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Cambridge Pre-U Certificate

FURTHER MATHEMATICS **9795/02**

Paper 2 Further Application of Mathematics **May/June 2017**

MARK SCHEME

Maximum Mark: 120

Published

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Question	Answer	Marks	Notes
1(i)	On average 95% of all identically constructed confidence intervals contain the parameter	**B1**	Use of "confident" without explanation: B0.
1(ii)	$\bar{x} = 39.72$	**B1**	
	$s_{n-1} = 3.30711$	**B1**	3.31 or 10.9
	$39.72 \pm 2.776 \times \dfrac{3.30711}{\sqrt{5}}$	**M1**	Needs $\sqrt{5}$ but allow s_n, z, or **FT** errors
		A1	All numbers correct (apart from s_{n-1}) soi
	= awrt (35.6, 43.8)	**A1**	Both correct to 3 SF. Condone wrong order.
2(i)	$\frac{1}{3}t^{-1} + \frac{2}{3}t^2$	**B2**	**B1** for each If x used in an otherwise correct expression then SC1. Condone further consistent use in part (ii).
2(ii)(a)	$G_{10}(t) = (\frac{1}{3}t^{-1} + \frac{2}{3}t^2)^{10}$ oe	**M1**	$[G(t)]^{10}$
	Differentiate (could be any wrong G) and put $t = 1$	**M1**	
	$G'_{10}(t) = 10(-\frac{1}{3}t^{-2} + \frac{4}{3}t)(\frac{1}{3}t^{-1} + \frac{2}{3}t^2)^9$	**A1**	Correct derivative in any form
	$G'(1) = 10$	**A1**	Answer 10 only. www SC: E(X) = 1 B1, E(T) = 10 "E(X)" B1ft. Max 2/4
2(ii)(b)	Attempt coefficient of t^8	**M1**	
	Coefficient of $t^8 = {}^{10}C_4\,(\frac{1}{3})^4(\frac{2}{3})^6$	**A1**	Correct expression (term or coefficient)
	= 4480/19683 or awrt 0.228	**A1**	SC: 6 2s and 4 –1s: ${}^{10}C_4\,(\frac{1}{3})^4(\frac{2}{3})^6$ M1 0.228 A1 (Max 2/3)
3(i)	$\frac{32}{100} \pm 2.576\sqrt{\frac{32}{100} \times \frac{68}{100} \div 100}$	**M1**	Correct form incl. 100. Allow $n/(n-1)$ for M1.
		A1	2.576
	= awrt (0.200, 0.440)	**A1**	Answer, limits correct to $\geqslant$3 sf. Condone wrong order
3(ii)	$2 \times 2.576 \times \sqrt{\dfrac{0.32 \times 0.68}{n}} = 0.04$ o.e.	**M1**	Correct equation, allow same wrong z or σ^2 as in (i), or 2 omitted. Allow n-1 for M1.
		M1	Solve including squaring
	$n_{\min} = 3610$	**A1**	3610 only

Question	Answer	Marks	Notes
4(i)	Use total area = 1 with integration	**M1**	
	$a + a\left[1 - \frac{1}{3}\right] = 1$	**A1**	Correct integration
	$a = \frac{3}{5}$	**A1**	0.6 or $\frac{3}{5}$ only
4(ii)	Integrate for one non-zero region (*a* can remain, constants can be omitted)	**M1**	
	$\begin{cases} \frac{3}{5}(x+1) & -1 \leqslant x < 0 \\ \frac{3}{5}(x - \frac{x^3}{3} + 1) & 0 \leqslant x \leqslant 1 \\ 0 & x < -1 \\ 1 & x > 1 \end{cases}$	**4**	**A1** One formula correct **A1** Other formula correct **A1** Ranges $-1 \leqslant x < 0$ and $0 \leqslant x \leqslant 1$ correct, allow $\leqslant$/$<$ **B1** 0 and 1
4(iii)	F(0.25) = 0.746875 < 0.75	**M1**	Evaluate F(0.25) *and* compare with 0.75
	UQ > 0.25	**A1**	Correct conclusion from correct values Alt method by finding UQ:$4u^3 - 12u + 3 = 0$ (oe) M1 Correct solution (e.g. by GC $u = 0.25556...$) (or sign change to demonstrate root between 0.25 and 1) and correct conclusion A1 Alt method: $u = 0.25 + u^3/3 > 0.25$ and conclusion B2
5(i)	Po(4)	**M1**	Po(4) and "1 –" in tables, e.g. 0.2149 or 0.0511
	1 – P(≤ 6) = awrt 0.111	**A1**	
5(ii)	$e^{-\lambda} = 0.6$	**M1**	(λ could be $t/180$ or $t/3$ or $20t$ depending on units)
	$\lambda = -\ln 0.6$ or awrt 0.511	**A1**	soi
	t = 92 seconds	**A1**	Answer 92 seconds or 1 minute 32 seconds only
5(iii)	Po(60) ≈ N(60, 60)	**M1A1**	Po(60) and N(60, …); N(60, 60) soi
		M1	Standardise
		A1	Correct √ and cc
	$\Phi\left(\frac{65.5 - 60}{\sqrt{60}}\right) = \Phi(0.71)$ = awrt 0.761	**A1**	Answer, a.r.t. 0.761 [no cc, 0.7406; wrong cc, 0.7193; 60 not √60, 0.5366]

