**Cambridge Assessment International Education**
Cambridge Pre-U Certificate

**FURTHER MATHEMATICS** **9795/02**

Paper 2 Further Application of Mathematics **May/June 2018**

MARK SCHEME

Maximum Mark: 120

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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This syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

| Question | Answer | Marks | Partial Marks |
|---|---|---|---|
| 1(i) | B(200, 0.2) ≈ N(40, 32) | **M1** | N(40, …) |
| | $\Phi\left(\frac{30.5-40}{\sqrt{32}}\right)=\Phi(-1.679)$ | **A1** | variance 32 soi |
| | | **M1** | standardising |
| | = 0.0466 | **A1** | answer in [0.046, 0.047] |
| 1(ii) | $Y$~B(200, 0.02) ≈ Po(... | **M1** | Poisson, any $\lambda$ |
| | ...4) | **A1** | $\lambda$ (or $np$) = 4 |
| | $P(Y \leqslant 3) = 0.433$ | **A1** | answer in [0.433, 0.434] |
| 2(i) | Breaks in transmission must occur independently of one another | **B1** | independent, in context |
| | and at constant average rate or constant expected rate | **B1** | 'constant average rate' or 'constant expected rate' or 'mean ∝ time int' |
| 2(ii) | $X \sim \text{Po}(7.5)$ | **M1** | Po(7.5) stated or implied |
| | $P(6 \leqslant X \leqslant 10) = P(X \leqslant 10) - P(X \leqslant 5)$ | **M1** | Po, $P(\leqslant 10) - P(\leqslant 5 \text{ or } 6)$ |
| | = 0.8622 – 0.2414 = 0.621 | **A1** | awrt 0.621 |
| 2(iii) | $P(0) = e^{-\lambda}$ | **M1** | correct use of P(0) formula |
| | $e^{-1.3T} = 0.6$ | **A1** | $e^{-kT}$ or $e^{-\lambda} = 0.6$ (provided not clearly incorrect Poisson e.g. Po(8)) |
| | $-1.3\ T = \ln(0.6)$ [= –0.5108] | **M1** | **dep** on first M mark<br>for taking logarithms |
| | $T$ (= 0.393 min) = 23.6 s | **A1** | 23.6 or 24 s or better |
| 3(i) | $M'(t) = 6(1-2t)^{-4}$ | **M1** | attempt to differentiate $M(t)$ |
| | $M'(0) = 6$ so $E(X) = 6$ | **A1** | 6 correctly obtained |
| | $M''(t) = 48(1-2t)^{-5}$ [$M''(0) = 48$] | **M1** | attempting $M''(t)$ |
| | Therefore $\text{Var}(X) = 48 - 6^2$ | **M1** | $M''(0) - [M'(0)]^2$ used |
| | = 12 | **A1** | 12 correctly obtained |

