MATHEMATICS

Paper 9794/01
Pure Mathematics 1

Key messages

Candidates should to be aware of the need to afford some justification for statements made in proofs or where formulating their own strategies and to be confident when employing technical vocabulary.

General comments

It is encouraging to note that candidates were able to demonstrate a confident understanding of almost all aspects of the specification examined. A generally excellent standard of presentation was observed and even weaker candidates showed their ability to produce fully detailed, well argued and accurate solutions. The early questions were written to allow all candidates to show positive achievement, and the vast majority achieved full marks on the early questions. There were also many innovative and superbly well handled solutions to the unstructured question at the end of the paper. It was encouraging to find that even the weaker candidates could often make some considerable headway in formulating their own strategies for answering some of the questions.

Comments on specific questions

Question 1

This was a very straightforward question with any lack of success due either to arithmetic slips or answering only one of the two requests. Candidates showed themselves to be familiar with the formulae required but an occasional candidate subtracted the two x and y values to find the midpoint instead of adding them.

Answer: (6, 4) and 10.

Question 2

This again proved to be a question in which candidates demonstrated great confidence. A few, however, lost the final mark by including the x^3 as part of the coefficient. Candidates must ensure that they have a secure grasp of the technical vocabulary.

Answer: −80

Question 3

This question was again very well done by the majority of candidates. Most candidates chose to find *PQ* by the cosine rule but equally successful attempts by the sine rule were also found, and it was pleasing to note that candidates were both clear and accurate in their use of the formulae. Very few indeed resorted to the unsatisfactory habit of first converting radians to degrees. However, a minority interpreted "the line *PQ*" to refer to the arc *PQ* and confusion over formulae led to some finding the area of the shaded region.

Answer: **(i)** 10.5 **(ii)** 22.4.

Question 4

This question proved one of the most accessible on the paper with very few incorrect responses seen.

Answer: 0.781

Question 5

Again, there was a high degree of success with this question with a very small number of candidates omitting the + c or getting confused as to which function the limits should be applied to.

Answer: **(i)** $x^3 - 2x^2 + 8x + c$ **(ii)** 26

Question 6

This question provided unexpected difficulties for many candidates. Many lost marks with inadequate sketches. Thus, most indicated a stationary point at $x = 0$ but did not provide a sufficient indication of one at $x = 2\pi$. Straight line segments could not gain credit. It has been a key theme in previous reports that candidates need to take care when sketching graphs to give a fully accurate representation of the major features of the graph and this certainly includes stationary points and avoiding distortions such as treating curves as straight lines.

Another key message is the need to master technical vocabulary and to use it confidently and accurately. Descriptions in terms of wavelengths and frequencies have their place in models of physical phenomena but are not mathematical definitions of the transformation. Words like “squash” or “squeeze” are not acceptable and neither does “enlargement” convey the idea of a uni-dimensional change. Candidates are expected to have mastered the appropriate definitions. Examiners would also like to stress the need for accuracy in statements. It is correct to say that a stretch occurs parallel to an axis but references to the stretch occurring “along”, or “in” the axis gave too inaccurate an idea to receive credit.

Answer: **(ii)** stretch with scale factor 0.5 parallel to the x-axis

Question 7

The use of Pythagoras to find $|z|$ was usually correct but the process of finding arg z was mostly not. Candidates are always advised to draw a diagram to guide their response. Very few did so and, as a result, answers such as –0.810 or 46.4 were only too common and received no marks. Those who were more successful used a variety of approaches such as $\pi + \tan^{-1}\left(\frac{-21}{20}\right)$, $\pi - \tan^{-1}\left(\frac{21}{20}\right)$, or $\frac{\pi}{2} + \tan^{-1}\left(\frac{20}{21}\right)$. Part **(iii)** was slightly better answered although some unexpected approaches were seen. Multiplication by the conjugate was the anticipated approach, but multiplication by 20 + 21i was allowed, as was an attempt to solve the equation $(-20 + 21\text{i})(x + \text{i}y) = 1$, although one may wonder why this was considered a desirable method to use.