Question	Answer	Marks	Notes
6(i)	Attempt $\int f(x)e^{xt}dx$, correct limits (could be later)	**M1**	
	$\int_{-1}^{1} f(x)e^{xt}\,dx = \int_{-1}^{1} \tfrac{1}{2}e^{xt}\,dx$	**A1**	Correct expression
	$= \dfrac{1}{2t}\left[e^{xt}\right]_{-1}^{1}$ **AG**	**A1**	Correct integral
	$= \dfrac{e^{t}-e^{-t}}{2t} = \dfrac{\sinh t}{t}$	**A1**	Correctly obtain given answer SC Using formula for MGF of uniform distribution from formula book. Use of formula and substituting a = –1, b = 1 M1 Correctly obtain given answer A1 Max 2/4
6(ii)	$\dfrac{1}{t}\left(t+\dfrac{t^3}{3!}+\dfrac{t^5}{5!}+\ldots\right)$	**M1**	Correct expansion of sinh t used
	$=1+\dfrac{t^2}{6}+\dfrac{t^4}{120}+\ldots$	**A1**	Correct after division by t (at least 3 terms) soi
	$E(X^2) = M''_X(0) = 2!\times$coeff of $t^2 = \frac{1}{3}$	**M1**	Use $2!\times$coeff of t^2 or attempt to diff twice
	$\mathrm{Var}(X) = \frac{1}{3}$	**A1**	$\frac{1}{3}$, distinction between $E(X^2)$ and $\mathrm{Var}(X)$ made,
	$E(X^4) = 4!\times$coeff of $t^4 = \frac{1}{5}$	**A1**	or $E(X)$ (or $M'_X(0)$) stated to be zero Correctly obtain 1/5
7(i)	N(570, …	**M1**	
	… 470)	**A1**	
	$1 - \Phi(1.845) = 0.0325$	**A1**	

Question	Answer	Marks	Notes
7(ii)	P(C > (P + J + C)/8)	M1A1	or equivalent e.g. $\frac{7}{8}C-\frac{1}{8}P-\frac{1}{8}J\sim \mathrm{N}(-\frac{5}{4},\frac{475}{32})$
	= P(7C – P – J > 0)	M1	
	7C – P – J ~ N(–10, 950)	M1A1	SC: Common error: forgetting that T (= P + J + C) and C are **not** independent
	P(> 0) = 1 – Φ((0– (–10))/√950)	M1	
	= 1 – Φ(0.3244) = awrt 0.373	A1	If $Y=k(8C-T)$ then $Y \sim N(-10k, 1110k^2)$ for non-zero k. Allow M1 A1. Then P(Y>0) (or P(Y<0) if k < 0) = = 1 – Φ((0– (–10))/√1110) = 1–Φ(0.3002) M1dep = awrt 0.382 A1 Max 4/7
8(i)	$\dot{\omega}=4.2$	B1	
8(ii)	$r\dot{\omega}=0.63$	B1	$r\dot{\omega}$ seen
	$r\omega^2$ = ±0.216	B1	$r\omega^2$ seen
	$\lvert\boldsymbol{a}\rvert$ = √($0.63^2 + 0.216^2$)	M1	Find |acceleration| for stated components
	= 0.666	A1	Answer, a.r.t. 0.666
9(i)	Trajectory formula quoted or obtained	M1	
	$y=x\tan\theta-\frac{x^2(1+\tan^2\theta)}{320}$	A1	Correct including 1 + $\tan^2\theta$, can be recovered (condone g/3200 in second term)
9(ii)	Substitute	M1	
	$y=72\tan\theta-\frac{72^2}{320}(1+\tan^2\theta)$	A1	
	α : $16.2t^2 - 72t + (16.2 + y) = 0$	M1	β: Differentiate w.r.t. θ
	$b^2 = 4ac \Rightarrow 72^2 = 4\times16.2(y + 16.2)$	M1	$72\sec^2\theta - 32.4\tan\theta\sec^2\theta = 0$
	$\Rightarrow y$ = 63.8	A1	Condone $y\leqslant$ [tan θ = 20/9], y = 63.8 [exact] Alt: Completing the square or using $-b/2a$ = 20/9
10(i)	WD = $P \times t$ = 75 × 60	M1	
	= 4500 kJ or4500000 J oe	A1	Allow 4500000 for A1 but not 4500

