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Cambridge Pre-U Certificate

MATHEMATICS **9794/03**

Paper 3 Applications of Mathematics **May/June 2017**

MARK SCHEME

Maximum Mark: 80

Published

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Question	Answer	Marks	Guidance
1(i)	Mean = 1365/18 = 75.8(33…)	**B1**	
	$\text{sd} = \sqrt{\frac{111381}{18} - 75.83...^2} = 20.9(07...)$	**B1**	Accept unbiased estimate = 21.5(14…)
1(ii)		**M1**	Either threshold found correctly.
	Lower limit = 75.833 – 2 × 20.907 = 34.0(17…) Upper limit = 75.833 + 2 × 20.907 = 117(.64…)	**A1**	Both thresholds found correctly. **FT** *their* mean and sd.
	∴ 19 is the only outlier.	**A1**	C.a.o., but **FT** *their* mean and sd provided 19 only is identified as an outlier.
2(i)	From the data: $n = 6$ $\Sigma x = 45$ $\Sigma x^2 = 495$ $\Sigma y = 23.64$ $\Sigma xy = 215.88$ $S_{xy} = 215.88 - \frac{45 \times 23.64}{6} = 38.58$	**M1**	
	$S_{xx} = 495 - \frac{45^2}{6} = 157.5$	**M1**	
	$b = \frac{S_{xy}}{S_{xx}} = \frac{38.58}{157.5} = 0.244(95\ldots) \approx 0.245$	**A1**	cao
	$\therefore a = \frac{23.64}{6} - 0.24495\ldots \times \frac{45}{6}$	**M1**	
	$= 3.94 - 0.24495\ldots \times 7.5 = 2.10(28\ldots)$ ≈ 2.10	**A1**	b = 0.245 gives a = 2.1025. **FT** *their* b.
2(ii)	For 2009 $r = 3.97 - (2.10 + 0.245 \times 9)$ $= 3.97 - 4.307(42\ldots)$	**M1**	"obs-calc". Allow SC B1 for "calc-obs".
	= –0.337(42…)	**A1**	a and b to 3sf give y = –0.335. **FT** *their* a and b.
2(iii)	In 2024, x = 24, y = 7.98(17…) ≈ 7.98 (millions)	**B1**	a and b to 3sf give y = 7.98. **FT** *their* a and b.
	2024 estimate is unreliable since it involves extrapolation.	**B1**	o.e.

Question	Answer	Marks	Guidance
3(i)	$k \times (10 + 12 + 12 + 10 + 6) = 1$	**M1**	Sum of 5 non-zero probabilities in terms of k equated to 1.
	$\therefore\ 50k = 1 \quad \therefore\ k = 1/50$	**M1**	Shown convincingly. Depends on previous mark.
	Alternative by verification: Sub k and all probs correct.	**M1**	
	Show $\Sigma p = 1$.	**M1**	Shown convincingly. Depends on previous mark.
3(ii)	$E(X) = \dfrac{0+12+24+30+24}{50} = \dfrac{9}{5}$	**B1**	
	$E(X^2) = \dfrac{0+12+48+90+96}{50}$ $= \dfrac{246}{50}$ or 4.92	**B1**	
		M1	Use formula for Var(X).
	$\text{Var}(X) = \dfrac{246}{50} - \left(\dfrac{9}{5}\right)^2 = \dfrac{84}{50}$ or 1.68	**A1**	**FT** *their* E(X) and/or E(X^2) provided variance is positive. Accept any equivalent form.
3(iii)	$P(X = 4 \setminus X > 0) = \dfrac{6/50}{40/50}$	**M1**	Conditional probability as a ratio with either numerator or denominator correct.
	$= \dfrac{4}{40}$	**A1**	Accept any equivalent form.
4(i)		**B1**	$\dfrac{10!}{\ldots}$
		M1	Reasonable attempt at denominator
	$\dfrac{10!}{3!3!2!} = 50\,400$	**A1**	cao
4(ii)	$\dfrac{8!}{3!2!}$	**M1**	Arrangements of 'TATISTIC': $\dfrac{8!}{\ldots}$
		M1	Denominator correct for repeated letters.
	$= 3360$	**A1**	cao