| Question | Answer | Marks | Partial Marks |
|---|---|---|---|
| 3(i) | **Alternative**<br>$M_X(t) = 1 + 6t + \dots$ | **M1** | attempt to expand:1st 2 terms |
| | $\mathrm{E}(X) = 6$ | **A1** | 6 correctly (soi) obtained |
| | $M_X(t) = \text{“}1 + 6t\text{”} + 24t^2 + \dots$ | **M1** | 3rd term and using 2! |
| | Therefore $\mathrm{Var}(X) = 2 \times 24 - 6^2$ | **M1** | $\mathrm{E}(X^2)) - [\mathrm{E}(X)]^2$ used |
| | $= 12$ | **A1** | 12 correctly obtained |
| 3(ii) | $Y = X_1 + X_2$: $\mathrm{M}_Y(t) = (1 - 2t)^{-6}$ | **M1** | $[\mathrm{M}_X(t)]^2$ |
| | | **M1** | attempt to find $\mathrm{M}_Y'''(t)$ |
| | $M_Y''' = 2688(1-2t)^{-9}$ | **A1** | correct expression |
| | Therefore $\mathrm{E}(Y^3) = M_Y'''(0) = 2688$ | **A1** | answer 2688 only |
| | **Alternative 1**<br>$Y = X_1 + X_2$: $\mathrm{M}_Y(t) = (1 - 2t)^{-6}$ | **M1** | $[\mathrm{M}_X(t)]^2$ |
| | | **M1** | using coefficient of $t^3$ |
| | $\frac{1}{3!}\mathrm{E}(Y^3) = (-2)^3 \frac{(-6)(-7)(-8)}{1\times 2\times 3}$ | **A1** | correct expression inc. 3! |
| | Therefore $\mathrm{E}(Y^3) = 6 \times 7 \times 8^2 = 2688$ | **A1** | answer 2688 only |
| | **Alternative 2**<br>$M_X(t) = \text{“}1 + 6t + 24t^2 +\text{”} + 80$ soi | **M1** | genuine attempt at next term |
| | | **M1** | **Dep** Attempt to expand and correct use of expectation algebra |
| | $\mathrm{E}((X_1 + X_2)^3) = 2\mathrm{E}(X^3) + 6\mathrm{E}(X^2)\mathrm{E}(X)$ oe | **A1** | |
| | $2 \times 480 + 6 \times 48 \times 6 = 2688$ | **A1** | answer 2688 only |
| 4(i) | $\mathrm{f}(x) = \frac{3}{8}x^2$ | **M1** | attempt to differentiate |
| | $\int_0^2 \frac{3}{8}x^2.x\,\mathrm{d}x = \left[\frac{3}{32}x^4\right]_0^2$ | **M1** | $\int x\mathrm{f}(x)\,\mathrm{d}x$ <u>and</u> limits 0, 2 |
| | $= 1\frac{1}{2}$ oe | **A1** | exact answer |

| Question | Answer | Marks | Partial Marks |
|---|---|---|---|
| 4(ii) | $Y = \frac{1}{X^2} : \mathrm{P}(Y \leqslant y) = \mathrm{P}\left(\frac{1}{X^2} \leqslant y\right)$ | **M1** | appropriate probability statement |
| | $= \mathrm{P}\left(X \geqslant \frac{1}{\sqrt{y}}\right)$ | **M1** | using inverse function correctly somewhere |
| | $= 1 - \mathrm{F}\left(\frac{1}{\sqrt{y}}\right)$ or $1 - \frac{1}{8}y^{-\frac{3}{2}}$ | **M1** | 1 – F(inverse function) |
| | | **M1** | differentiating their F($y$) (wrt correct variable) |
| | $\Rightarrow \quad \mathrm{f}(y) = \frac{3}{16}y^{\frac{-5}{2}}$ | **A1** | correct formula for f($y$), cwo |
| | for $y \geqslant \frac{1}{4}$ (must be correct variable) | **B1** | correct range for non-zero f |
| | **Alternative**<br>$\mathrm{f}_Y(y) = \mathrm{f}_X(\mathrm{g}^{-1}(y))\left\lvert\frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{g}^{-1}(y))\right\rvert$ | **B1** | statement |
| | $\mathrm{g}^{-1}(y) = \frac{1}{\sqrt{y}} = y^{-\frac{1}{2}}$ | **B1** | correct inverse function (condone ±) |
| | $\mathrm{f}_X(\mathrm{g}^{-1}(y)) = \frac{3}{8}(y^{-\frac{1}{2}})^2 = \frac{3}{8}y^{-1}$ | **B1** | |
| | $\frac{\mathrm{d}}{\mathrm{d}x}(\mathrm{g}^{-1}(y)) = -\frac{1}{2}y^{-\frac{3}{2}}$ | **B1** | |
| | $\mathrm{f}_Y(y) = \frac{3}{8}y^{-1}\left\lvert-\frac{1}{2}y^{-\frac{3}{2}}\right\rvert = \frac{3}{16}y^{-\frac{5}{2}}$ | **B1** | |
| | for $y \geqslant \frac{1}{4}$ (must be correct variable) | **B1** | correct range for non-zero f |