Question	Answer	Marks	Notes
10(ii)	$v = 24\ \text{ms}^{-1}$ so $F = P/v$	**M1***	Find resistive force (could be found in (i))
	$= 3125$	**A1**	3125 seen
		M1*	N2 with P/v (used here or later) and component of weight
	$\dfrac{75000}{v} - 0.05 \times 4000g - 3125 = 0$	**A1**	Correct equation
		M1dep*	Solve for v
	Hence $v = 14.6\ \text{ms}^{-1}$ $75000 = 0.05v \times 4000g + 3125v$ $75000 = 5125v$	**A1**	Answer, in range [14.6, 14.7] Alternative approach to N2 equation considering energy instead of force: (if 1s considered but could be multiplied throughout by eg 60). M1 for energy balance equation involving only PE, work done against resistance and work done by engine A1 correct
11(i)	Moments about A:	**M1**	Take moments about A involving a component of P and weight. Must be force×distance.
	$1.25 \times P \sin\alpha = 0.4 \times 1.6\text{g}$	**A1**	Correct equation, $P \sin\alpha$ needs deriving
	$P \sin\alpha = 5.12$ **AG**	**A1**	Correctly obtain given answer
11(ii)	N2($\uparrow$): $P \sin\alpha + N = 1.6\text{g}$	**M1A1**	3 forces with a component of P.
	N2($\rightarrow$): $P \cos\alpha = F$	**B1**	Correct equation (soi)
	$F \leqslant \frac{6}{17}N$ or 3.84	**M1**	Use $F \leqslant \mu N$ or $F = \mu N$
	$N = 10.88$, $P\cos\alpha \leqslant 3.84$ $P^2(\sin^2\alpha + \cos^2\alpha) \leqslant 40.96$	**M1**	Value for $P \cos\alpha$ and eliminate α (allow from $\alpha = 53.1°$)
	$P \leqslant 6.4$ **AG**	**A1**	Correctly obtain AG, inequalities correct (NB $\tan\alpha \geqslant 4/3$) throughout (or convincing argument for changing equation to inequality www)
12(i)	$0.6v\dfrac{\mathrm{d}v}{\mathrm{d}x} = -10 \times 0.6 - 0.024v^2$	**M1**	Use $v\ \mathrm{d}v/\mathrm{d}x$ for a and two other terms in $F = ma$
	Hence $v\dfrac{\mathrm{d}v}{\mathrm{d}x} = -10 - 0.04v^2$	**A1**	AG, completely correct, signs not fudged

Question	Answer	Marks	Notes
12(ii)	$\int \frac{v}{10+0.04v^2}\,\mathrm{d}v = -\int \mathrm{d}x$	**M1**	Separate variables correctly (allow a multiplicative constant error) (ignore $\int$ signs)
	$\frac{1}{0.08}\ln(10+0.04v^2) = -x + c$ oe	**A1 A1**	Correct indefinite integrals, ignore *c*/limits here
	$x = 0,\ v = u \Rightarrow c = \frac{1}{0.08}\ln(10+0.04u^2)$	**M1**	Find *c* in terms of *u*
	$x = \frac{1}{0.08}\ln\left(\frac{10+0.04u^2}{10+0.04v^2}\right)$	**A1**	Correct formula for *v* and *x*, aef
	$v = 0, \quad x = \frac{1}{0.08}\ln\left(1+0.004u^2\right)$	**M1**	Substitute $v = 0$ and solve for *u*
	$x = 50, \quad u = \sqrt{\frac{(e^4-1)}{0.004}} = 116\ \mathrm{ms}^{-1}$	**A1**	awrt 116 ms^{-1}.
	Alternative I $x = 50,\ v = 0 \Rightarrow c = \frac{1}{0.08}\ln(10) + 50$	**M1**	Find numerical *c*
	$x = 50 + \frac{1}{0.08}\ln\left(\frac{10}{10+0.04v^2}\right)$	**A1**	Correct formula for *v* and *x*, aef
	$x = 0, v = u, \quad 50 = \frac{1}{0.08}\ln\left(1+0.004u^2\right)$	**M1**	Substitute $x = 0$, $v = u$ and solve for *u*
	$u = \sqrt{\frac{(e^4-1)}{0.004}} = 116\ \mathrm{ms}^{-1}$	**A1**	awrt 116 ms^{-1}.