Answer: **(i)** 29 **(ii)** 2.33 rad or 134° **(iii)** $-\frac{20 + 21i}{841}$

Question 8

On the whole, this question was successfully answered by many candidates. Very few candidates attempted to use some other method like decimal search to achieve the result. These few naturally received no credit even if they reached the correct answer. What should be emphasized is the importance of demonstrating a correct argument and conclusion when asked to “show” a result. In part **(i)** Examiners expected not only to see that f(1) and f(2) resulted in –1 and 5 but, at the very least, a reference to the fact that this implied that the curve crossed the x-axis and hopefully also that it indicated the existence of a root between $x = 1$ and $x = 2$. Those who obtained –1 and 5 without making any conclusion from these calculations were penalized.

Answer: **(ii)** 1.325

Question 9

Many candidates lost marks in part **(i)**. They need to appreciate that when asked to "show" a result, it is completely insufficient merely to write down $R\cos\alpha = 1$ and $R\sin\alpha = \sqrt{3}$ without demonstrating where these equations came from. Similarly $R = 2$ needs justification. Candidates must understand that mathematics involves logical argument and is not simply the performance of routines. In part **(ii)** an angle was frequently obtained but knowledge of how to obtain the angle in the required range did not seem so readily available. Some also believed that the angle θ should be given in degrees, which did not receive credit.

Answer: **(i)** $R = 2$, $\alpha = \frac{\pi}{3}$ **(ii)** 1.68

Question 10

This question proved very challenging to a large number of candidates. It was expected that candidates would make use of two items in the specification, first to change the cartesian form of the line equations into vector form and then to find the point of intersection of the two lines in vector form. Some candidates did adopt this approach and found the point of intersection successfully and with minimum effort. However, some then lost a mark by leaving their answer as a vector and not in the co-ordinate form requested.

Unfortunately, most candidates decided to approach this task algebraically. A minority of these easily found two correct simultaneous equations but a large majority either tried to form equations in three variables with an inevitable descent into a welter of algebra which got nowhere, or merely equated the x, y and z components directly.

Answer: (1, 8, 12)

Question 11

Most candidates could handle the first part of the question very well but, in the remaining parts, candidates struggled to eliminate the trigonometric expressions. This was particularly so in part **(iii)** but there were quite a few in part **(ii)** who did not achieve numerical values for the gradient and co-ordinates. At the same time, it was pleasing to see alternative approaches to the normally expected method being adopted successfully. For example, in part **(iii)** integrating $\frac{dy}{dx} = -x$ gave the equation almost directly.

Answer: **(ii)** $y = 3 - 2x$ **(ii)** $y = 1 - \frac{x^2}{2}$

Question 12

Although candidates were able to tackle part **(i)** successfully, most did not know how to start part **(ii)** and those who did make some effort at this proof had very tentative appreciation of what a limit was. This should have been a standard piece of bookwork since it is clearly included in the specification but it was not to be. It was also clear that understanding was not helped by confusing and muddled notation. In general, candidates should make a clear distinction between when functional notation and that due to Leibniz is easier and not to mix the two. They should also be aware that in proofs of this sort, the use of functional notation is much to be preferred.

Question 13

In unstructured questions of this sort, the steps in the strategy adopted should be justified at least to some extent. For example, candidates who found f(1) = 0 and stated nothing further about $x = 1$ were penalized. A conclusion which stated that either $x = 1$ was a root or $(x - 1)$ was a factor would have been an adequate justification. Allusion to mathematics being a process of argument which requires some rigour has already been made. Even more unsatisfactory therefore, is it to perform a long division and leave it to the Examiner to deduce that a factor may be involved. That having been said, Examiners were extremely impressed by the elegance and originality of the solutions offered to this problem. More than one valid form of partial fraction was seen, the more unusual being $\frac{A}{x+3}+\frac{Bx+C}{(x-1)^2}$ as well as highly sophisticated algebraic solutions which did not use partial fractions at all. Almost all of these methods, some including the need for integration by reverse substitution, were carried through with confidence and success.

Answer: $\ln(x + 3) + \ln(x - 1) - \frac{1}{x-1} + c$

MATHEMATICS

Paper 9794/02
Pure Mathematics 2

Key Messages

In questions with given answers, it is important that candidates make sure to give sufficient steps at the end of the solution to demonstrate how the given answer is achieved. In order to do well on this paper, candidates should be careful to present complete arguments when answering questions that include the words "show", "prove" and "verify", and should make sure to have a good grasp of appropriate technical terminology.