| Question | Answer | Marks | Partial Marks |
|---|---|---|---|
| 5(i) | $\overline{t}_B = 17, \quad \overline{t}_G = 22.29$ | **B1** | both sample means seen |
| | $S_B^{\ 2} = 64$ ($S_B = 8$), $S_G^{\ 2} = 12.35$ ($S_G = 3.514$) | **M1** | method for finding both $S$s<br>(or $S_{BB}$ (=768) and $S_{GG}$ (= 1210/7)) |
| | $\hat{\sigma}^2 = \dfrac{12S_B^2 + 14S_G^2}{12+14-2} = 39.20$ ($\hat{\sigma} = 6.261$) | **M1** | proper attempt at pooled variance (no working and wrong **M0**) |
| | $\overline{t}_G - \overline{t}_B \pm 2.064\sqrt{\hat{\sigma}^2(\frac{1}{12}+\frac{1}{14})}$ | **A1** | correct variance estimate |
| | | **M1** | CI with $t$ or $z$ and $\frac{1}{12}+\frac{1}{14}$ soi<br>$(5.2857 \pm (2.064 \times 2.463))$ |
| | | **B1** | $t = 2.064$ used in a CI |
| | = (0.202, 10.4) (3sf or better (cwo)) | **A2** | **A1** for each end-point, allow if both negative (i.e. (–10.4, –0.202)) |
| | Assume distributions of boys' and girls' times normally distributed with common variance oe | **B1** | |
| 5(ii) | No as 0 is not in the confidence interval | **B1** | **FT** |
| 6(i) | $\mathrm{P}(X = r) = \dfrac{1}{N}$ | **B1** | PDF $\dfrac{1}{N}$ stated or implied |
| | $\mathrm{E}(X) = \displaystyle\sum_{x=1}^{N} \frac{1}{N}x = \frac{N(N+1)}{2N}$ | **M1** | using $\Sigma x\mathrm{f}(x)$ and $\frac{1}{2}N(N+1)$ |
| | $= \frac{1}{2}(N+1)$ **AG** | **A1** | correctly obtaining **AG** |
| | $\mathrm{E}(X^2) = \displaystyle\sum_{x=1}^{N} \frac{1}{N}x^2 = \frac{(N+1)(2N+1)}{6}$ | **M1** | using $\Sigma x^2\mathrm{f}(x)$ and $\frac{1}{6}N(N+1)(2N+1)$ |
| | $\mathrm{Var}(X) = \dfrac{(N+1)(2N+1)}{6} - \left(\dfrac{N+1}{2}\right)^2$ | **M1** | subtracting their $[\mathrm{E}(X)]^2$ |
| | $= \frac{1}{12}(N+1)[2(2N+1) - 3(N+1)]$<br>$= \frac{1}{12}(N^2 - 1)$ **AG** | **A1** | correctly obtaining **AG** |

| Question | Answer | Marks | Partial Marks |
|---|---|---|---|
| 6(i) | **Alternative**<br>PGF $G(t) = (t + t^2 + ... + t^N) / N$<br>so $G'(t) = (1 + 2t + ... + Nt^{N-1}) / N$ | M1 | Correct PGF and differentiation |
| | $E(X) = G'(1) = \sum_{x=1}^{N} \frac{1}{N} x = \frac{N(N+1)}{2N}$ | M1 | equating and using $\Sigma x f(x)$ and $\frac{1}{2}N(N+1)$ |
| | $= \frac{1}{2}(N+1)$ **AG** | A1 | correctly obtaining **AG** |
| | $Var(X) = G''(1) + G'(1) - [G'(1)]^2$<br>$G''(t) = (2 + 3 \times 2t + 4\times3t + ...N(N-1)t^{N-2}) / N$ | M1 | for correct formula <u>and</u> G" (t) |
| | $Var(X) = \frac{1}{N}\sum_{x=1}^{N} x(x-1) + \frac{1}{N}\sum_{x=1}^{N} x - \left(\frac{N+1}{2}\right)^2$<br>$Var(X) = \frac{1}{N}\sum_{x=1}^{N} x^2 - \frac{1}{4}(N+1)^2$<br>$Var(X) = \frac{(N+1)(2N+1)}{6} - \frac{1}{4}(N+1)^2$ | M1 | **dep** for simplifying <u>and</u> using $\frac{1}{6}N(N+1)(2N+1)$ |
| | $= \frac{1}{12}(N+1)[2(2N+1) - 3(N+1)]$<br>$= \frac{1}{12}(N^2 - 1)$ **AG** | A1 | correctly obtaining **AG** |
| 6(ii) | $E(A) = \frac{1}{2}(N+1)$ | M1 | correct statement of $E(A)$ |
| | Therefore $E_1 = 2A - 1$ (is an unbiased estimator) | A1 | correct UE |
| | $Var(A) = \frac{N^2 - 1}{12 \times 40}$ aef | B1 | correct variance of $A$ soi |
| | $E_1 \sim$ Normal($N$, any $\sigma^2$) or (any $\mu$, $\frac{1}{120}(N^2 - 1)$ ) | M1 | Normal stated, correct $\mu$ or $\sigma^2$ |
| | $\sigma^2 = Var(E_1) = 4Var(A) = \frac{1}{120}(N^2 - 1)$ | A1 | **FT** for correct mean and variance of UE in **(ii)** only |