Question	Answer	Marks	Guidance
4(iii)	$\dfrac{\frac{8!}{3!2!}}{\frac{10!}{3!2!2!}} = \dfrac{3360}{50400}$	**M2**	**M1** Numerator; allow *their* (ii). **M1** Denominator; allow *their* (i).
	$= \dfrac{1}{15}$	**A1**	**FT** *their* (ii) and/or *their* (i).
5(i)	**Alternative version 1** $P(X > n) = 1 - P(X \leqslant n)$		
	$= 1 - \{p + pq + \ldots + pq^{n-1}\}$	**M1**	1 – list of first n probabilities
	$= 1 - \dfrac{p(1-q^n)}{1-q}$	**M1**	Sum of GP used correctly.
	$= 1 - (1-q^n) = q^n$	**A1**	Simplified convincingly.
	Alternative version 2 $P(X > n) = pq^n + pq^{n+1} + pq^{n+2} + \ldots$	**M1**	List of subsequent probabilities.
	$= q^n\{p + pq + pq^2 + \ldots\}$	**M1**	Sum of infinite GP used.
	$= q^n \times 1 = q^n$.	**A1**	Sum in {} = 1 (property of Geo(p)).
	Alternative version 2 If $X > n$ then …	**M1**	
	… must “fail” on first n attempts.	**M1**	
	$\therefore P(X > n) = P(\text{“Fail” } n \text{ times}) = q^n$.	**A1**	
5(ii)	$P(X \geqslant 4) = P(X > 3) = q^3 = 0.216$ $\therefore q = 0.6$	**M1**	Use q^3 and find q.
	$\therefore p = 0.4$	**A1**	cao
	$P(X \leqslant 8) = 1 - P(X > 8) = 1 - 0.6^8$	**M1**	$1 - q^8$.
	$= 1 - 0.01679616 = 0.98320384 \approx 0.983$	**A1**	**FT** 1 – *their* q^8.
5(iii)	$E(X) = \dfrac{1}{0.4} = 2.5$	**B1**	**FT** *their* p.
	$\text{Var}(X) = \dfrac{0.6}{0.4^2} = 3.75$	**B1**	**FT** *their* p.

Question	Answer	Marks	Guidance
6(i)	Diagram of crate with weight and tension in the cable shown.	**B1**	
6(ii)	$220a = 220g - T$ $\therefore T = 220(10 - a)$	**M1**	Correct application of N2 used at least once.
	$a = 1.5$ $\therefore T = 1870$ (N)	**A1**	cao
	$a = 0$ $\therefore T = 2200$ (N)	**B1**	cao
	$a = -0.75$ $\therefore T = 2365$ (N)	**A1**	cao
6(iii)	Trapezium (middle portion horizontal), one vertex at the origin, fourth vertex on the t axis.	**B1**	
	Third part less steep than first. Axes labelled t and v; horizontal section at v = 3.	**B1**	
6(iv)	Acceleration and deceleration stages take 2 + 4 = 6 sec.	**B1**	For acceleration time.
		B1	For deceleration time.
	If t = total time of descent then $s = \frac{1}{2} \times 3\left(t + (t-6)\right) = 15$	**B1**	Area of trapezium …
		M1	… equated to 15.
	$\therefore t = 8$ (sec)	**A1**	cao
7(i)	Vertical: $-5t^2 = -33.8$	**B1**	Allow absence of both minus signs.
	Alternative version 1 $\therefore t = \sqrt{\frac{33.8}{5}} = 2.6$ (sec)	**B1**	cao
	Horizontal: $2.6u = 31.2$	**M1**	
	$\therefore u = 12$ (ms^{-1})	**A1**	**FT** *their t*.

