10$	M1	Comparing real and imaginary parts
	e.g. eliminating one variable and solving for the other	M1	Allow implied by e.g. $a = 5, b = 2$ (or v.v.)
	$a = \pm 5, b = \mp 2$	A1	Ignore any complex answers
2	$\Sigma\alpha = -2$ and $\Sigma\alpha\beta = 3$	B1	Both ($\alpha\beta\gamma = -7$ not required)
	$\alpha^2 + \beta^2 + \gamma^2 = (\Sigma\alpha)^2 - 2\Sigma\alpha\beta = -2$	M1A1	FT
	1 real and 2 complex (conjugate) roots	B1	Accept any comment that “not all roots are real
	Alternative Form an equation with roots $\alpha^2, \beta^2, \gamma^2$; $y^3 + 2y^2 - 19y - 49 = 0$	M1A1	
	$\Sigma\alpha^2 = -\frac{b}{a} = -2$	B1	FT
	1 real and 2 complex (conjugate) roots	B1	Accept any comment that “not all roots are real
3(i)		B3	B1 Starts at (1, 0) B1 Decreasing spiral B1 All (essentially) correct
3(ii)	$\text{Area} = \frac{1}{2} \int_0^{2\pi} \frac{1}{(1 + \theta)^2} d\theta$	M1	Attempt to integrate $k(1 + \theta)^{-2}$
	$= \frac{1}{2} \left[\frac{-1}{1 + \theta} \right]_0^{2\pi}$	A1	Correct integration
	$= \frac{1}{2} \left(1 - \frac{1}{1 + 2\pi} \right) \text{ or } \frac{\pi}{1 + 2\pi}$	A1	Correct answer

Question	Answer	Marks	Part Marks
4	$\dot{x} = t - \frac{1}{t}$ and $\dot{y} = 2$	B1	at least \dot{x} correct
	$(\dot{x})^2 + (\dot{y})^2 = t^2 - 2 + \frac{1}{t^2} + 4$	M1	attempted
	$= \left(t + \frac{1}{t}\right)^2$	A1	Here or in the integral for S (2 nd fraction of line below)
	$S = 2\pi \int_1^4 2t \cdot \left(t + \frac{1}{t}\right) dt$	M1	Use of formula (Ignore limits until final answer)
	$= 4\pi \int_1^4 (t^2 + 1) dt$	A1	In a form ready to integrate
	$= 4\pi \left[\frac{t^3}{3} + t \right]_1^4$	B1	Correct integration (FT provided it is polynomial)
	$= 96\pi$	A1	
5(i)	$y = \tanh^{-1} x \Leftrightarrow \tanh y = x = \frac{e^{2y} - 1}{e^{2y} + 1}$	M1	
	$xe^{2y} + x = e^{2y} - 1 \Leftrightarrow 1 + x = e^{2y}(1 - x)$	M1	Identifying e^{2y}
	$y = \tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$	A1	Legitimately obtained by taking logs Allow verification by substitution of given result
5(ii)	Method I $t + \frac{1}{t} = 4 \Rightarrow t^2 - 4t + 1 = 0$	M1	Creating a quadratic in $\tanh x$
	$\Rightarrow t = 2 \pm \sqrt{3}$	M1	Solving