| Question | Answer | Marks | Partial Marks |
|---|---|---|---|
| 6(iii) | $E_2 = \frac{27}{40}B$ | **B1** | but not $X$ or $N = \frac{27}{40}B$ ) |
| | $\text{Var}(E_2) = \left(\frac{27}{40}\right)^2 \times \alpha N^2$ | **B1** | **FT** $q^2\alpha N^2$ from $E_2 = qB + c$ only |
| | $\left(\frac{27}{40}\right)^2 \times \alpha N^2 > \frac{1}{120}(N^2 - 1) \Rightarrow \frac{2187}{40}\alpha > 1 - \frac{1}{N^2}$ oe | **M1** | inequality <u>and</u> correct useful simplification (e.g. to $a\alpha > b\left(1 - \frac{1}{N^2}\right)$ ) |
| | True for all $N$ iff $\frac{2187}{40}\alpha \geqslant 1$ | **B1** | convincingly dealing with all $N$. Condone > |
| | $\alpha \geqslant \frac{40}{2187}$ [= awrt 0. 0183] | **A1** | correct final inequality, even if decimals used. Condone > |
| 7(i) | $\frac{1}{2}.800(20^2 - 10^2) + 800g \times 16$ | **M1** | use of both KE and GPE |
| | | **A1** | correct formulation |
| | = 248 kJ from correct working | **A1** | |
| | **Alternative**<br>$20^2 = 10^2 + 2a \times \frac{16}{\sin\theta}$<br>and $D - 800g\sin\theta = 800a$ | **M1** | correct use of <u>both</u> $v^2 = u^2 + 2as$ <u>and</u> NII with correct distance |
| | Eliminating $a$ leads to<br>WD = $D \times \frac{16}{\sin\theta} = 248000\text{J}$ | **A1** | correct answer |
| | Assuming that $a$ is constant | **A1** | giving assumption |
| 7(ii) | | **M1** | $\frac{189000}{v}$ soi |
| | $\frac{189000}{v} - 7v^2 = 0$ | **A1** | correct equation |
| | $v = \sqrt[3]{27000} = 30$ (ms$^{-1}$) | **A1** | for $v = 30$ |
| 8(i) | $mg = \frac{\lambda}{l}e$ | **M1** | $mg = \frac{\lambda}{l}e$ used |
| | $\Rightarrow e = 0.1$ | **A1** | |