Question	Answer	Marks	Guidance
7(i)	**Alternative version 2** Horizontal: $ut = 31.2$	**B1**	
	$33.8 = 5 \times \dfrac{31.2^2}{u^2}$	**M1**	Eliminate t.
	$\therefore u = \sqrt{\dfrac{5 \times 31.2^2}{33.8}} = 12 \text{ (ms}^{-1}\text{)}$	**A1**	
7(ii)	$v_x = 12$		
	Either $v_y = (0) - 10 \times 2.6 = -26$ Or $v_y = \sqrt{(0^2) + 2 \times 10 \times 33.8} = (-)26$	**B1**	**FT** *their* t. Allow absence of minus sign.
	$\therefore \lvert v \rvert = \sqrt{12^2 + (-26)^2} = \sqrt{820} = 28.635\ldots$	**M1**	
	$\approx 28.6\left(\text{ms}^{-1}\right)$	**A1**	**FT** *their* v_y and/or u.
	$\theta = \tan^{-1}\left(\dfrac{-26}{12}\right)$	**M1**	
	$= -65.2°$	**A1**	Must be negative or have reference to the horizontal, e.g. "below …". **FT** *their* v_y and/or u.
8(i)	A fully labelled triangle of forces, including angles (θ and 30°) and arrows.	**B1**	Triangle is ambiguous. Candidates not expected to consider/show this here.
8(ii)	$\dfrac{10}{\sin 30} = \dfrac{16}{\sin\phi}\left(= \dfrac{P}{\sin\theta}\right)$ $\therefore \sin\phi = \dfrac{16\sin 30}{10} = \dfrac{4}{5}$	**M1**	Sine rule or Lami's Theorem used. ϕ is the third angle (= 180 – 30 – θ).
	$\therefore\ \phi = 53.1°$ or $126.9°$	**A1**	Either value correct.
	$\therefore\ \theta = 150 - \phi = 96.9$ or 23.1	**A1**	Both correct values required.
	$\therefore P = \dfrac{10\sin\theta}{\sin 30}\left(= 20\sin\theta\right)$	**M1**	Sine rule or Lami's Theorem used or resolve horizontally.
	$\therefore\ P = 19.856\ldots$ or $7.856\ldots$	**A1**	cao Both values required.

Question	Answer	Marks	Guidance
8(ii)	**Alternative methods**, involving resolving and/or the cosine rule: Correct elimination of either P (or θ).	M1	
	Either value of (e.g. $\theta + 30$ or $\theta - 60$ or $\cos\theta$) (or P) correct.	A1	
	Both correct values of θ (or P).	A1	
	Use of $P = 20\sin\theta$ as above.	M1	
	Both values of P (or θ).	A1	cao. NB Beware of spurious values of θ.
9(i)	$3.6 = \frac{1}{2}(0+v)\times 18$	M1	Use of appropriate '*suvat*' formula.
	$\therefore\ v = 0.4$ (ms^{-1})	A1	cao
9(ii)(a)	$t = 0$ or 18.	B1	Both values required.
9(ii)(b)		M1	Integrate v.
	$x = \int \frac{1}{270}\left(18t - t^2\right)\mathrm{d}t = \frac{1}{270}\left(9t^2 - \frac{t^3}{3}\right) + c$	A1	All terms correct; condone omission of "+ c". Allow definite integral as alternative.
	When $t = 0$, $x = 2$, $\therefore\ c = 2$	M1	Deal with c correctly or apply limits of definite integral.
	When $t = 18$ $I = \frac{1}{270}\left(9\times 18^2 - \frac{18^3}{3}\right) = \frac{2916 - 1944}{270}$ $= \frac{972}{270} = 3.6$ $\therefore\ x = 3.6 + 2 = 5.6$	A1	Evaluate for $t = 18$ or add 2 if definite integral used. Convincingly shown.
9(iii)	In Model 2, $v = 0$ when the particle reaches Q.	B1	

Question	Answer	Marks	Guidance
10	$4(t+5)=(0)+\frac{1}{2}\times\frac{1}{2}\times t^2$ Where t = time from bus setting off.	**M1**	s at constant v for cyclist equated to ...
		A1	… s at constant a for bus. Allow $t = 0$ as cyclist passes bus.
	$\therefore t^2-16t-80=0$ $\therefore (t-20)(t+4)=0$	**M1**	Solve quadratic equation which must involve 3 non-zero terms.
	$\therefore\ t = 20$ s (not –4)	**A1**	cao. A0 if final answer contains both values of t. $t = 0$ as cyclist passes bus gives $t = 25$ (and 1); must now subtract 5. SR If M0M0, allow B1 for $t = 20$ obtained without any wrong working, e.g. by trial and error.
	$v=(0)+\frac{1}{2}\times 20=10\,\text{ms}^{-1}$	**B1**	**FT** *their t*.