	Using $\frac{1}{2} \ln \left(\frac{1+t}{1-t} \right)$ with $t = 2 - \sqrt{3}$ and/or $2 + \sqrt{3}$	M1	(NB since $ \tanh x < 1$, it must be $t = 2 - \sqrt{3}$)
	$x = \frac{1}{2} \ln \left(\frac{3 - \sqrt{3}}{-1 + \sqrt{3}} \times \frac{1 + \sqrt{3}}{1 + \sqrt{3}} \right) = \frac{1}{2} \ln(\sqrt{3})$	M1	By rationalising denominator or direct observation (possibly from calculator use)
	$= \frac{1}{4} \ln(3)$	A1	Must be in this form

Question	Answer	Marks	Part Marks
5(ii)	Method II $\frac{\text{sh}}{\text{ch}} + \frac{\text{ch}}{\text{sh}} = 4$	M1	
	$\Rightarrow \text{ch}^2 + \text{sh}^2 = 4\text{sh.ch} \Rightarrow \cosh(2x) = 2 \sinh(2x)$	M1	Conversion to double-“angles”
	$\Rightarrow \tanh(2x) = \frac{1}{2}$	A1	
	$\Rightarrow 2x = \frac{1}{2} \ln\left(\frac{\frac{3}{2}}{\frac{1}{2}}\right)$	M1	Use of $\tanh^{-1} x$ formula from (i)
	$\Rightarrow x = \frac{1}{4} \ln(3)$	A1	Must be in this form
	Method III $\frac{e^{2x}-1}{e^{2x}+1} + \frac{e^{2x}+1}{e^{2x}-1} = 4$	M1	
	$\Rightarrow (e^{2x}-1)^2 + (e^{2x}+1)^2 = 4(e^{2x}-1)(e^{2x}+1)$	M1	
	$\Rightarrow e^{4x} - 2e^{2x} + 1 + e^{4x} + 2e^{2x} + 1 = 4(e^{4x} - 1)$	A2	A1 LHS A1 RHS
	$\Rightarrow 6 = 2e^{4x} \Rightarrow x = \frac{1}{4} \ln(3)$	A1	Must be in this form
6(i)	HA $y = 1$ VA $x = -1$	B2	B1 for each
6(ii)	$y = \frac{x^2+1}{(x+1)^2}$ or $y = 1 - \frac{2x}{(x+1)^2}$ $\Rightarrow \frac{dy}{dx} = \frac{(x+1)^2(2x) - (x^2+1).2(x+1)}{(x+1)^4}$ or $-\frac{(x+1)^2.2 - 2x.2(x+1)}{(x+1)^4} = \frac{2(x-1)}{(x+1)^3}$	M1A1	Attempted; correct unsimplified
	$\Rightarrow \frac{dy}{dx} = 0$ when $x = 1, y = \frac{1}{2}$	A2	A1 for each
6(iii)		3	G1 for graph in 2 bits, separated by a (FT) vertical asymptote and all positive G1 for y-intercept at (0, 1) and MIN. in (approx. FT) correct place G1 for correct asymptotic behaviour

Question	Answer	Marks	Part Marks
7(i)	$y = kx \sin 2x \Rightarrow \frac{dy}{dx} = 2kx \cos 2x + k \sin 2x$	M1	attempt using the <i>Product Rule</i>
	and $\frac{d^2y}{dx^2} = kx \cdot -4 \sin 2x + 2k \cos 2x + 2k \cos 2x$	M1	attempt using the <i>Product Rule</i>
	$= -4y + 4k \cos 2x$	M1	for substn. into given d.e. or comparison
	$\Rightarrow k = 2$	A1	
7(ii)	Comp. Fn. from $m^2 + 4 = 0$	M1	
	$\Rightarrow y_c = A \cos 2x + B \sin 2x$	A1	Or $R \cos(2x - \alpha)$ etc.