| Question | Answer | Marks | Partial Marks |
|---|---|---|---|
| 8(ii) | $\frac{\lambda}{2l}(x-0.2)^2$ | M1 | using formula for EPE |
| | $\frac{\lambda}{2l}(x-0.2)^2 = mgx$ | M1 | equating EPE and GPE |
| | $20(x-0.2)^2 = 4x$ | A1 | correct equation |
| | $25x^2 - 15x + 1 = 0$ oe | A1 | correct simplified quadratic |
| | $x = \frac{3+\sqrt{5}}{10} = 0.524$ (or $x \geqslant 0.524$) | A1 | [0.523, 0.524] or $\frac{1}{10}\left(3+\sqrt{5}\right)$ oe |
| | **Alternative 1 (from slack position)**<br>$\frac{\lambda e^2}{2l}$ | M1 | using formula for EPE |
| | $\frac{\lambda e^2}{2l} = mg(e+0.2)$ | M1 | equating EPE and GPE |
| | $20e^2 = 4(e+0.2)$ | A1 | correct equation |
| | $25e^2 - 5e - 1 = 0$ oe | A1 | correct simplified quadratic |
| | $x = 0.2 + \frac{1\pm\sqrt{5}}{10} = 0.524$ (or $x \geqslant 0.524$) | A1 | [0.523, 0.524] or $\frac{1}{10}\left(3+\sqrt{5}\right)$ oe |
| | **Alternative 2 (from eqm position)**<br>$\frac{\lambda}{2l}(h+0.1)^2$ | M1 | using formula for EPE |
| | $\frac{1}{2}\frac{\lambda}{l}(h+0.1)^2 = mg(h+0.3)$ | M1 | equating EPE and GPE |
| | $20(h+0.1)^2 = 4(h+0.3)$ | A1 | correct equation |
| | $20h^2 - 1 = 0$ oe | A1 | correct simplified quadratic |
| | $x = 0.3 + \frac{1}{\sqrt{20}} = 0.524$ (or $x \geqslant 0.524$) | A1 | [0.523, 0.524] or $\frac{1}{10}\left(3+\sqrt{5}\right)$ oe |

| Question | Answer | Marks | Partial Marks |
|---|---|---|---|
| 8(ii) | **Alternative 3 (using SHM equation)**<br>$0.4g - \frac{8(x-0.2)}{0.2} = 0.4\ddot{x}$ leading to | M1 | deriving SHM formula in recognisable form |
| | $\ddot{x} + 100x = 30$<br>0.4 × 10 × 0.2 = $\frac{1}{2}$ × 0.4× $v^2$ => $v$ =2 | M1 | **Dep** on previous M.<br>Energy consideration leading to $v$ as string goes slack |
| | $x = A\cos 10t + B\sin 10t + 0.3$ or<br>$x = R\cos(10t+\phi) + 0.3$ | A1 | full correct solution with $\omega = 10$ and 2 arbitrary constants |
| | $R\cos\phi = -0.1$ and $10R\cos\phi = 2$ and attempt to solve simultaneously | M1 | using $t = 0$, $x = 0.2$ and $v = 2$ in correct equations to find $A$ and $B$ or $R$ and $\psi$ |
| | $R = \sqrt{A^2+B^2} = \frac{\sqrt{5}}{10} = 0.224$ so $x = 0.524$ | A1 | cao |
| | **Alternative 4 (using SHM energy equation)**<br>$0.4g - \frac{8(x-0.2)}{0.2} = 0.4\ddot{x}$ leading to | M1 | deriving SHM formula in recognisable form |
| | $\ddot{x} + 100x = 30$<br>0.4 × 10 × 0.2 = $\frac{1}{2}$ × 0.4× $v^2$ => $v$ =2 | M1 | **Dep** on previous M.<br>Energy consideration leading to $v$ as string goes slack |
| | $\omega = 10 \Rightarrow v^2 + 100(x-0.3)^2 = 100R^2$ | A1 | value of $\omega$ soi and fully correct energy equation with 3 terms and an arbitrary constant |
| | When $x = 0.2$, $v = 2$ so $100R^2 = 5$ | M1 | using $x = 0.2$ and $v = 2$ in correct equation to find $A$ |
| | $\therefore x = \frac{\sqrt{5}}{10} + 0.3 = 0.524$ | A1 | cao |
| 9(i) | cos/sin$\theta$ = 0.8/0.6 | B1 | 3:4:5 triangle implied |
| | $T\cos\theta = T\sin\theta + 0.2g$ | M1 | resolving and balancing forces vertically (trig values not nec) |
| | $0.8T = 0.6T + 2$ | A1 | correct equation with values |
| | $T = g = 10$ N | A1 | $T = g$ or 10 |