Question	Answer	Marks	Notes
12(ii)	**Alternative II** $\left[\frac{1}{0.08}\ln(10+0.04v^2)\right]_u^0 = \left[-x\right]_0^{50} = -50$	**B1**	Correct limits and evaluation on x side
	$\ln(10)-\ln(10+0.04u^2)$ or $\ln\left(\frac{10}{10+0.04u^2}\right)$ oe	**M1**	Correctly dealing with limits on v side
	$\frac{1}{0.08}\left(\ln(10)-\ln(10+0.04u^2)\right) = -50$	**M1**	Solve for u
	$u=\sqrt{\frac{(e^4-1)}{0.004}} = 116\ \text{ms}^{-1}$	**A1**	awrt 116 ms^{-1}.
13(i)	At top, $mg\ (+\ T) = mv^2/r$	**M1**	N2(↓) at top
	so $v^2 \geqslant 0.6g$ or 6	**M1A1**	Obtain inequality (or equation); correct, a.e.f. *[No mention of T: M1M0A1]*
	C of E: $\frac{1}{2}mv^2 + 2mgr = \frac{1}{2}mu^2$	**M1A1**	Use conservation of energy; correct equation
	$v^2 = u^2 - 4gr$; $u^2 \geqslant 5gr = 30$,	**M1**	Solve for u
	$\Rightarrow$ $u_{\text{min}} = 5.48$	**A1**	Answer, awrt 5.48. Condone $\sqrt{30}$ Common Error: $v_{\text{min}} = 0$ at top leading to $u_{\text{min}}=\sqrt{24} = 4.90$. M0M0A0M1A1M1A0 or SC3.
13(ii)	$mg\cos\theta + T = mv^2/r$	**M1**	Resolve inwards at general angle, T not needed (NB $-mg\cos\theta$ if with downward vertical). Condone sign errors or v^2/r for M1
	$v^2 = gr\cos\theta$	**A1**	Correct condition
	$\frac{1}{2}mv^2 = \frac{1}{2}m5^2 - 0.6mg(1+\cos\theta)$	**M1A1**	C of E for general angle; correct equation. $-\cos\theta$ if with downward vertical. Condone mgh for M1.
	$u^2 = gr(2 + 3\cos\theta)$ $[25 = 6(2 + 3\cos\theta)]$	**M1**	Find value for $\cos\theta$ [$= \pm 13/18$]
	$\theta = \cos^{-1}(0.7222) = 43.8°$	**A1**	Answer in range [43.7. 43.8]
14(i)	$0.05\ddot{x} = -\frac{0.6}{1.2}x$ or $\ddot{x} = -10x$	**M1**	Use $ma = -\lambda x/l$. If two non-zero tensions, M0
		A1	Condone $a = -10x$

Question	Answer	Marks	Notes
14(ii)	$v = \omega\sqrt{(a^2 - x^2)} = \omega a$	**M1**	Use $v = \omega\sqrt{(a^2 - x^2)}$
	$= 0.316$	**A1**	$\omega = \sqrt{10}$ and $a = 0.1 \Rightarrow v = \sqrt{10}/10$ or awrt 0.316
	Alternative I $\frac{1}{2}\,0.05v^2 = 0.6\times0.1^2 / (2\times1.2)$	**M1**	Use C of E: $\frac{1}{2}mv^2 = \lambda x^2/2l$
	$\Rightarrow v = 0.316$	**A1**	$x = 0.1$ and given values $\Rightarrow v =$ awrt 0.316
	Alternative II $x = 0.1\cos(t\sqrt{10})$	**M1**	Solving SHM equation with initial conditions
	$\dot{x} = -0.1\sqrt{10}\sin(\sqrt{10}t) \Rightarrow v = \pm 0.316$	**A1**	Differentiating and finding max $\Rightarrow v =$ awrt 0.316
14(iii)	One complete period: $2\pi/\omega$	**M1**	Use $2\pi/\omega$ somewhere
	$= 1.987$ s	**A1**	Correct time for SHM parts. Allow $2\pi/\sqrt{10}$
	2×0.4m at constant speed v:	**M1**	Or one way: 0.4m at constant speed v
	2.53 s or $0.8\sqrt{10}$ o.e.	**A1**	1.26s
	Total 4.52	**A1**	Allow $\dfrac{\sqrt{10}(4+\pi)}{5}$ oe