General Comments

The demand of the paper was much the same as last year. It was very pleasing to see many good responses to the early parts of the paper, with some excellent presentation of algebraic manipulation shown by the better candidates towards the end. The level of communication was generally very good. All parts of the paper were accessible to some candidates, though only the strongest produced complete solutions to **Questions 9** and **10**.

Many candidates used graph paper unnecessarily for the sketches in **Questions 3** and **4**. This was not penalised, but it will have cost those candidates time in the exam. All that is required for **Question 3** is a hand drawn graph in the standard answer booklet with correct graph shape and approximately consistent scales, showing all the necessary features. **Question 4** simply required two axes and three labelled points in approximately the right relative positions.

Comments on Specific Questions

Question 1

These two requests were answered correctly by the vast majority of candidates.

(i) Occasional sign errors were all that led to a loss of marks here.

(ii) The most common error was to find $|\mathbf{u}+\mathbf{v}|^2$ rather than $|\mathbf{u}+\mathbf{v}|$. This was condoned where no incorrect statements such as $|\mathbf{u}+\mathbf{v}| = 65$ were seen.

Answers: **(i) u + v =** $\begin{pmatrix}1\\8\end{pmatrix}$ **u – v =** $\begin{pmatrix}7\\4\end{pmatrix}$

Question 2

These three requests about sequences were correctly done by most candidates, showing a good knowledge of the basic skills required by this part of the syllabus. In part **(i)** a few candidates found the sum for the first twenty-one terms rather than the twenty-first term itself. With regard to part **(iii)** candidates need to make sure that they have learnt the appropriate terminology; many gave long descriptive accounts of the behaviour where a single technical term would have been sufficient.

Answers: **(i)** 43 **(ii)** 243 **(iii)** –1, 3, –1, 3 The sequence is periodic (or equivalent)

Question 3

The process of completing the square, and using that form for graph sketching, was well-understood by most candidates. In part **(ii)** some candidates did not seem to understand the term "vertex"; many of them then showed the graph as asymmetric with a vertex at (0, –3). Many candidates chose to produce a *plot* on graph paper rather than a *sketch*. Since this involved more work than a sketch and produced a graph with all the necessary features, it was possible to gain full marks from this approach. A simple hand drawn parabola with the necessary features marked is all that is necessary; many of the strongest candidates produced very nicely presented sketches of this nature.

Answers: **(i)** $x^2 + 2x - 3 = (x + 1)^2 - 4$

Question 4

Almost all candidates showed a good grasp of the essentials of complex numbers in this question.

(i) To "verify" requires a clear demonstration that substituting $z = -1$ in the expression gives zero. This necessarily involves some evaluation beyond the substitution step.

Some candidates divided $z^3 + 5z^2 + 9z + 5$ by $z + 1$ and then applied the factor theorem. This is valid, but a lot of work for one mark; candidates should be careful to make sure that the method they use is commensurate with the reward offered.

(ii) The most common method seen was long division followed by use of the quadratic formula. Many candidates did not manage to simplify correctly leading to incorrect answers.

Some of the stronger candidates simply factorised out a factor of $z + 1$ by eye, then solved the resulting quadratic equation by completing the square. This proved to be very efficient and less prone to small errors and omissions.

(iii) The Argand diagram was completed correctly by nearly all candidates. There are many ways of presenting such a diagram, all of which were allowed, but candidates should be careful not to draw a line from the origin *through* the given point rather than *to* that point.

Answers: **(ii)** $-2 + \mathrm{i}$ and $-2 - \mathrm{i}$

Question 5

This example of implicit differentiation and its application was very well done by most candidates in both parts. Candidates need to take care to leave the final answer of part **(ii)** in the given form. Some leniency was given as to the order of terms, but all coefficients should be integers.

Answers: **(ii)** $8x - 7y + 5 = 0$

Question 6

This question was tackled well only by the best candidates. Most produced the required graph in part **(ii)** but the details of the method did not seem familiar to all.

(i) The logarithmic transformation was often seen correct, but the full argument for a linear relationship was only rarely seen in full. To achieve full marks requires candidates to identity which are the constant *and* variable parts of the linear equation.