| Question | Answer | Marks | Partial Marks |
|---|---|---|---|
| 9(ii) | $r = 0.48$ | **B1** | radius 0.48, can be implied or seen in **(i)** (or correct cancellation) |
| | $T\sin\theta + T\cos\theta = mr\omega^2$ or $mv^2/r$ | **M1** | NII horizontally $= mr\omega^2$ |
| | $0.6T + 0.8T = 0.2\times0.48\omega^2$ or $0.2v^2/0.48$ | **A1** | **FT** for correct equation |
| | $\omega = 12.1$ or $v = 5.80$ (or 5.81 or $\sqrt{33.6}$) | **A1** | correct $v$ or $\omega$ |
| | Time $2\pi \div \omega = 0.520$ seconds | **A1** | 0.52 or awrt 0.520 |
| 10(i)(a) | $-mg\sin\theta = mr\ddot{\theta}$ cwo | **M1** | NII tangentially. Ignore radial |
| | $\ddot{\theta} = -\frac{g}{r}\sin\theta \approx -\frac{g}{r}\theta$ so approx. SHM | **A2** | **A1** for $\ddot{\theta}$ correct including signs<br>**A1** for $\sin\theta \approx \theta$ and stating SHM |
| | Period $2\pi \div \omega = 2.35(095)$ (seconds) | **B1** | awrt 2.35 or $\frac{1}{5}\pi\sqrt{14}$ |
| | **Alternative 1 (Hor & Vert)**<br>$-T\sin\theta = ma\cos\theta$ and | **M1** | NII (↕) and( ↔) |
| | $T\cos\theta - mg\sin\theta = ma\sin\theta$ cwo | **A1** | both fully correct |
| | $\ddot{\theta} = -\frac{g}{r}\sin\theta \approx -\frac{g}{r}\theta$ so approx. SHM | **A1** | eliminating $T$ and using $\sin\theta \approx \theta$ and $a = r\ddot{\theta}$ and stating SHM |
| | Period $2\pi \div \omega = 2.35(095)$ (seconds) | **B1** | awrt 2.35 or $\frac{1}{5}\pi\sqrt{14}$ |
| | **Alternative 2 ($x$)**<br>$-mg\sin\theta = m\ddot{x}$ cwo | **M1** | NII tangentially. Ignore radial |
| | $\ddot{x} = -g\sin\theta \approx -g\theta = -\frac{g}{r}x$ so approx. SHM | **A2** | **A1** for $\ddot{x}$ correct including signs<br>**A1** for using $\sin\theta \approx \theta$ and $x = r\theta$ (or just $x \approx r\sin\theta$) and stating SHM |
| | Period $2\pi \div \omega = 2.35(095)$ (seconds) | **B1** | awrt 2.35 or $\frac{1}{5}\pi\sqrt{14}$ |

| Question | Answer | Marks | Partial Marks |
|---|---|---|---|
| 10(i)(a) | **Alternative 3 (energy)**<br>$\frac{1}{2}mv^2 + mgr - mgr\cos\theta$ = const | **M1** | adding KE and PE (condone sign errors) |
| | $v = r\dot{\theta}$ (or this and $\dot{v} = r\ddot{\theta}$) used to derive DE | **M1** | eliminating $v$ (and/or $\dot{v}$) |
| | $mr^2\dot{\theta}\ddot{\theta} + mgr\sin\theta\dot{\theta} = 0 \Rightarrow \ddot{\theta} + \frac{g}{r}\theta \approx 0$ so approx. SHM | **A1** | differentiating implicitly, using $\sin\theta \approx \theta$ and rearranging and stating SHM |
| | Period $2\pi \div \omega = 2.35(095)$ (seconds) | **B1** | awrt 2.35 or $\frac{1}{5}\pi\sqrt{14}$ |
| 10(i)(b) | $\dot{\theta}^2 = \omega^2({\theta_0}^2 - \theta^2)$ | **M1** | SHM energy equation in terms of $\theta$ and $\dot{\theta}$ used |
| | $= \frac{10}{1.4}(0.3^2 - 0.2^2)$ | **A1** | **FT** for correct equation with their $\omega$ |
| | $v = r\dot{\theta} = 1.4 \times 0.5976...$ | **M1** | using $v = r\dot{\theta}$ |
| | $v = 0.836(66...)$ or $\sqrt{70}/10$ or awrt 0.837 | **A1** | converting to $v$ |
| | **Alternative 1 ($x$)**<br>$v^2 = \omega^2(a^2 - x^2)$ | **M1** | SHM energy equation in terms of $v$ and $x$ used |
| | $x = 1.4\theta$ soi | **M1** | either $x = 0.28$ or $a = 0.42$ |
| | $v^2 = \frac{10}{1.4}(0.42^2 - 0.28^2)$ | **A1** | **FT** for correct equation with their $\omega$ |
| | $v = 0.836(66...)$ or $\sqrt{70}/10$ or awrt 0.837 | **A1** | |
| | **Alternative 2 ($\theta$ solution)**<br>Solution to SHM equation is<br>$\theta = A\cos\omega t \quad (+B\sin\omega t)$<br>so initial conditions $\Rightarrow \theta = 0.3\cos\omega t$ | **M1** | particular solution, their $\omega$ |
| | Substituting $\omega t = \cos^{-1}\frac{2}{3}$ into $\dot{\theta} = \pm 0.3\omega\sin\omega t$ | **M1** | Award if $\sin^{-1}$ into $\pm 0.3\omega\cos\omega t$ |
| | $v = r\dot{\theta} = 1.4 \times 0.5976...$ | **M1** | using $v = r\dot{\theta}$ |
| | $v = 0.836(66...)$ or $\sqrt{70}/10$ or awrt 0.837 | **A1** | converting to $v$ (must be +ve) |