	Gen. Soln. is thus $y = A \cos 2x + (B + 2x) \sin 2x$	B1	FT
	Then $\frac{dy}{dx} = -2A \sin 2x + 2(B + 2x) \cos 2x + 2 \sin 2x$ OR $= 2(B + 2x) \cos 2x$ if found after A (correctly) evaluated	B1	
	Subst ^g . in given initial conditions	M1	
	$A = 1$ from $x = 0, y = 1$	A1	FT from an incorrect $x \sin 2x$ term in y
	$B = \frac{1}{2}$ from $x = 0, \frac{dy}{dx} = 1$ i.e. soln. is $y = \cos 2x + (2x + \frac{1}{2}) \sin 2x$	A1	FT from an incorrect $x \cos 2x$ term in y' Withhold final A mark if in e^{\wedge} complex form
8(i)(a)	$\cos \theta = \frac{12 + 2 + 6}{3 \times 7} = \frac{20}{21}$	M1A2	A1 scalar product; A1 both moduli Give B1s for correct scalar product; both moduli if $\sin \theta = \dots$ used
8(i)(b)	Subst ^g . $(2\lambda, -\lambda, 2\lambda)$ into $6x - 2y + 3z = 35$	M1	
	$\Rightarrow \lambda = \frac{7}{4} \Rightarrow \mathbf{p} = \frac{7}{4} \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$	A1A1	Second A1 is FT
8(i)(c)	SD O to $\Pi_1 = OP \cos \theta = \frac{7}{4} \times 3 \times \frac{20}{21} = 5$	M1A1	A1FT

Question	Answer	Marks	Part Marks
8(i)(c)	Alternative I $(6\lambda, -2\lambda, 3\lambda)$ in plane $\Rightarrow 36\lambda + 4\lambda + 9\lambda = 35$	M1	$\Rightarrow \lambda = \frac{5}{7}$
	$\Rightarrow SD = \lambda \sqrt{6^2 + 2^2 + 3^2} = 5$ cao	A1	
	Alternative II Quote formula: $SD = \frac{\left \frac{d}{\ \mathbf{n}\ } \right }{\text{cao}} = \frac{35}{\sqrt{6^2 + 2^2 + 3^2}} = 5$	M1A1	
8(ii)	Similar working gives $\lambda_1 = -\frac{21}{40}$	B1	
	Planes parallel, <i>and on opposite sides of O</i> , so total distance is $3\left(\frac{7}{4} + \frac{21}{40}\right) \cos \theta = \frac{13}{2}$	M1A1	
	Alternative I Π_2 has equation $\mathbf{r} \cdot \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} = -\frac{21}{2}$	B1	
	\Rightarrow SD to Π_2 is $-\frac{3}{2}$	B1	
	Planes parallel, <i>and on opposite sides of O</i> , so distance between them is $5 - -\frac{3}{2} = \frac{13}{2}$	B1	FT
	Alternative II Quote Sh. Dist. formula for $P\left(\frac{7}{4}, -\frac{7}{2}, \frac{7}{2}\right)$ to Π_2	M1	or using distance from any point in Π_1 or Π_2 to other plane
	$SD = \frac{\left 12\left(\frac{7}{4}\right) - 4\left(-\frac{7}{2}\right) + 6\left(\frac{7}{4}\right) + 21 \right }{\sqrt{12^2 + 4^2 + 6^2}} = \frac{91}{14} = \frac{13}{2}$	A1A1	
9(i)	Full elimination of x : $I = \int \frac{1}{\cosh^2 \theta \cdot \sinh \theta} \cdot \sinh \theta d\theta$	M1	
	$\Rightarrow I = \int \text{sech}^2 \theta d\theta$	A1	
	$= \tanh \theta (+ C)$	A1	
	$= \frac{\sqrt{x^2 - 1}}{x} (+ C)$ from $\frac{\sinh \theta}{\cosh \theta}$	A1	(AG)

Question	Answer	Marks	Part Marks
9(ii)	$\sec y = x \Rightarrow \sec y \tan y \frac{dy}{dx} = 1$	M1A1	
	Use of $\tan y = \sqrt{\sec^2 y - 1}$	M1	