(ii) This graph was well produced by most candidates with an appropriate line of best fit.

Candidates were asked to use their line of best fit to estimate the values of *a* and *b*. Candidates who had found the correct linear form in part **(i)** were able to do this very easily, others found the values by substituting two points on their line into the exponential form which was also accepted. Using values from the table was not condoned except in cases where the line of best fit happened to pass through those points, or no line of best fit had been drawn.

(iii) and (iv) Candidates with values of *a* and *b* were almost universally able to continue through these parts. Those with reasonably accurate values of *a* and *b* were more likely to gain the B1 and A1 marks, but all had access to the M1 marks in part **(iv)**. In both parts candidates were awarded the accuracy marks where their answers fell in the given ranges *and* followed correctly from their a and b. In part **(iv)** candidates were asked to predict the year in which the population first exceeds 500 breeding pairs. The value of t found had to be interpreted in the context, i.e. as a calendar year, for more than 1 mark to be awarded. In this particular context the value of t needed to be rounded up to the nearest whole number since the variable t is discrete. Most candidates who found a value for t did this accurately.

(v) This part was only answered well by a minority of candidates. The most common misconceptions were to make comment on the *accuracy* of the line of best fit or on whether extrapolation was valid in general. Observations leading to the award of this mark include that exponential models are not suitable for population models because they exhibit unrestricted growth, that the population would tend to level out due to competition.

Answers: **(i)** $\log N = \log a + t\log b$ where $\log N$ and t are variables and $\log a$ and $\log b$ are constants
(ii) a between 5.5 and 6.5, b between 1.32 and 1.42
(iii) 2008 value between 50 and 95, 2020 value between 1400 and 5500
(iv) between 2013 and 2017

Question 7

The processes involved here were well understood by most candidates. While most managed to correctly find the exact coordinates in part **(ii)**, full marks were not often awarded in part **(i)**. The issue here is in how much detail to give; candidates must be careful to show enough steps to justify reaching the given answer. In this case, clear evidence of use of the product rule (or quotient rule) was needed since the result of using that rule can easily be obtained by multiplying out the brackets in the given answer.

Answers: **(ii)** (0, 0) and $(4, e^{-2})$

Question 8

The summation expressions here were not well understood by many candidates and only the best obtained full marks. Some of the weaker candidates tried to apply the formulae for sums of arithmetic and geometric sequences directly. Candidates who wrote out a few terms and looked for which ones cancelled were usually successful in obtaining the right answers.

Answers: **(i)** $\frac{31}{30}$ **(ii)** 50

Question 9

(i) This proof was attempted by many candidates and some very well presented and efficient solutions were seen. Candidates who converted $\operatorname{cosec}x$ and $\cot x$ into $\sin x$ and $\tan x$ (or $\sin x/\cos x$), combined the two fractions into a single term and converted functions of $2x$ into functions of x with double angle formulae as three distinct steps were usually successful. All three steps can be seen from the statement of the identity and candidates should be encouraged to plan their pathway through such a proof before starting.

Most candidates saw the relevance of the identity for evaluating $\tan(\frac{3}{8}\pi)$, but not all could give enough evidence that they had not simply found the exact value on a calculator.

(ii) The connection with part **(i)** was spotted by almost all candidates.

It was pleasing to see how many of the strongest candidates appeared to know the integral of $\tan^2 x$ as standard bookwork and could produce a very concise and neat solution using the Pythagorean identity. Those that did not had many potential pathways including integration by parts, by substitution of $u = \tan x$ or even by multiplying out the given form and using a different Pythagorean identity. All could lead to the correct solution, though the integrations by parts involved some difficult manipulation that not many managed to navigate fully correctly.

Answers: **(i)** $\tan(\frac{3}{8}\pi) = 1 + \sqrt{2}$ **(ii)** $\sqrt{2} - \frac{1}{8}\pi$

Question 10

(i) Very few candidates were awarded full marks in this part. The best solutions wrote down that $\frac{dV}{dt} \propto \sqrt{h}$ where h is the depth of water, then showed that h is proportional to V or substituted $h = V/A$ where A is the cross-sectional area and argued from there. Many candidates appeared to have misread the stem as "..at a rate which is proportional to the square root of the volume of the water." This greatly simplified the request and no marks were awarded.