| Question | Answer | Marks | Partial Marks |
|---|---|---|---|
| 10(i)(b) | **Alternative 3 ($x$ solution)**<br>Gen sol to SHM eqn is $x = A\cos\omega t\ (+B\sin\omega t)$ | **M1** | |
| | so initial conditions $\Rightarrow x = 0.42\cos\omega t$ | **A1** | using $t = 0$ => both $v = 0$ and $x = 1.4 \times 0.3$ |
| | Substituting $\omega t = \cos^{-1}\frac{2}{3}$ into $\dot{x} = \pm 0.42\omega\sin\omega t$ | **M1** | Award if $\sin^{-1}$ in $\pm 0.42\omega\cos\omega t$ |
| | $v = 0.836(66...)$ or $\sqrt{70}/10$ or awrt 0.837 | **A1** | must be +ve |
| 10(ii) | | **M1** | KE gained = PE lost |
| | $\frac{1}{2}mv^2 = mg(1.4\cos 0.2 - 1.4\cos 0.3)$ | **A1** | correct equation |
| | $[v^2 = 0.692]$ $v = 0.832(13)$ | **A1** | awrt 0.832 |
| | % difference 0.544% | **A1** | awrt 0.544 (could be from correct calculations of $\dot{\theta}$) |
| 11(i) | $U_y = -6$ soi | **B1** | $\perp$ component of $U$ |
| | $\therefore u_y = 3$ | **M1** | ± 0.5 × perp <u>component</u>. |
| | Impulse = $0.2(3 - -6) = 0.2(3 + 6)$ | **M1** | $0.2 \times (\lvert u_y\rvert + \lvert U_y\rvert)$ |
| | = 1.8 Ns | **A1** | ignore units |
| | perpendicular to and away from the plane | **B1** | |
| 11(ii) | $u_x = 8$, $a_y = (-)8$ | **B1** | both of these soi in **(ii)** |
| | $y = 3t - 4t^2$ (or $0 = 3 - 8 \times 0.5t$ or $-3 = 3 - 8t$) | **M1** | $y$-equation used, their $u_y$, $a_y$ |
| | = 0 at $t = 0.75$ (or $t$ (= 2 × 3/8) = 0.75) | **A1** | obtaining $t = 0.75$ validly |
| | $x = 8t - 3t^2 = 4.3125$ or awrt 4.31 | **A1** | using $x$-equation to obtain $4\frac{5}{16}$ oe |
| 11(iii) | $v_y = -3$, $\therefore u'_y = 1.5$ | **M1** | $\lvert u'_y\rvert = \lvert 0.5v_y\rvert$ |
| | $1.5t - 4t^2 = 0$ or $0 = 1.5 - 8 \times 0.5t$ or<br>$-1.5 = 1.5 - 8t$ | **M1** | $y$-eqn., new $u_y$ (and old $a_y$) |
| | $t = 0.375$ | **A1** | finding new $t$ |
| | Total time is 0.75 + 0.375 = 1.125 cwo | **A1** | awrt 1.13 or $1\frac{1}{8}$ |