	to get $\frac{dy}{dx} = \frac{1}{x\sqrt{x^2 - 1}}$	A1	AG Ignore lack of reason for taking the +ve sq.rt. (e.g. from +ve gradient of \sec^{-1} curve)
9(iii)	$\int \sec^{-1} x \cdot \frac{1}{x^2} dx$ $= \sec^{-1} x \cdot \frac{-1}{x} - \int \frac{-1}{x} \cdot \frac{1}{x\sqrt{x^2 - 1}} dx$ $= \frac{-\sec^{-1} x}{x} + \int \frac{1}{x^2 \sqrt{x^2 - 1}} dx$	M1A2	By parts
	$= \frac{-\sec^{-1} x}{x} + \frac{\sqrt{x^2 - 1}}{x} (+C)$	A1	using (i)
	Alternative Use $u = \sec^{-1} x \Rightarrow \frac{du}{dx} = \frac{1}{x\sqrt{x^2 - 1}}$ $\Rightarrow \sec u \tan u du = dx$	M1	
	$\Rightarrow \int \sec^{-1} x \cdot \frac{1}{x^2} dx = \int u \sin u du$	A1	
	2-stage integration by parts: $\int u \sin u du = -u \cos u + \int \cos u du$ $= -u \cos u + \sin u (+C)$	M1	
	Correctly turning this back into $= \frac{-\sec^{-1} x}{x} + \frac{\sqrt{x^2 - 1}}{x} (+C)$	A1	
10(i)	$\frac{1}{(k-1)k(k+1)} \equiv \frac{A}{k-1} + \frac{B}{k} + \frac{C}{k+1}$	M1	Correct form
	Equating terms / substn. / cover-up	M1	Method for determining constants
	$\equiv \frac{\frac{1}{2}}{k-1} - \frac{1}{k} + \frac{\frac{1}{2}}{k+1}$	A1	

Question	Answer	Marks	Part Marks
10(ii)	$\sum_{k=3}^n \frac{1}{(k-1)k(k+1)} \equiv \frac{1}{2} \sum_{k=3}^n \frac{1}{k-1} + \frac{1}{2} \sum_{k=3}^n \frac{1}{k+1} - \sum_{k=3}^n \frac{1}{k}$	M1	Splitting up
	$\equiv \frac{1}{2} \left\{ \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n-1} \right\} + \frac{1}{2} \left\{ \frac{1}{4} + \dots + \frac{1}{n-1} + \frac{1}{n} + \frac{1}{n+1} \right\} - \left\{ \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n-1} + \frac{1}{n} \right\}$	M1	Attempt at cancelling of terms
	$\equiv \frac{1}{2} \left\{ \frac{1}{2} + \frac{1}{3} \right\} + \frac{1}{2} \left\{ \frac{1}{n} + \frac{1}{n+1} \right\} - \left\{ \frac{1}{3} + \frac{1}{n} \right\}$	A1	Correct ones clearly identified
	$\equiv \frac{1}{12} - \frac{1}{2} \left\{ \frac{1}{n} - \frac{1}{n+1} \right\} \equiv \frac{1}{12} - \frac{1}{2n(n+1)}$	A1	Legitimately shown (AG)
	Limit (S_n) as $n \rightarrow \infty$ is $S = \frac{1}{12}$	B1	FT
	Alternative $\sum_{k=3}^n \frac{1}{(k-1)k(k+1)} \equiv \frac{1}{2} \sum_{k=3}^n \frac{1}{k(k-1)} - \frac{1}{2} \sum_{k=3}^n \frac{1}{k(k+1)}$	M1	
	$= \frac{1}{2} \left(\frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{n(n-1)} \right) - \frac{1}{2} \left(\frac{1}{12} + \frac{1}{20} + \dots + \frac{1}{n(n-1)} + \frac{1}{n(n+1)} \right)$	M1	Clear listing of terms
	All correct and ready to cancel	A1	
	$= \frac{1}{12} - \frac{1}{2n(n+1)}$	A1	Legitimately shown (AG)
	Limit (S_n) as $n \rightarrow \infty$ is $S = \frac{1}{12}$	B1	FT
10(iii)	$k^3 > k^3 - k = k(k-1)(k+1)$ $\Rightarrow \frac{1}{k^3} < \frac{1}{(k-1)k(k+1)}$	B1	