(ii) The method of separation of variables was well understood by many candidates. Most of the distinction candidates were able to separate and integrate successfully. The following manipulation, using two conditions to find two unknown constants in terms of a third, was found difficult by most and very few fully correct solutions were seen. Successful candidates set out their work very clearly and carefully, simplifying expressions as they went along to mitigate some of the difficulty.

(iii) Most candidates made some attempt at this as it stands independent of any previous part and most managed a reasonable argument. The best approach was to set $V = 0$ in the equation in part **(ii)** and solve for t then convert clearly into hours and minutes.

Verification was condoned on this occasion, although it does not show the given result, only that V is very small after 3 hours and 25 minutes and it certainly does not imply that V could not have been zero at any other preceding time.

MATHEMATICS

Paper 9794/03
Applications of Mathematics

Key Message

Candidates need to understand what is meant by a question that asks them to "Show that …" (as in **Questions 3**, **7** and **9**). Because they are given the answer, the explanation has to be detailed and cover every step. The onus is on the candidate to ensure that their answer is complete and convincingly obtained, and that the reader is not expected to fill in any gaps. When asked to "Derive …" (as in **Question 8**), a rigorous argument is needed, not merely the quotation of the result.

General comments

This paper appeared to have been well received by candidates and there were many high scoring scripts. There was no evidence to suggest that candidates were short of time. The Probability Section proved to be largely accessible to most candidates while parts of the Mechanics Section were found to be more demanding. Candidates are reminded that they should take heed of the instruction about the accuracy of final answers, while at the same time bearing in mind the consequences of premature approximation.

Comments on specific questions

Section A: Probability

Question 1

This question was answered correctly by most candidates. Some candidates' attempts to find the standard deviation revealed the importance of learning the correct formula and how to use it. Candidates should be encouraged to make sensible and efficient use of the built-in statistics functions of their calculator. It is appropriate to show a certain amount of working in order to demonstrate an understanding of the method but there should be no need for candidates to do the calculations manually.

Answers: Mean 1.92 Standard deviation 1.09

Question 2

Most candidates were able to quote and use correctly the probability rules needed to answer this question. Some used Venn diagrams or two-way tables to help them, and were usually successful.

Answers: **(i)** $\frac{1}{8}$ **(ii)** $\frac{11}{24}$

Question 3

(i) There were very many good answers to this part: generally the working was set out clearly and the formula for the correlation coefficient was quoted and used correctly.

(ii) In this part candidates really needed to think about what it means to *show* that the gradient and intercept of the regression line are as given in the question, correct to 2 decimal places. In order to achieve this it is necessary for them to quote their answers to the calculations to at least 3 decimal places and so to be able to demonstrate that, when rounded, they agree with the given values. Furthermore, this ensures that they show that they have carried out the calculation and have not relied on the assumption that their calculation will give the right answer. Then, having found the gradient, say, it was necessary to use the unrounded value in the next calculation to find the intercept. On this occasion, premature rounding was likely to result in an answer that did not agree to 2 decimal places.

(iii) Many candidates seemed to confuse "validity" with "accuracy". Comments were expected to consider whether or not it was valid to use the regression equation at all, based on the age of the person in relation to the data used to derive the equation, and not on the perceived accuracy of the answers.

Answers: **(i)** 0.799 **(ii)** Before rounding: grad 0.831(22…), intercept 41.284(60…) **(iii)** 82.8 valid, 50 is within data range; 95.2 not valid, 65 is beyond data range.

Question 4

(i) This was well answered by almost all candidates.

(ii) There were many correct answers to this part of the question. It seemed to be well recognised that the distribution needed was B(6, p), where p is the answer to part **(i)**. For some, more care was needed in setting up the two terms and/or remembering to include the binomial coefficient '6' in the probability of 1 underweight tomato.

(iii) In this part the correct strategy was usually employed, but as in **Question 3(iii)**, premature approximation was an issue for some. A further matter that candidates should bear in mind is that expectation or expected value is the same as the mean. Consequently, it is not appropriate to round to the nearest whole number simply because the variable in question happens to be discrete.

Answers: **(i)** 0.0668 **(ii)** 0.944 **(iii)** 14.0

Question 5

(i) This was well answered by almost all candidates.