| Question | Answer | Marks | Partial Marks |
|---|---|---|---|
| 12(i) | M($B$): $4F = 5g\times 2\sin 60°$<br>M($A$): $2 \times 5g\cos 30° = 4 \times T\cos 30°$<br>M($C$): $2 \times F = 2\times T\cos 30°$<br>NII($\perp AB$): $F + T\cos 30° = 5g\cos 30°$<br>(NII($//AB$): $T\sin 30° + N = 5g\sin 30°$)<br>(NII($\leftrightarrow$): $N\cos 30° = F\sin 30°$)<br>(NII($\updownarrow$): $N\sin 30° + F\cos 30° + T = 5g$) | **M3** | **M1** for attempt to take moments <u>and</u> attempt to derive one other equation by NII or moments about another point<br>**M1** for deriving one useful equation<br>**M1** for deriving a second useful equation (or 2nd and 3rd if $N$ is involved) |
| | $F = 12.5\sqrt{3} = 21.65$ N cwo | **A1** | [21.6, 21.7] or $12.5\sqrt{3}$ oe |
| | $T = 25N$ cwo | **A1** | 25 or awrt 25.0 |

| Question | Answer | Marks | Partial Marks |
|---|---|---|---|
| 12(ii) | | **M1** | **dep** for attempt at moments equation |
| | **One of:**<br>M($C$): $2N_B\cos30°$<br>$= 2N_A\cos60° + 2F_A\cos30° + 2F_B\cos60°$<br>M($A$): $2W + 4F_B \sin30° = 4N_B\sin60°$<br>M($B$): $2W = 4N_A \sin30° + 4F_A\sin60°$ | **A1** | one correct moments equation |
| | **Then completing either:**<br>Both of:<br>M($C$)<br>NII(↔): $N_A\sin60°$<br>$= F_A\sin30° + N_B\sin30° + F_B\sin 60°$<br>**or:**<br>Three of:<br>One of M(C) & NII(↔)<br>M($A$) and/or M($B$)<br>NII($//\Pi_A$): $F_A + N_B = W\cos30°$<br>NII($\perp\Pi_A$): $N_A = F_B + W\sin30°$<br>NII(↕): $W + F_B\cos60°$<br>$= N_A\cos60° + F_A\cos30° + N_B\cos30°$ | **M1** | **Dep** on first M1 for other equation(s) to complete: *either* two equations, neither involving $W$ (=5$g$), *or* three equations, some involving $W$.<br>In each equation condone sign errors and 60/30 or sin/cos errors and calc of $mg$. But correct forces or moments 'resolved' where necessary, required for 2nd **M1**. Mark the equations used or, if both not used, the 2 that give the most marks. |
| | $F_A = \mu_A N_A$ and $F_B = \mu_B N_B$ | **B1** | Can be on diagram or in working |
| | $N_B(\sqrt{3} - \mu_B) = N_A(1 + \sqrt{3}\mu_A)$<br>$N_B(1 + \sqrt{3}\mu_B) = N_A(\sqrt{3} - \mu_A)$ | **M1** | **Dep** on second M1 for simplifying to 2 sim equations in 2 eliminatable 'unknowns' oe elimination |
| | $\dfrac{\sqrt{3}-\mu_B}{1+\sqrt{3}\mu_B} = \dfrac{1+\sqrt{3}\mu_A}{\sqrt{3}-\mu_A}$ | **M1** | **Dep** on second M1 for obtaining single equation in $\mu_A$ and $\mu_B$ only |
| | $\sqrt{3}(\mu_A + \mu_B) + \mu_A\mu_B = 1$ | **M1** | **Dep** on second M1 for making $\mu_B$ subject of formula and using exact values of cos/sin |
| | $\mu_B = \dfrac{1-\sqrt{3}\mu_A}{\sqrt{3}+\mu_A}$ | **A1** | final answer in given form, cwo<br>($\alpha = \sqrt{3}$)<br><br>Note: Form of answer given so must see full working with factorisation leading to single term in $\mu_B$ for final **M1A1**<br>Equation must have $\mu_A$, $\mu_B$ and $\mu_A\mu_B$ terms for **M1**. |