10(iv)	$\sum_{k=1}^{\infty} \frac{1}{k^3} > 1 + \frac{1}{8} = \frac{9}{8} = \frac{27}{24}$	B1	Given result justified
	$\sum_{k=1}^{\infty} \frac{1}{k^3} = 1 + \frac{1}{8} + \sum_{k=3}^{\infty} \frac{1}{k^3} < 1 + \frac{1}{8} + \sum_{k=3}^n \frac{1}{(k-1)k(k+1)}$	M1	
	$= 1 + \frac{1}{8} + \frac{1}{12} = \frac{29}{24}$	A1	Given result justified
11(i)(a)	$\mathbf{AB} = \begin{pmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{pmatrix}$	B1	
	$\det \mathbf{A} = ad - bc \text{ and } \det \mathbf{B} = eh - fg$	B1	

Question	Answer	Marks	Part Marks
11(i)(b)	$\det(\mathbf{AB}) = (ae + bg)(cf + dh) - (af + bh)(ce + dg)$ and some attempt to multiply out	M1	
	$= acef + adeh + bcfg + bdgh$ $\quad - acef - bceh - adfg - bdgh$ $= adeh - bceh - adfg + bcfg$ $= (ad - bc)(eh - fg)$	A1	Legitimately shown
11(ii)	<i>CLOSURE</i> : $\mathbf{A}, \mathbf{B} \in S \Rightarrow \det \mathbf{A} = \det \mathbf{B} = 1$	M1	Attempted
	and above result $\Rightarrow \det \mathbf{AB} = 1 \Rightarrow \mathbf{AB} \in S$ <i>(ASSOCIATIVITY: given)</i>	A1	Convincing
	<i>IDENTITY</i> : $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \in S$ since $\det \mathbf{I} = 1.1 - 0.0 = 1$	B1	Must show why $\mathbf{I} \in S$ and not just say that \mathbf{I} is the identity
	<i>INVERSES</i> : $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in S \Rightarrow \mathbf{A}^{-1}$ $= \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \in S$	B1	for stating \mathbf{A}^{-1} (or explaining that it exists)
	Since $da - (-b)(-c) = ad - bc = 1$ Hence (S, \times_M) is a group, G .	B1	for justifying its membership of S
11(iii)(a)	$\det \mathbf{K} = 1.0 - i.i = -i^2 = 1$ (so $\mathbf{K} \in S$)	B1	
11(iii)(b)	Attempt at powers of \mathbf{K} ; \mathbf{K}^2 & \mathbf{K}^3	M1	
	$\mathbf{K}^2 = \begin{pmatrix} 0 & i \\ i & -1 \end{pmatrix}$ and $\mathbf{K}^3 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$	A1	
	NB $\mathbf{K}^4 = \begin{pmatrix} -1 & -i \\ -i & 0 \end{pmatrix}$ and $\mathbf{K}^5 = \begin{pmatrix} 0 & -i \\ -i & 1 \end{pmatrix}$ $\Rightarrow \mathbf{K}^6 = \mathbf{I}$ and H has order $n = 6$	A1	
11(iii)(c)	e.g. The set of rotations about O through multiples of 60° OR $(\mathbf{K}^*) =$ group generated by $\begin{pmatrix} 1 & -i \\ -i & 0 \end{pmatrix}$	B1	FT for any n
	Justifying the two are isomorphic	B1	e.g. stating both are cyclic, etc.

Question	Answer	Marks	Part Marks
12(i)	Method I $F_{n+2}(\theta) - \frac{1}{4} \sin^2(2\theta) F_{n+1}(\theta)$ $\equiv (c^2 + s^2)(c^{2n+4} + s^{2n+4})$ $- \frac{1}{4}(2sc)^2(c^{2n+2} + s^{2n+2})$	M2	M1 all F_n terms M1 $\sin 2\theta$ form
	$\equiv c^{2n+6} + c^2 s^{2n+4} + s^2 c^{2n+4} + s^{2n+6}$ $- c^2 s^2 (c^{2n+2} + s^{2n+2})$	A1	
	$\equiv c^{2n+6} + s^{2n+6} \equiv F_{n+3}(\theta)$	A1	AG