(ii) The best answers to this question came from candidates who thought to break the problem down to 3 cases: no 1s, one 1 and two 1s. For each case, it was then a matter of selecting the appropriate number of cards from the remaining five, working out the corresponding number of arrangements and finally adding the results. A common mistake involved thinking that there are 21 ways to select 5 cards from the 7, and hence that there are 10, instead of 5, ways that use only one 1. Overall, while there were quite a few well explained solutions, many candidates would have benefitted from taking the trouble to explain their working more carefully.

Answers: **(i)** 2520 **(ii)** 1320

Section B: Mechanics

Question 6

(i) Most candidates had little difficulty in finding the times when the particle is at rest. In contrast, there was considerable room for improvement in the quality of sketching the velocity-time graph. In a significant minority of cases the cubic graph consisted of straight lines and it seemed to be assumed that the maximum and minimum points were at t = 1 and 3, respectively. Furthermore, candidates at this level ought to be able to *sketch* a cubic graph without the need for graph paper.

(ii) On the whole this part was answered correctly: only a few appeared not to realise that they should differentiate and then substitute.

(iii) Most candidates realised that they should integrate the expression for velocity, and did so successfully. There was some carelessness concerning the arbitrary constant "c", which, more often than not, was either ignored completely or assumed, without justification, to be zero. For the most part, the distance travelled and the average speed in the first 2 seconds were both obtained easily.

Answers: **(i)** 2, 4 (sec) **(ii)** –4 (ms^{-2}) **(iii)** 4 (m), 2 (ms^{-1})

Question 7

(i) On the whole this question was answered better than the equivalent question on this topic in 2012. However, it remains the case that success depends on the careful application of the two main principles: the Conservation of Linear Momentum and Newton's Experimental Law. Thereafter, with care, it should be a straightforward matter to solve the resulting pair of linear equations.

(ii) Following correct solutions in part **(i)**, most answers to this part were correct. In cases where part **(i)** was not correct, candidates might have been able to rescue the situation by looking at their answers here more critically: for instance, it was not unusual for the speed of *A* to be greater than the speed of *B*.

(iii) While many obtained the correct answers to this part, surprisingly few noticed the connection with part **(i)** and the chance to save themselves a lot of work.

(iv) As in part **(ii)**, this part gave candidates an opportunity to review their answers thus far and to check that they were at least plausible. In answering the question it was necessary to consider the possibilities of a further collision between *A* and *B*, and between *B* and *C*. If everything else was correct then there would be no further collisions.

Answers: **(i)** $v_B = \frac{2}{3}(1+e)u$ **(ii)** $\frac{1}{2}u$, u **(iii)** $\frac{1}{2}u$, $\frac{1}{2}u$, u

Question 8

(i) The danger in this question is that, having written down the expressions for x and y, candidates can be too economical with the subsequent derivation of the (presumably well known) Cartesian equation of a projectile.

(ii) Candidates used either the Cartesian equation to obtain a quadratic equation in $\tan(\theta)$ or, less frequently, the formula for the range of a projectile to obtain an equation in $\sin(2\theta)$. Only a few candidates responded to the wording of the question by pointing out that a quadratic equation would have two solutions and hence there would be two trajectories. Either way the angles of projection were usually found easily in the end.

(iii) Many possible advantages of each trajectory were offered. The most popular of those deemed acceptable were along the lines that the lower trajectory would be faster while the higher would be easier to track.

Answers: **(ii)** 15° or 75°

Question 9

(i) A clear, carefully drawn and labelled force diagram was presented by most candidates. This made a substantial difference to the quality of response in the subsequent parts of the question. Candidates should be aware that, however helpful they think it might be to them, to have both the tension and its components shown on the same diagram risks the diagram being seen as incorrect.

(ii) There were many correct solutions to this question. As with so many topics in Mechanics, carefully justified statements at the outset, based on resolving forces and on the friction law, provide candidates with the best opportunity to succeed.

(iii) Many good candidates rose to the challenge of this part. Some began by differentiating the whole expression for T while the more astute realised that it was sufficient to maximise the denominator in order to minimise T. Of these, the majority favoured rewriting the denominator using the form $R\sin(\theta + \alpha)$ as opposed to differentiation.

Answers: **(iii)** 36.9°