	Method II $\equiv c^{2n+4} + s^{2n+4} - s^2 c^2 (c^{2n+2} + s^{2n+2})$	M1	Use of $\sin 2\theta$ form
	$\equiv c^{2n+4} + s^{2n+4} - s^2 c^{2n+4} - c^2 s^{2n+4}$	A1	
	$\equiv (1-s^2)c^{2n+4} + (1-c^2)s^{2n+4}$	M1	
	$\equiv c^{2n+6} + s^{2n+6} \equiv F_{n+3}(\theta)$	A1	AG
12(ii)(a)	Use of $z = c + is$ and $z^{-1} = c - is$	M1	
	$z + z^{-1} = 2c$ and $z - z^{-1} = 2is$	A2	A1 for each
12(ii)(b)	Method I $(2c)^6 = (z + z^{-1})^6 = z^6 + 6z^4 + 15z^2 + 20$ $+ 15z^{-2} + 6z^{-4} + z^{-6}$	M1	
	$= 2\cos 6\theta + 12\cos 4\theta + 30\cos 2\theta + 20$	A1	
	$-(2s)^6 = (z - z^{-1})^6 = z^6 - 6z^4 + 15z^2 - 20$ $+ 15z^{-2} - 6z^{-4} + z^{-6}$ $= 2\cos 6\theta - 12\cos 4\theta + 30\cos 2\theta - 20$	B1	FT (Must have – sign)
	Subtracting: $64(c^6 + s^6) = 12(z^4 + z^{-4}) + 40$ $= 12 \cdot 2\cos 4\theta + 40$	M1	
	Dividing by 8: $8(c^6 + s^6) = 3\cos 4\theta + 5$	A1	AG
	Use of $\cos 4\theta = 2\cos^2 2\theta - 1$ and $1 = \cos^2 2\theta + \sin^2 2\theta$	M1	
	$\Rightarrow c^6 + s^6 = \frac{3}{8}(2\cos^2 2\theta) + \left(-\frac{3}{8} + \frac{5}{8}\right)(\cos^2 2\theta + \sin^2 2\theta)$ $= \cos^2 2\theta + \frac{1}{4}\sin^2 2\theta$	A1	AG

Question	Answer	Marks	Part Marks
12(ii)(b)	Method II $\cos 4\theta = \operatorname{Re}(c + is)^4$	M1	
	$= c^4 - 6c^2s^2 + s^4 = c^4 - 6c^2(1-c^2) + (1-c^2)^2$ $= 8c^4 - 8c^2 + 1$	A1	
	$c^6 + s^6 = c^6 + (1-c^2)^3 = c^6 + 1 - 3c^2 + 3c^4 - c^6$	M1	
	$= 3c^4 - 3c^2 + 1$	A1	
	so that $8(c^6 + s^6) = 3\cos 4\theta + 5$	A1	AG
	Use of $\cos 4\theta = \cos^2 2\theta - \sin^2 2\theta$ and $1 = \cos^2 2\theta + \sin^2 2\theta$	M1	
	$\Rightarrow 8(c^6 + s^6) = 3\cos 4\theta + 5$ $= 3(\cos^2 2\theta - \sin^2 2\theta) + 5(\cos^2 2\theta + \sin^2 2\theta)$ $\Rightarrow c^6 + s^6 = \cos^2 2\theta + \frac{1}{4}\sin^2 2\theta$	A1	AG
12(iii)	Case for $n = 1$ established in (ii) (b):	B1	noted explicitly (possibly at end)
	Assume $c^{2k+4} + s^{2k+4} \leq \cos^2 2\theta + \frac{1}{2^{k+1}}\sin^2 2\theta$	B1	i.e. the case for $n = k$
	A clear statement of the result must be given, possibly within what follows Then $c^{2k+6} + s^{2k+6} =$ $c^{2k+4} + s^{2k+4} - \frac{1}{4}\sin^2 2\theta(c^{2k+2} + s^{2k+2})$	M1	attempt at $n = k + 1$ case using (i)'s identity
	$\leq \cos^2 2\theta + \frac{1}{2^{k+1}}\sin^2 2\theta - \frac{1}{4}\sin^2 2\theta(c^{2k+2} + s^{2k+2})$	M1	use of the induction hypothesis (i.e. the $n = k$ case)
	$= \cos^2 2\theta + \frac{1}{2^{k+2}}\sin^2 2\theta - \frac{1}{4}\sin^2 2\theta\left(c^{2k+2} + s^{2k+2} - \frac{1}{2^k}\right)$	M1A1	splitting up the $\sin^2 2\theta$ term into two equal parts
	$\leq \cos^2 2\theta + \frac{1}{2^{k+2}}\sin^2 2\theta$ Proof follows by induction since $\sin^2 2\theta \geq 0$ and given result that $c^{2k+2} + s^{2k+2} \geq \frac{1}{2^k}$	A1	